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Bond stress development at a surface coating-substrate interface due to the action of a nucleus of thermo-elastic strain

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Abstract

The class of problems dealing with surface reinforced elastic media is encountered in many areas of materials engineering, notably in connection with surface layers that are used to provide protection to an otherwise softer substrate. These problems are of particular importance to the assessment of the mechanical behaviour of thin films and other forms of industrial coatings. This paper examines the problem related to the flexure of a plate-like surface layer that is bonded to an elastic halfspace region, and where the flexure of the coating is induced by a nucleus of thermo-elastic strain acting within the halfspace region.

Keywords: Interface stresses in a coating; Thermo-elastic delamination stresses; Plate flexure model of coating; Bond stresses; Analytical results

1. Introduction

Bonded surface coatings are often used to protect either a weaker or a softer substrate and to enhance the load carrying capacity of the combined system. Applications of coating techniques have been extensively discussed in literature dealing with nano-technology and functionally graded materials used in materials engineering and biomechanics [1–7]. One of the objectives of the reinforcing surface layer is to minimize the detrimental effects of contact-induced stresses that can be generated at the surface of the substrate. For example, when the surface coating has the ability to diffuse applied loads through its *flexural action*, the detrimental effects of even highly localized loads can be minimized. Consider the problem of an elastic halfspace region where the surface is subjected to a Boussinesq-type concentrated force. In this case, the displacements and stresses in the halfspace region at the point of application of the load will be *singular*. If the same concentrated loading is applied to the surface of a halfspace region that is reinforced with a coating with flexural properties and bonded to its surface, both the displacement field and the stresses in the substrate at the point of application of the concentrated force will be *finite*. This ability to diffuse the applied localized loads is a characteristic feature of the surface reinforcement effect. Examples of this diffusion action have been discussed by Selvadurai [8] in connection with the classical problem dealing with plates on elastic foundations and has been extended to include effects of bonding at the interface [9], and interaction with internally applied loads [10]. In this paper we consider a further example dealing with the problem of a halfspace that is reinforced with a flexible coating and which involves the loading action due to a nucleus of thermo-elastic strain applied at the interior of the halfspace region. The problem can be identified as an idealization of the action of, say, microwave heating of the interior of an elastically deformable halfspace region with heat conduction characteristics sufficiently low to justify the modelling of the problem by appeal to the steady state thermo-elastic model. The transient effects of bond stress development during heat transfer are an important issue but attention here is restricted to the thermal shock-type situation where no heat conduction takes place. This type of modelling represents the worst case scenario and any provision for heat transfer will moderate the thermo-elastic effects due to the re-

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distribution of the temperature field consistent with the time history of the heat generation and the thermal conductivity parameters of both the surface coating and the substrate. Other applications of the influences of thermal loadings on thin films and coatings are given in [11–15]. The modelling of nuclei of strain, thermo-elastic or otherwise, in elastic materials has a long and rich history dating back to the works of Dougall [16] and Love [17] (see also Timoshenko and Goodier [18] and Lur'e [19]) who examined the nuclei of strain associated with elastic domains of infinite extent. These applications were extended by Mindlin and Cheng [20] to examine the nuclei of strain problem related to the halfspace region and by Dundurs and Guell [21] to include bimaterial elastic regions. Extension of these results to include thermo-elastic effects commences with the classic paper by Goodier [22], related to the integration of the thermo-elastic equations (see also Boley and Wiener [23]). These studies were extended by Sen [24] to include nuclei of thermo-elastic strain in halfspace regions. In this paper we adopt the solution for the nuclei of thermo-elastic strain in an *infinite space region* to develop an *analytical solution* to the problem of the internal heating of a halfspace region, where the surface is reinforced by the bonding action of a flexible coating (Fig. 1) and the flexural behaviour of the coating is modelled by the Germain–Poisson–Kirchhoff thin plate theory (Timoshenko and Woinowsky–Krieger [25], Selvadurai [26]). The applicability of the thin plate model for the study of thin films and coatings is discussed by a number of investigators and references to these can be found in the articles [1–7] and [11–15] cited previously and in the studies by Ramsay et al. [27], McGurk and Page [28], Huang and Suo [29], Carvalho et al. [30], Rochat et al. [31] and Li [32]. The paper develops analytical estimates for the deflection of the surface coating and the bond stresses that are generated at the plate-elastic halfspace interface due to the action of the nucleus of thermo-elastic strain.

