ground water

Technical Note/

On Cavity Flow Permeability Testing of a Sandstone

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Abstract

This paper describes a laboratory experiment designed to measure the bulk permeability of a cuboidal sample of sandstone measuring ~450 mm^2 in plan area and 508 mm in height. The relatively large dimensions of the sandstone specimen allow the determination of the permeability of the material by creating a central cavity that can be pressurized to maintain a constant flow rate. The paper describes the experimental details and the test procedure, and discusses the computational and analytic approaches that have been used to interpret the test results.

Introduction

The choice of scale is an important factor in the measurement and interpretation of the permeability characteristics of low-permeability intact rocks. Crustal scales can be of the order of 0.5 to 5.0 km, borehole scales can range from 30 to 300 m, and laboratory scales can range from 5 to 15 cm. The bulk permeability of a geological medium can thus be influenced by fractures, fissures, inclusions, and other inhomogeneities that are dominant at the selected scale. Despite this limitation, procedures need to be devised whereby representative volume elements of the geologic material can be tested to yield estimates of permeability that can be used in engineering calculations of ground water flow, contaminant transport, ground water hydrology, and water resources management. The choice of the representative volume element that can be used in the assessment of permeability of a geologic material in a laboratory setting is influenced by both the ease with which intact samples can be obtained from a geological stratum and the capabilities of laboratory devices to accommodate the sample to be tested. Conventional laboratory samples used in permeability testing of geologic materials are generally cylindrical with diameters ranging from 25 to 150 mm. The lengths of the samples are generally three times the diameter, dictated largely by the sizes required for other tests used to assess the mechanical properties of the geologic material. Recently, Selvadurai et al. (2005) presented the results of an experimental study involving relatively large cylinders of Barre Granite measuring 450 mm in diameter, which would correspond to permeability estimates representative of the borehole scale. The results of a *transient pressure pulse technique* presented in the study by Selvadurai et al. (2005) can be interpreted through an analytic result to provide estimates for the permeability of the granite. References to further studies are given by Lama and Vutukuri (1978) and Selvadurai et al. (2005).

This paper deals with the use of a constant flow rate test for measuring the permeability of a large specimen of intact gray sandstone from Quebec, Canada. The dimensions of the cuboidal sample of sandstone used in the experiment were 450 mm^2 in plan area and 508 mm in height. The cuboidal sample contains a cylindrical borehole of diameter 63.5 mm that can be pressurized to maintain a constant flow rate. The plane ends of the cuboidal specimen are sealed to ensure the development of plane fluid flow through the sandstone sample normal to the axis of the borehole. The interpretation of the experimental results is achieved through a finite-element analysis of the plane flow problem applicable to a sector of the flow domain. Both computational results and approximate analytic relationships are used to estimate the permeability characteristics of the sandstone.

The Test Facility

The test facility (Figures 1 and 2) consists of a Plexiglas container that is attached to an aluminum base and

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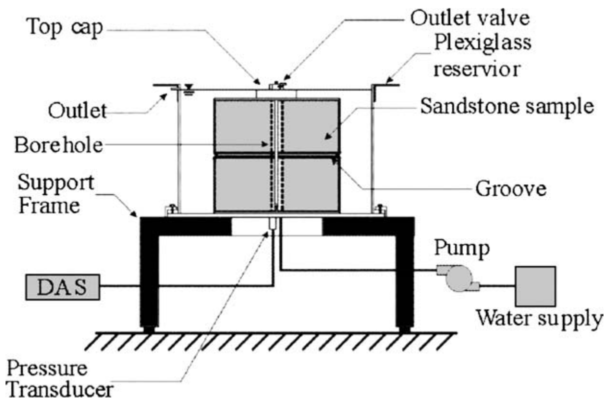

Figure 1. Schematic view of the apparatus.

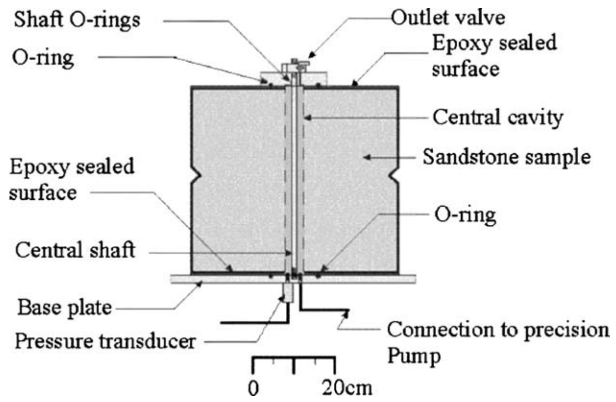

Figure 3. Cross-sectional view of sample.

serves as the external reservoir. The outlet overflow allows a constant hydraulic potential to be maintained on the outer surface of the sandstone sample throughout any experiment. The upper and lower plane ends of the cuboidal sandstone sample are sealed with an epoxy resin, which provides completely impervious barriers on surfaces normal to the axis of the borehole. The cavity of the cuboidal sample is sealed with a set of O-rings that are located on the base plate of the apparatus and at the underside of the top cap (Figure 3). A stainless steel central rod, which is tightened through a screw connection to the base plate, provides the restraint required for sealing the central cavity. In addition, O-ring seals are also provided in the upper cap to form a seal between the central rod and the cap. Manual tightening of the central rod is sufficient to provide a seal for the purpose of pressurizing the central cavity to initiate steady flow. The base plate contains a pressure transducer with the active surface open to the central fluid-filled cavity. The upper cap of the sealing attachment contains an outlet valve that can be used for de-airing the cavity during the filling of the central cavity.

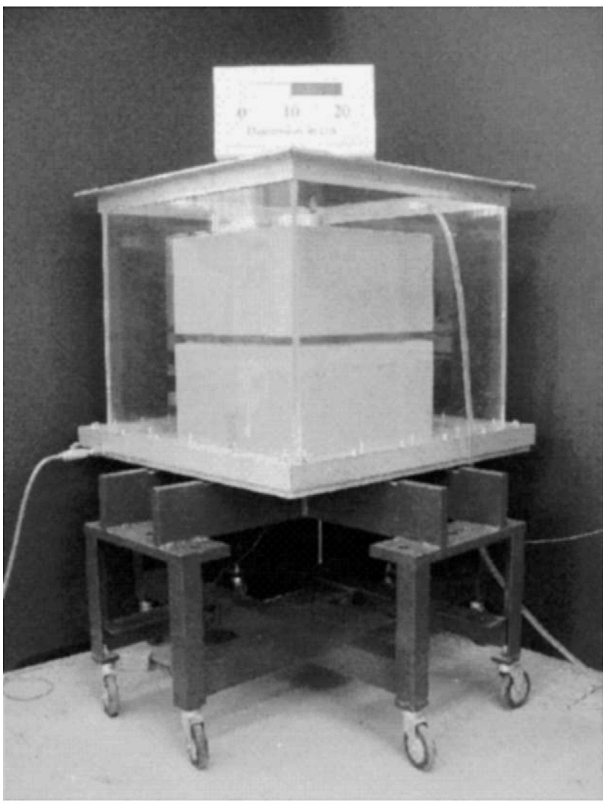

Figure 2. The test facility.