2. Basic equations

The result of Goodier [22] indicates that if an elemental region of volume $d\Omega$ surrounding a point P and located in an isotropic elastic solid of infinite extent, is raised to a temperature T while the rest of the solid is at a different temperature T^*, the displacement field at any point in the infinite space region is given by

$$\mathbf{u} = \frac{\alpha_s(T - T^*)(1 + \nu_s)d\Omega}{4\pi(1 - \nu_s)} \nabla\left(\frac{1}{r}\right) \tag{1}$$

where ν_s is Poisson's ratio, α_s is the coefficient of linear expansion, $\mathbf{r}$ is the distance of any point under consideration from P and ∇ is the gradient operator. As discussed previously, this is an idealization void of transient effects. Restricting attention to the state of axial symmetry associated with the nucleus of thermo-elastic strain, we can evaluate from Eq. (1) the displacement components $u_r(r, z)$ and $u_z(r, z)$ applicable to the cylindrical polar coordinate system. Let us restrict attention to the case where the nucleus of thermo-elastic *expansion* is placed at the location $z=c$; in this case, the displacement in the axial direction is given by

$$u_z^{(e)}(r,z) = \frac{\Delta(z - c)}{\left[r^2 + (z - c)^2\right]^{3/2}} \tag{2}$$

where

$$\Delta = \frac{\alpha_s(T - T^*)(1 + \nu_s)d\Omega}{4\pi(1 - \nu_s)} \tag{3}$$

and Δ is a constant. Implicit in Goodier's solution is the requirement that the region that undergoes thermo-elastic dilatation is thermo-elastically identical to the surrounding elastic region and that the dimensions of the region $d\Omega$ are substantially small in comparison to the dimension $\mathbf{r}$. Ideally, the region that undergoes thermo-elastic dilatation should be modelled as a finite domain with elasticity and thermal properties that are different from the rest of the domain. Such a problem, for example, a spherical inclusion region in a halfspace region, involves the use of a bi-spherical coordinate system, which is more complicated and perhaps not altogether

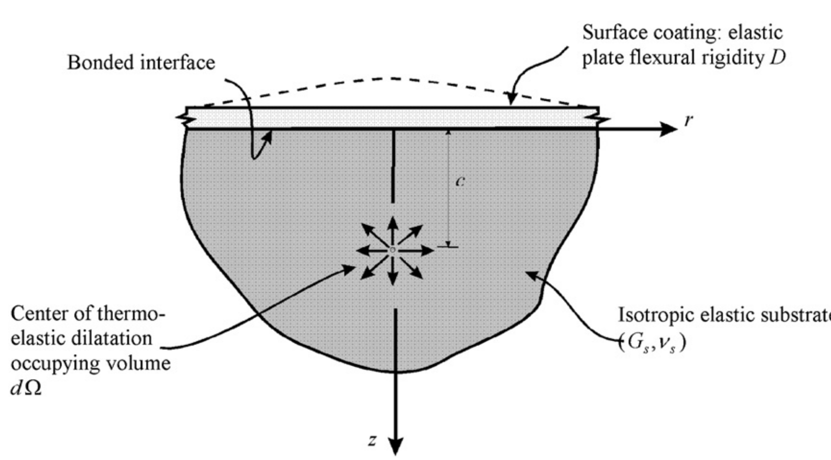

Fig. 1. Thermo-elastic loading of a bonded surface coating.

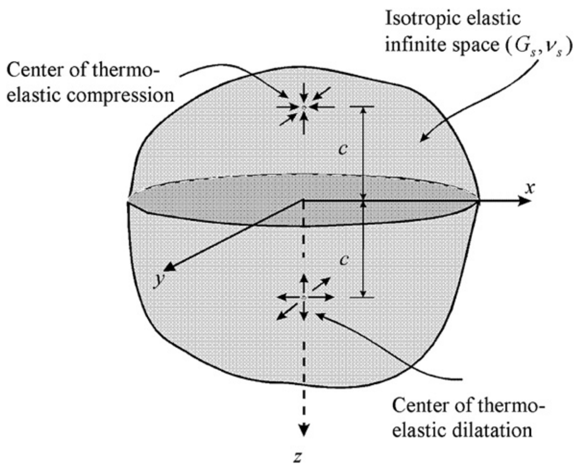

Fig. 2. Action of nuclei of thermo-elastic strain in an infinite space.

consistent with the simplified modelling of the surface coating as a thin plate, as opposed to an elastic layer of finite thickness. A result similar to Eq. (2) can be derived for $u_r(r,z)$, the displacement in the radial direction, but this is not relevant to the discussion that follows. If we now consider the problem of a nucleus of thermo-elastic *compression* placed at the location $z=-c$, the corresponding displacement in the elastic medium can be obtained by replacing c in (2) by $-c$. Now consider the simultaneous action of the combination of nuclei of thermo-elastic dilatation and compression placed at distances $z=c$ and $z=-c$, along the z-axis respectively (Fig. 2). These give the following displacements on the plane $z=0$:

$$u_r^c(r,0)=0;\quad u_z^c(r,0)=-\frac{2\Delta c}{(r^2+c^2)^{3/2}} \tag{4}$$

and the null radial displacement results from the state of asymmetry arising from the combination of the nuclei of thermo-elastic strain. In addition we note that

$$\sigma_{zz}(r,0)=0;\quad \sigma_{rz}(r,0)\neq 0 \tag{5}$$

the non-zero shear stresses arising from the null radial displacement condition at the plane $z=0$. The result (4) therefore corresponds to the displacement of a *halfspace with an inextensibility surface constraint* that is subjected to a nucleus of thermo-elastic strain of strength Δ that is located at the position $z=c$. In view of the axial symmetry of the adhesive contact problem for a surface coating that is being examined, it is convenient to utilize a Hankel transform development of two key results: the first relates to the axisymmetric loading of the surface of a halfspace with an inextensibility constraint by a normal traction distribution $q(r)$, and the second refers to the Hankel transform for the surface displacement of the halfspace with an inextensibility constraint and induced by the nucleus of thermo-elastic dilation placed at the location $z=c$. From the developments presented by Selvadurai [26] (see also Sneddon [33] and Gladwell [34]) it can be shown that

$$\bar{u}_z^q(\xi)=\frac{(3-4\nu_s)}{4G_s(1-\nu_s)\xi}\,\bar{q}(\xi) \tag{6}$$

where G_s is the linear elastic shear modulus and

$$\bar{u}_z^q(\xi)=\int_0^\infty r u_z^q(r,0)J_0(\xi r)dr \tag{7}$$

$$\bar{q}(\xi)=\int_0^\infty r q(r)J_0(\xi r)dr \tag{8}$$

Similarly, the Hankel transform of the surface displacement of the surface constrained halfspace due to the nucleus of thermo-elastic dilatation is given by

$$\bar{u}_z^c(\xi)=-2\Delta e^{-\xi c} \tag{9}$$

3. The bonded coating problem

Attention is now focused on the problem of an elastic surface coating that is bonded to an isotropic elastic substrate and the flexure of the coating is induced by a nucleus of thermo-elastic dilatation located at a distance $z=c$ along the axis of symmetry. The flexural behaviour of the surface coating is characterized by the Germain–Poisson–Kirchhoff thin plate theory [25,26], which permits for flexure of the plate without any *stretching* of its mid-plane. The differential equation governing flexure of the *externally unloaded surface coating* is given by

$$D\,\tilde{\nabla}^2\,\tilde{\nabla}^2 w(r)+q(r)=0 \tag{10}$$

where $D(=E_c t^3/12(1-\nu_c^2))$ is the flexural rigidity of the coating, E_c and ν_c are elastic constants of the coating material, t is the coating thickness, $q(r)$ is the contact normal stress at the bonded interface and

$$\tilde{\nabla}^2=\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr} \tag{11}$$

Operating on Eq. (10) with the zeroth-order Hankel transform we obtain

$$D\xi^4\,\bar{w}(\xi)+\bar{q}(\xi)=0 \tag{12}$$

The relationship between the surface displacement of the halfspace due to the combined action of $q(r)$ and the nucleus of thermo-elastic strain can be obtained by combining Eqs. (6) and (9); further, by observing the compatibility of the deformations of the bonded coating and the halfspace, we have

$$\bar{w}(\xi)=\bar{u}_z^q(\xi)+\bar{u}_z^c(\xi)=\frac{(3-4\nu_s)}{4G_s(1-\nu_s)\xi}[\bar{q}(\xi)-\bar{s}(\xi)] \tag{13}$$

where

$$\overline{s}(\xi) = \frac{8G_s\Delta(1-v_s)}{(3-4v_s)}\xi e^{-\xi c} \quad (14)$$

We can eliminate $\overline{q}(\xi)$ between Eqs. (12) and (13) to obtain an expression for $\overline{w}(\xi)$. Inverting the result we obtain

$$\frac{w(r)}{t} = \Theta \int_0^\infty \frac{\lambda e^{-(\lambda c/t)} J_0(\lambda r/t) d\lambda}{\left[1+\Phi\lambda^3\right]} \quad (15)$$

where

$$\Theta = \frac{\alpha_s(T-T^*)(1+v_s)d\Omega}{2\pi(1-v_s)t^3}; \; \Phi = \frac{1}{24}\left(\frac{E_c}{E_s}\right)\left(\frac{(1+v_s)(3-4v_s)}{(1-v_s)\left(1-v_c^2\right)}\right) \quad (16)$$

and Θ and Φ are both non-dimensional parameters. Similar expressions can also be derived for the contact stresses at the surface coating-elastic substrate interface and for the flexural moments developed in the flexible coating that is modelled as a thin plate. We first consider the expression for the variation of the deflection of the bonded coating for limiting values of the flexural rigidity of the coating. It is evident that for a completely rigid coating, $\Phi\to\infty$, and the displacements of the surface of the halfspace with the bonded coating will reduce to zero for $\forall r\in(0,\infty)$. Similarly, as $\Phi\to 0$, the integral (15) gives the result (4) for the displacement of the surface of a substrate that is only constrained in an inextensible fashion in the radial direction. A result of some interest relates to the maximum deflection experienced by the bonded surface coating when the nucleus of thermo-elastic strain migrates close to the surface of the substrate. In this case the deflection of the coating is given by