The Test Procedure

The realization of two-dimensional planar flow from the cavity depends on two factors: the hydraulic homogeneity of the sample and the sealing action of the plane ends of the sandstone sample. The first factor can only be gauged through a visual inspection of the sandstone sample. The tested sample did not contain any obvious inhomogeneities and stratifications that would indicate the presence of a hydraulic inhomogeneity. A more precise assessment of this assumption can only be realized by conducting permeability tests that initiate flows from selected regions of the surface of the sample (Tidwell and Wilson 1997). In the current series of experiments, it is assumed that the planar flow assumption will provide an estimate for the average hydraulic conductivity characteristics of the sandstone sample during flow emanating from the pressurized cavity. The sealing of the plane ends is achieved by coating both the upper and lower plane ends with epoxy resin (Figure 3). During this procedure, the central cavity is filled with foam rubber to ensure that the resin does not seal any part of the central cavity through which plane flow is established. To obtain a smooth finish, the resin is applied as evenly as possible, and a smooth surface is obtained by allowing it to cure for 24 h under a nominal weight transmitted through a thin metal (or Teflon®) sheet, roughly the size of the plane surface of the sample. The success of the experiment largely depends on the ability to maintain a pressurized central cavity without leakage from the O-ring seals. The O-rings are maintained in a clean condition and lightly lubricated with vacuum grease. The sandstone sample contains a central groove (Figure 3) that is intended for creating a fracture for subsequent experiments. A special sample holder that fits into the groove is used to place the sandstone specimen inside the Plexiglas reservoir. Accurate positioning of the sample ensures that the active surface of the pressure transducer and the port for supplying the water are open to the central cavity region. The outer reservoir is first filled with distilled water to ensure wetting saturation of the sandstone sample. This saturation process is observed through the release of air bubbles to the outer reservoir region. The central cavity region is then sealed using the central stainless steel rod, and the reservoir is filled with distilled water, keeping the outlet valve open. Once the central cavity is filled with water, the constant flow rate generated by the precision pump is used to fully saturate the sandstone sample with distilled water. The saturation time can vary and the release of air bubbles into the reservoir is indicative of the progress of saturation. The development of cavity pressure during the saturation process and the subsequent attainment of a steady-state condition are recorded using a computerized data acquisition system. Figure 4 shows the results for the time-dependent development of cavity pressure during a prescribed flow rate. Time to attain a steady-state cavity fluid pressure for a given fluid flow rate can vary; in the current series of tests, a steady flow rate of 50 mL/min was maintained over variable periods. The test A shown in Figure 4 corresponds to the case where the flow is initiated through the sandstone block when the sandstone block is allowed to imbibe water from the reservoir from its dry state. As can be observed, the cavity pressure does not reach a stable value even after a test duration of 2 h. Test B was conducted after leaving the sample in the reservoir for ~1 h. Here again, the cavity pressure exhibits a time-dependent increase during the application of a steady flow rate of 50 mL/min. The condition of the water percolating through the sample during these tests was also observed. Initially, the water leaving the sample had a murky coloration, but in subsequent tests, a clear fluid discharge was observed. Test 1 is the first of the tests in which a steady cavity pressure of 161 kPa was observed during application of a steady flow rate of 50 mL/min. In subsequent tests (tests 2 to 4), the cavity fluid pressure reached stable values during the application of a flow rate of 50 mL/min. Considering these four tests, the cavity pressure varies between 140 to 152 kPa. It could be argued that the lowest of the cavity fluid pressures, 140 kPa, should be chosen because this would represent the most plausible situation where there is a high likelihood of a greater volume of the sandstone block achieving saturation. For the purposes of the final calculations, however, the cavity pressure corresponding to the steady flow rate of 50 mL/min is chosen as 146 kPa, which represents the average steady cavity pressures derived from tests 2 to 4.

Modeling

For Darcy flow, the flow potential φ consists of the pressure and datum potentials (i.e., $\varphi = \varphi_d + \varphi_p$ with unit L). In a hydraulically homogeneous and isotropic porous medium, Darcy's law for the velocity vector is given by:

$$\mathbf{v}(\mathbf{x}) = -k\nabla\varphi \tag{1}$$

where k is the hydraulic conductivity (with units L/T), $\mathbf{x}$ is a position vector, and $\nabla\varphi$ is the hydraulic gradient. Combining Darcy's law with the equation for mass conservation gives rise to the following partial differential equation for the flow potential $\varphi(\mathbf{x})$:

$$\nabla^2\varphi(\mathbf{x}) = 0 \tag{2}$$

The previous partial differential equation can be solved by specifying appropriate Dirichlet and Neumann boundary conditions applicable to the flow domain (Harr 1962; Bear 1972; Selvadurai 2000). The flow net for a