$$\left[\frac{w(0)}{t}\right]_{(c/t)\to 0} = \Theta \int_0^\infty \frac{\lambda d\lambda}{\left[1+\Phi\lambda^3\right]} = \frac{2\pi\Theta}{3\sqrt{3}\sqrt[3]{\Phi^2}}. \quad (17)$$

where $\lambda(=\xi t)$ is a non-dimensional integration parameter. It should be noted, however, that in the developments presented previously, the nucleus of thermo-elastic strain is assumed to occupy a small but finite volume, with the result that the limiting case of $\overline{c}\to 0$, even though it exists, is not a physically consistent limit. The acceptable limit for the parameter c/t can thus be assigned through an inspection of the value of $(d\Omega/t^3)$. Alternatively, $c/\sqrt[3]{d\Omega} >> 1$.

Here again, if the flexural stiffness of the surface coating is finite, this moderates the deflections of the surface of the substrate from attaining a singular value. In the absence of the stiffening effects of the coating, the result (1) indicates that the displacements will be singular at the point of application of the nucleus of thermo-elastic dilatation. Once the parameters Θ, Φ and (c/t) are specified, the expression (15) can be numerically evaluated to any required accuracy, using the integration procedures applicable to oscillatory integrals available in mathematical software such as MATHEMATICA™, MATLAB™ and MAPLE™ .

4. Numerical results

A result of particular importance relates to the interfacial stresses generated at the bonded interface between the surface coating and the substrate due to the action of the nucleus of thermo-elastic strain. In particular, the development of tensile stresses can lead to detachment at the interface, which will render the stiffening layer ineffective. The tensile or compressive nature of the interface stresses at the bonded contact will depend on either the relative increase or lowering of the temperature in the region $d\Omega$ in relation to the remainder of the halfspace region. If the region $d\Omega$ is subjected to cooling, then regions of the bonded interface in the vicinity of the nucleus of strain will experience tensile stresses, while heating of the region $d\Omega$ will invariably induce compressive stresses at the bonded interface in the vicinity of the nucleus of strain.

The non-dimensional form of the normal bond stresses $\sigma(\rho)$ at the surface coating-substrate interface can be obtained by combining the results (10) and (15) and the resulting integral form for the bond stress can be expressed in the *non-dimensional* form

$$\sigma(\rho) = \frac{|q(r)|}{E_c\Theta/12\left(1-v_c^2\right)} = \int_0^\infty \frac{\lambda^5 e^{-(\lambda\overline{c})} J_0(\lambda\rho) d\lambda}{\left[1+\Phi\lambda^3\right]} \quad (18)$$

where

$$\rho = \frac{r}{t}; \; \overline{c} = \frac{c}{t} \quad (19)$$

and the non-dimensional parameters Θ and Φ are defined by Eq. (16).

The result (18) can be integrated explicitly for $\rho=1$, with additional constraints $\overline{c}$ 0 and $\Phi\neq\pi$; but this result involves the *Meier G function*, which in itself requires a contour integration, excluding identified poles, of the product of a sequence of *Gamma functions*. This evaluation needs to be performed by appeal to numerical procedures. Also, an inspection of Eq. (18) indicates that $\sigma(0)\to\infty$ as $\overline{c}\to 0$. Here again, the comments made previously concerning the acceptable lower limit of $\overline{c}$ should be based on the requirement that $c/\sqrt[3]{d\Omega} >> 1$. The numerical results derived from Eq. (18) should therefore be viewed with this constraint in mind. The result (18) was evaluated using the integration routines available in MATHEMATICA™; the integration is performed for a finite range of λ, ensuring that, for convergence, an increase in the interval chosen does not result in a change in the value of the infinite integral. As remarked previously, the sense of the stress state (i.e. whether the interface stresses are compressive or tensile) needs to be interpreted by considering the relative increase or decrease in the temperature in the region undergoing thermo-elastic deformation in relation to the remainder of the halfspace region. As is evident, if the nucleus of thermo-elastic strain corresponds to a center of compression, (i.e. reduction in temperature in $d\Omega$) in relation to the rest of the substrate region, then tensile stresses can be developed at the interface as the position of the center of dilatation approaches the bonded interface between the surface coating and the substrate. In this case, compressive interface stresses will materialize only in regions remote from axis of symmetry. Conversely, for a center of dilatation, the largest interface stresses are compressive and tensile stresses would occur only in regions remote from the axis of symmetry. An alternative interpretation of these results is as follows: consider the uniform heating of the substrate with a surface coating, which raises the temperature in the entire region, except for a localized inhomogeneity in the vicinity of the bonded interface that does not experience the same uniform thermal expansion as in the rest of the substrate region. In this case tensile bond stresses can be induced at the interface, which can contribute to development of delamination.

While results of a generalized nature for the interface normal stresses can be presented through the evaluation of the integral (18), it is instructive to apply the analysis to typical surface coating-substrate systems that are reported in the literature. For this purpose, we utilize the material parameters presented, for example, by McGurk and Page [28], Carvalho et al. [30] and Souza et al. [35] for coatings of *Niobium Nitride* and *Titanium Nitride* applied to an *Aluminum substrate*. Since the integrals are presented in non-dimensional forms, the thickness of the coating does not explicitly enter the calculations. The thickness of the coating (t) as well as the volume over which the nucleus of thermo-elastic strain acts within the elastic substrate region is needed only for calculating the actual stresses. For the purposes of illustration of the distribution of interface stresses at the interface, it is necessary to introduce a length parameter. As indicated in Eq. (19), both the radial distance (r) and the depth of location of the nucleus of thermo-elastic strain (c) are normalized with respect to the thickness of the coating (t). The material and geometric parameters used in the investigation are as follows:

Aluminum Substrate: Young's modulus (E_s)=70 GPa; Poisson's ratio (ν_s)=0.33
Coefficient of thermal expansion (α_s)=23×10^{-6}/°C
Niobium Nitride Coating: Young's modulus (E_c)=460 GPa; Poisson's ratio (ν_c)=0.30
Coefficient of thermal expansion (α_c)=10^{-5}/°C
Coating thickness (t)=2.8 µm.
Titanium Nitride Coating: Young's modulus (E_c)=700 GPa; Poisson's ratio (ν_c)=0.30
Coefficient of thermal expansion (α_c)=10^{-5}/°C
Coating thickness (t)=1.0 µm.

Fig. 3 illustrates the distribution the *non-dimensional normal stresses* at the interface between the NbN coating and the aluminum substrate. For the combination of the NbN coating and an aluminum substrate, the relative stiffness parameter $\Phi\approx1$. The trends exhibited by the numerical results are consistent with remarks indicated previously. As the depth of location of the nucleus of thermo-elastic strain exceeds value c 10t, the nucleus of thermo-elastic strain has little influence on the development of interface stresses (Fig. 4). This conclusion is valid only for the case involving the localized nucleus of thermo-elastic strain. It is plausible that in situations where a distribution of nuclei of thermo-elastic strain acts over a region within the substrate, the influence of such thermal loadings will be noticeable even when such thermal anomalies are located beyond distances greater than 10t. Figs. 5 and 6 illustrate analogous results for the case involving a