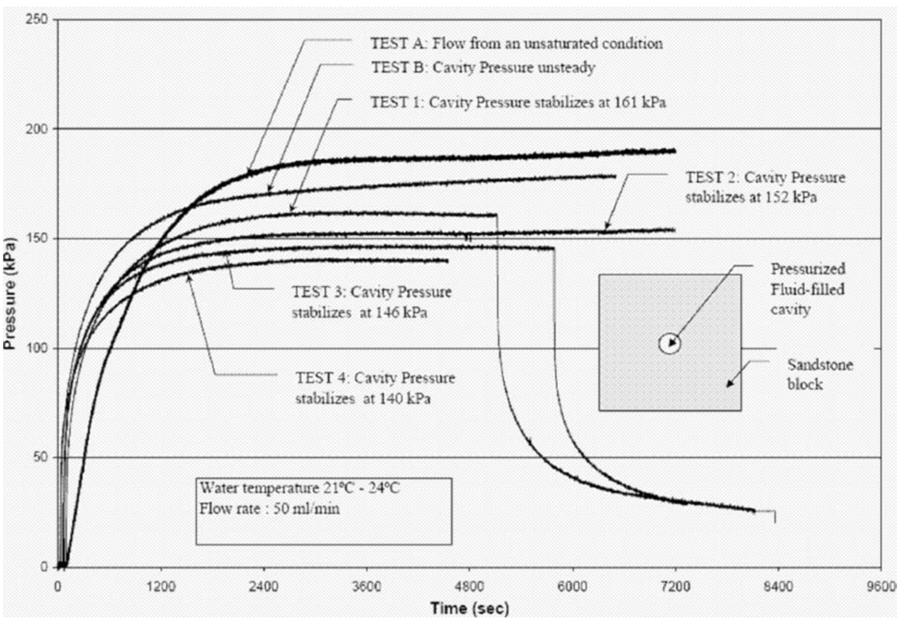

Figure 4. Time-dependent evolution of cavity fluid pressure during attainment of steady flow.

sector of the flow domain, derived from a finite-element analysis of Equation 2 (Anderson and Woessner 1991; Zienkiewicz and Taylor 2000), along with the associated boundary and symmetry conditions is shown in Figure 5.

Computational Modeling

The total flow rate can be calculated by computing the flow volume per unit time across any surface connecting the planes on which the Neumann boundary conditions are prescribed. If we denote the boundary of the circular inner region of the modeled sector area by S^*, the fluid flow rate is given by:

$$Q = -8kH_0 \int_{S^*} \mathbf{n}\nabla\varphi \mathrm{d}S \tag{3}$$

where H_0 is the height of the sandstone specimen, **n** is the outward unit normal to S^*, and the factor 8 accounts for the number of sectors. Although in principle the result Equation 3 is invariant of choice of location of the surface, the computational evaluations of Equation 3 can be influenced by the mesh discretization, particularly at boundaries. An alternative is to use the flow net to compute the flow rate. Using standard developments (Harr 1962; Bear 1972; Verruijt 1982), it can be shown that the permeability of the material K (with units L^2), can be estimated from the result:

$$K = \frac{k\mu}{\gamma_w} = \frac{Q\mu}{\gamma_w H_0(\varphi_i - \varphi_e)}\left(\frac{N_d}{N_f}\right) \tag{4}$$

where Q is the flow rate (units L^3/T), μ is the dynamic viscosity of water at the tested temperature (units M/LT), γ_w is the unit weight of water (units M/L^2T^2), $(\varphi_i - \varphi_e)$ is the potential difference initiating flow (units L), and N_d and N_f are, respectively, the number of potential drops and the total number of flow channels associated with the complete flow net. A further approximate relationship for estimating the permeability of the sandstone can be obtained by observing the near radial flow pattern occurring

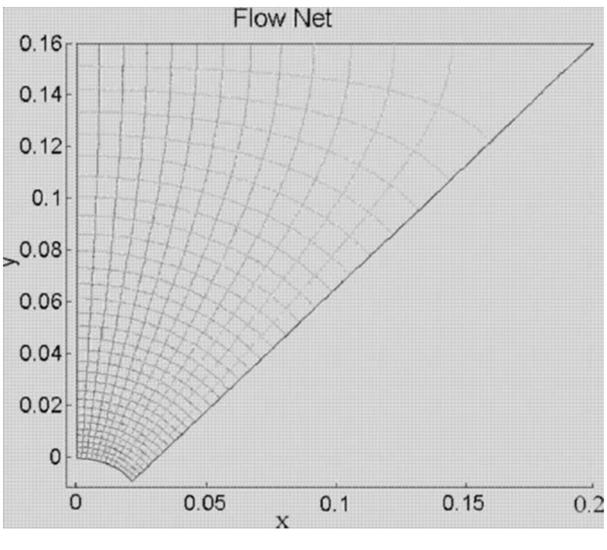

Figure 5. The flow net applicable to the sector domain.