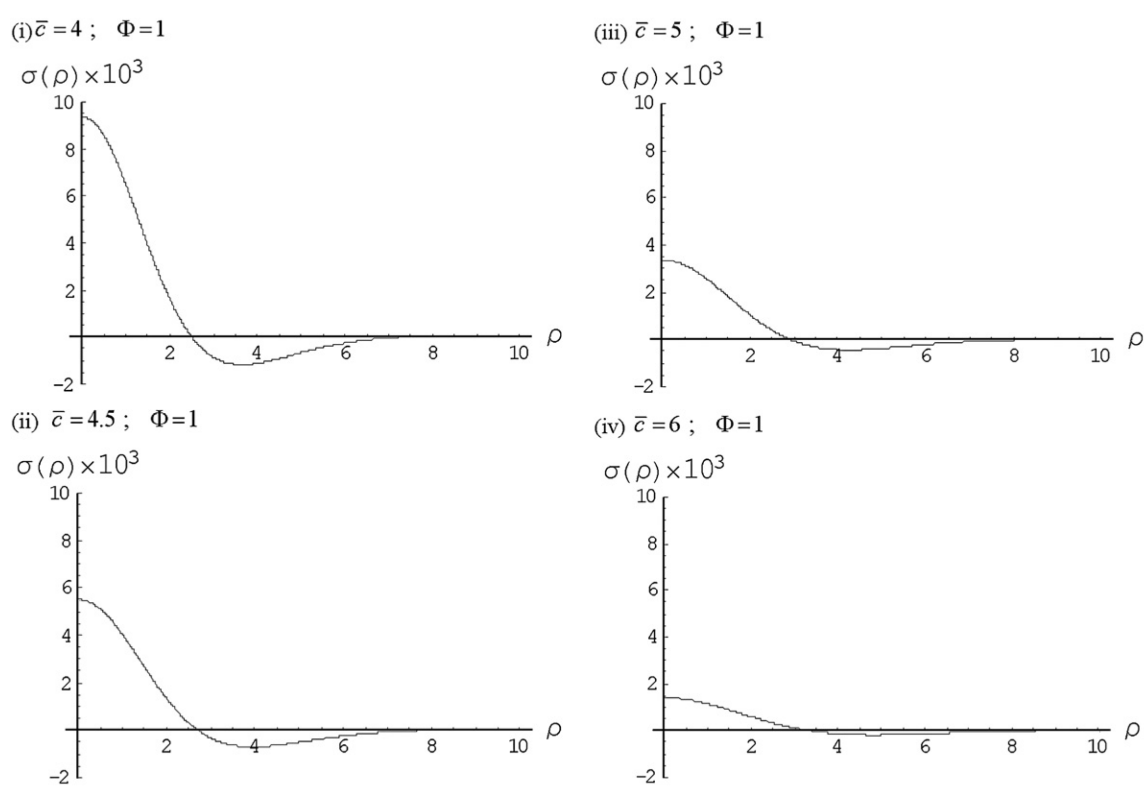

Fig. 3. Interface normal stresses at the bonded surface coating due to the action of a nucleus of thermo-elastic strain — NbN coating on aluminum substrate.

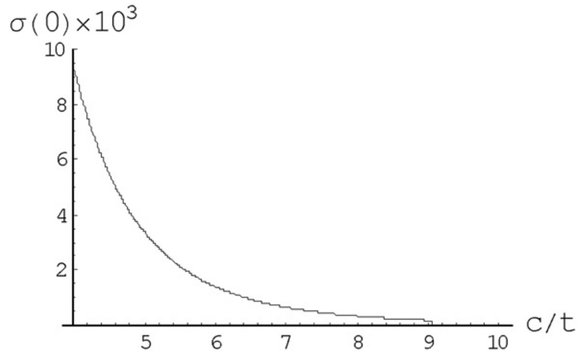

Fig. 4. Influence of the depth of location of the nucleus of thermo-elastic strain on the maximum interface stress at the bonded surface coating-NbN coating on an aluminum substrate.

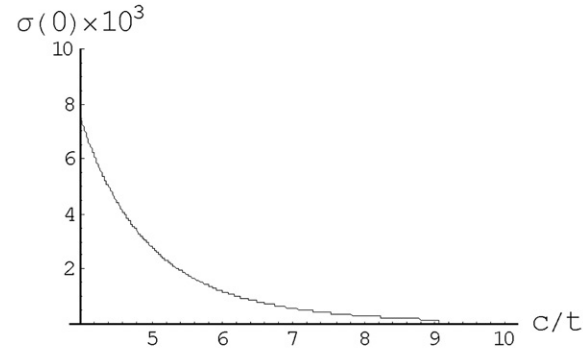

Fig. 6. Influence of the depth of location of the nucleus of thermo-elastic strain on the maximum interface stress at the bonded coating — TiN coating on an aluminum substrate.

TiN coating and an aluminum substrate. For the combination of a TiN coating and an aluminum substrate, the relative stiffness parameter $\Phi \approx 1.5$. The results are similar in character to those observed in connection with the previous example. As an example, consider the problem where a region of a TiN coating (1 μm)-aluminum substrate system experiences a temperature rise of 200 °C. A region of the substrate of volume $d\Omega \approx 0.125\ \mu m^3$, the center of which is located at 0.75 μm, does not experience the temperature rise. In this case, the interface between the coating and the substrate can develop a maximum tensile stress of approximately 14.58 MPa, immediately above the location of the thermo-elastic anomaly. [A reviewer has drawn the authors' attention to the possibility of the surface coating being under severe in-plane compression due to fabrication processes. This is an important issue particularly if the surface coating experiences flexure or even buckling due to in-plane compressive stresses. In such a case, since there are no normal tractions applied to the free surface of the coating, the normal stresses that can develop at the interface between the surface coating and the substrate will be self-equilibrating. The additional influences of the nucleus of thermo-elastic strain can either be additive or subtractive depending on the nature of the thermo-elastic strain (i.e. dilative or contractive) and the nature of the deformed shape of the surface coating due to the in-plane compression (i.e. outward or inward flexure) The combined influences of the separate stress states on the net bond stresses needs to be determined through considerations of these influences. The relative magnitudes of the bond stress development due to residual in-plane compression of the surface coating will depend on the relative stiffness parameter as indicated by the parameter Φ given by Eq. (16).]

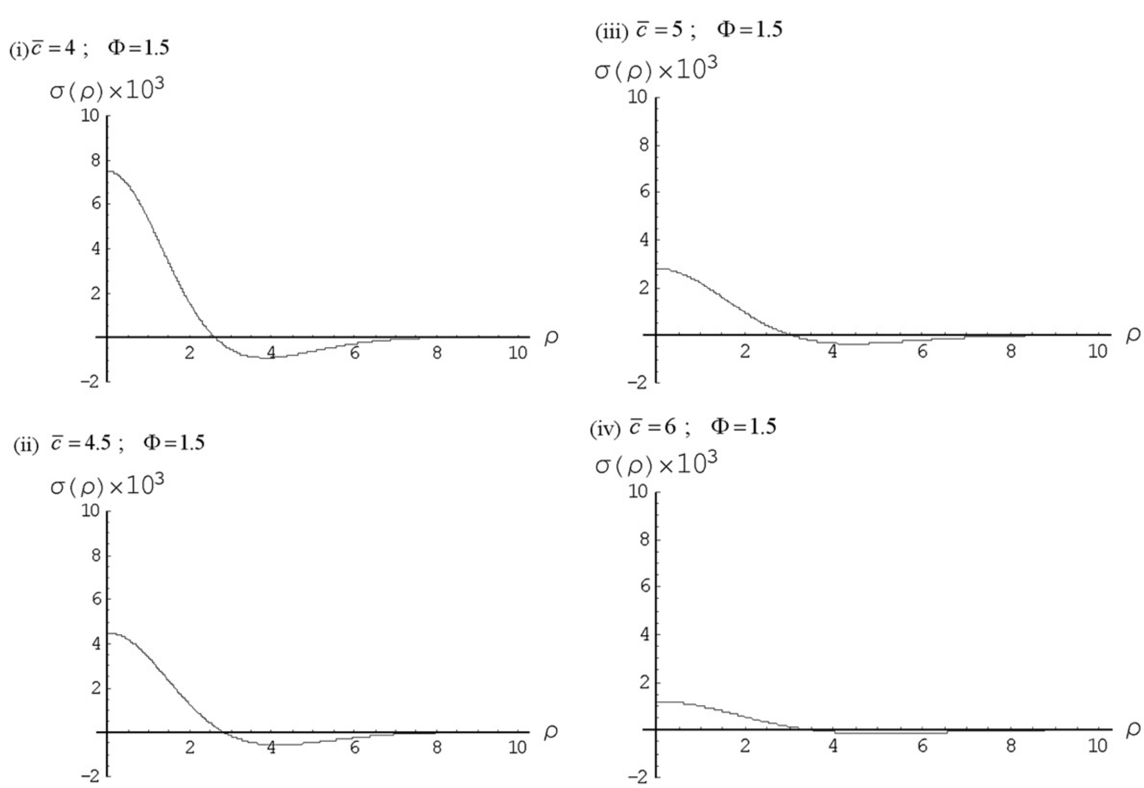

Fig. 5. Interface stresses at the bonded surface coating due to the action of a nucleus of thermo-elastic strain-TiN coating on an aluminum substrate.

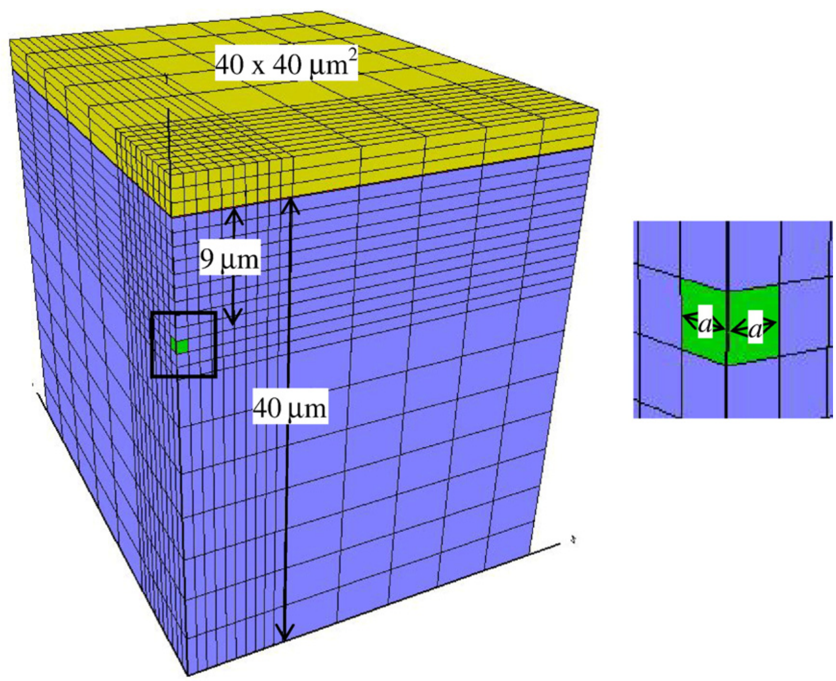

Fig. 7. Finite element discretization of the coating-substrate system.