in a substantial part of the flow domain shown in Figure 5. The analytic result for purely radial flow in a porous domain bounded by two concentric circles can therefore be adopted for estimating the permeability corresponding to the radially symmetric case. Selvadurai (2004) has shown that equivalence between the two flow domains (i.e., square region of side length $[2b]$ and an annular region of outer diameter $[2b^*]$) is best estimated by selecting relationships corresponding to surfaces through which flow takes place rather than by considering volumetric equivalence of the flow domain. Considering equivalence in the outer surface areas, we can show that:

$$b^* = \frac{4}{\pi}b \tag{5}$$

For equivalence in the total volume between the cuboidal sample and the hollow cylindrical sample, we have:

$$b^* = \frac{2}{\sqrt{\pi}}b \tag{6}$$

Restricting equivalence to the surface areas, the relevant analytic expression that can be used to estimate the permeability of the sandstone is given by (Harr 1962):

$$K = \frac{k\mu}{\gamma_w} = \frac{Q\mu}{\gamma_w H_0(\varphi_i - \varphi_e)}\left(\frac{\log_e(4b/\pi a)}{2\pi}\right) \tag{7}$$

The experiments conducted on the sandstone sample furnished the following experimental data: $\varphi_i = 14.6$ m, $\varphi_e = 0.5$ m, $Q = 50$ mL/min, $a = 31.75$ mm, $b = 225$ mm, $H_0 = 508$ mm. From physical tables, the properties of the water at standard temperature (~20°C) and atmospheric pressure are as follows: $\gamma_w = 10$ kN/m^3, $\mu = 10^{-6}$ kN s/m^2. From considerations of the properties of a flow net, we have $N_f \approx 104$ and $N_d \approx 31$.

Using these results in Equation 4, we obtain $K \approx 3.46 \times 10^{-15}$ m^2. Similarly, from Equation 6, we obtain the estimate $K \approx 4.07 \times 10^{-15}$ m^2. The estimates for permeability of sandstone available in the literature vary a great deal and range from 1.0×10^{-15} to 1.5×10^{-10} m^2 (e.g., Farmer 1968; Serafim 1968; Winterkorn and Fang 1975; Jaeger and Cook 1976; Lama and Vutukuri 1978; Costa and Baker 1981). Recently, Lock et al. (2003) investigated the use of the image analysis of the pore structure for the estimation of the permeability of sandstone. Their results suggest the variation of permeability of sandstone between 2.0×10^{-15} to 1.4×10^{-12} m^2. The experimental results conform to the lower estimate for permeability, indicative of a relatively intact bulk volume of the sandstone.

Conclusions

Permeability of a naturally occurring geological material is a scale-dependent parameter that can be influenced by defects and anomalies, which can either enhance or impede the flow of fluids through the medium. For this reason, permeability evaluation should ideally be conducted in situ. The laboratory evaluation of permeability characteristics of a geological material should be regarded only as a supplement to the adequate planning and execution of in situ tests such as borehole packer tests and other pumping tests, which involve either extraction or recharge. In this context, laboratory tests can be conducted on small diameter core samples recovered from a site investigation program. When substantially larger samples of the geological material can be recovered, laboratory tests can be carried out using specialized testing techniques. The in-plane flow test presented in this paper is a convenient method for testing intact rock samples of large dimensions. The limitations on the sample size used in the test are dictated largely by material handling capabilities and provisions for initiating a flow pattern that is amenable to convenient theoretical evaluation. The experimental configuration for in-plane flow outlined in this paper is a novel approach that provides a convenient technique for estimating the permeability characteristics of sandstone and other cemented geological materials in which steady flows can be established without the application of substantial fluid pressures. A key factor that can influence the measured permeability of the sandstone is the degree of saturation of the sample. There is no convenient way of addressing this issue without resorting to more complicated techniques for estimating the degree of saturation through indirect techniques. The alternative of vacuum saturation of the sandstone block is not a practical option, and the results would prove to be too artificial for practical applications. The results of the current experimental investigations give a reasonably repeatable set of data where the maximum cavity pressures during attainment of steady-state conditions do not differ by more than 10%. Such data can be readily used to estimate the permeability characteristics of the sandstone. The experimental procedures can also be extended to include the testing of samples of a geological material where the surfaces are partially sealed to create preferential fluid migration pathways. Results from such tests can, in turn, be used in conjunction with an inverse analysis to estimate hydraulic inhomogeneities within the sample.

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