The extension of the problem to include a thermal anomaly that occupies a finite region can be done either through an integration of the basic solution(18) for the loading by a nucleus of thermo-elastic strain or through a computational approach. The latter technique is more suitable particularly in situations where either the thermal anomaly does not exhibit spatial symmetry or a regular geometry or the domain of interest has a finite geometry. Here we use the finite element code CAPA-3D developed at TU-Delft [36] to examine the problem of the loading of the bonded TiN coating of thickness, $t=2.8$ μm. by a square region, which represents a thermo-elastic anomaly. The substrate is assumed to be aluminum and the material parameters associated with the numerical study are those presented previously. The finite element discretization employed in the computational modeling is shown in Fig. 7. In the computational simulation, the semi-infinite substrate is represented by an extended domain where the external dimensions are substantially large in comparison with the dimensions of the thermo-elastic anomaly. To provide a basis for comparison with the thin plate model for the surface coating, the interface between the surface coating and the substrate is incorporated with interface elements that provide an inextensibility constraint consistent

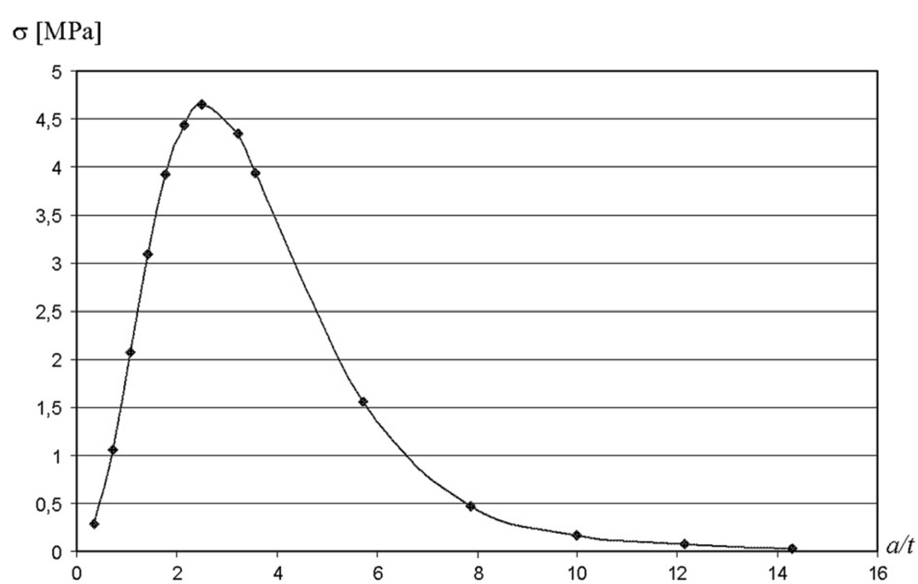

Fig. 8. Adhesive stresses at the bonded interface.

with the inextensible mid-plane behaviour inherent in the classical thin plate assumption. The accuracy of the computational model has been verified through comparisons with the analytical results presented in the paper. Fig. 8 illustrates the magnitude of the adhesive stresses that are generated at the bonded interface when the entire region experiences a temperature rise of 200 °C, except for the square-shaped anomaly of thickness 1 μm and plan area a μm × a μm that is located at a distance of 9 μm from the bonded interface. The results of the computational modeling indicate trends that are consistent with analytical results. It is also important to note that the geometry of the thermo-elastic anomaly has an influence on the magnitude of the bond stresses developed and the maximum bond stresses are influenced by both the position and area extent of the anomaly.

5. Conclusions

This paper presents a compact mathematical solution to the adhesive contact problem related to a surface coating that is bonded to an isotropic elastic substrate, and subjected to loading by a nucleus of thermo-elastic strain located at a finite distance from the bonded interface. It is shown that the solution to the problem can be obtained in explicit form in terms of infinite integrals of the Hankel-type. Results for the deflection of the surface coating applicable to specific situations can also be obtained in exact closed form. We note that in view of the thermo-elastic nucleus model used in the developments, the solutions presented here are applicable to situations where the dimensions of the region of thermo-elastic deformation are small in comparison to the dimensions at which it is located within the substrate. Alternatively, the solution can be regarded as the Green's function for the loading of a substrate with a surface coating that is loaded by a nucleus of thermo-elastic strain. An integration of the solution can be used to generate results for any arbitrary distribution of centers of thermo-elastic strain, including line, area and volume distributions of centers of thermo-elastic strain. The companion computational results indicate that the area distribution of the thermo-elastic anomaly has an influence on the magnitude of the bond stresses and as the area of the anomaly increases the bond stresses generated are attenuated. The analysis presented here can only address the evaluation of the stress state likely to initiate debonding failure at the interface according to a specified critical stress criterion. The problem related to the extension of an interface delamination requires the solution of the analogous stress analysis problem related to the presence of a delaminated region of arbitrary dimension at the interface and to establish, through an interface crack extension criterion, the conditions that will either arrest or promote the growth of the defect, which will lead to further extension of the interface delamination, as a result of thermal loading. It should also be noted that significant in-plane compressive stresses can exist in the surface coating due to the fabrication process. These in turn can generate additional bond stresses that can either promote or hinder delamination depending upon the mode of deformation of the surface coating induced by the in-plane compressive stresses.

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