Lateral Loading of a Rigid Rock Socket Embedded in a Damage-Susceptible Poroelastic Solid

A. Shirazi[1] and A. P. S. Selvadurai[2]

Abstract: The paper presents a computational assessment of the influence of damage on the behavior of a rigid rock socket embedded in a fluid-saturated poroelastic solid. The iterative computational scheme takes into consideration the irreversible alteration in the fluid transport characteristics that takes place as a result of damage and the stress dependency of the isotropic damage process. Numerical results illustrate the influence of the stress-state-dependent damage process on transient consolidation response of the rock socket head displacement.

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CE Database subject headings: Damage; Poroelasticity; Consolidation; Hydraulic conductivity; Lateral loads; Rocks.

Introduction

Structural elements such as rock sockets are used quite frequently to transmit axial, lateral and torsional loads to the interior of the supporting rock mass. Examples of applications of rock sockets and descriptions of the practical aspects are given by Parkin and Donald (1975), Pells and Turner (1979), Rowe and Pells (1980), Donald et al. (1980), Glos and Briggs (1983), Rowe and Armitage (1987), Whitworth and Turner (1989), Douglas and Williams (1993), and Leong and Randolph (1994). The relative flexibility of the rock socket is determined through a combination of parameters that include the dimensions of the rock socket [e.g., the length (L) to diameter (d) ratio] and the elastic modular ratio between the rock socket material and the geomaterial. As indicated by Poulos and Davis (1980), when the relative stiffness

$$R = \frac{\pi}{64}\frac{E_P}{E_S}\left(\frac{d}{L}\right)^4$$

is large (e.g., $R > 100$) the flexibility of the rock socket in the bending mode is negligible and the rock socket can be modeled as a rigid element. The rigid rock socket idealization is a useful limiting case of the behavior of the poroelastic medium–pile interaction problem for most anchor piles (see, e.g., Glos and Briggs 1983; Rowe and Armitage 1987; Leong and Randolph 1994). Furthermore, the absence of flexibility effects makes the analysis considerably easier to perform. The focus of the paper is therefore on the examination of the behavior of a rigid anchoring rock socket that is embedded at the surface of a geomaterial half-space region, which is saturated with a pore fluid. The constitutive behavior of the geomaterial in which the rock socket is embedded is represented by the poroelastic model proposed by Biot (1941). Geomaterials such as fluid saturated sandstone, mudstone, shale, and other soft rocks can be represented by the classical poroelasticity model. The purely poroelastic analysis of, say, a half-space region containing a rigid rock socket, can be examined quite conveniently, using existing approaches to computational modeling of poroelastic media. In this paper, we extend the modeling of the poroelastic medium to include the generation of stress-induced continuum damage in the porous skeleton. The incorporation of damage mechanics (Kachanov 1958; Krajcinovic 1984; Wohua and Valliapan 1998a,b; Shao and Desai 2000; and Voyiadjis and Deliktas 2000) within the context of the poroelasticity theory for geomaterials, is relatively new and the approach has been applied to examine the poroelastic behavior that involves damage-induced reduction in the elasticity characteristics of the poroelastic medium. This work employs the process of isotropic damage to examine the response of a brittle poroelastic medium. The analysis of experimental observations in soft rocks without any preferred orientation, presented by Schulze et al. (2001) indicates that at stress levels well below the development of macrodefects such as fracture, the nonlinearity can be modeled by appeal to the isotropic elastic continuum damage concept. Mahyari and Selvadurai (1998) extended these studies to include not only elasticity reduction but also hydraulic conductivity enhancement during damage evolution. Experimental evidence, albeit limited, exists to support both the assumption concerning the reduction in elasticity characteristics and increase in the hydraulic conductivity characteristics of granite and soft rocks, such as sandstone (e.g., Zoback and Byerlee 1975; Shiping et al. 1994; Kiyama et al. 1996; Schulze et al. 2001). Literature on further experimental investigations concerning the alteration in hydraulic conductivity of porous geomaterials has been presented by Selvadurai (2004). The work on poroelastic media susceptible to damage was extended by Shirazi and Selvadurai (2002) and Selvadurai and Shirazi (2004) to include, respectively, indentation and fluid inclusion problems associated with fluid-saturated, damage susceptible porous media.

In this paper, the computational modeling is applied to examine the problem of a laterally loaded rigid rock socket that is

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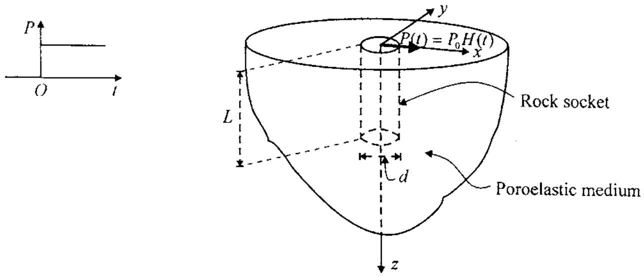

Fig. 1. Laterally loaded rigid rock socket embedded in poroelastic geomaterial

embedded at the surface of a damage-susceptible poroelastic half-space. In addition to the consideration of the alteration in the elasticity and hydraulic conductivity characteristics with strain, we also consider the influence of the stress-state dependency on the evolution of damage. In this latter approach, the isotropic damage evolution depends on the sense (tensile/compressive) of the first invariant of the effective stress tensor. Such an approximation is in keeping with common experimental observations conducted on soft rocks that damage cannot be initiated through an increase in the compressive effective confining stress that can be applied to a geomaterial. This assumption is in agreement with the experimental observations in the geological porous media with an interconnected network of pores including soft rocks and subjected to stress levels well below the failure. The studies by Katti and Desai (1995) and Park and Desai (2000), that use the disturbed state concepts indicate that intergranular bonded materials including overconsolidated clays and saturated sands with potential for exhibiting microstructural instability in the form of liquefaction, when subjected to high stress levels, can experience damage even at stress states that are compressive. The specific problem examined in the paper is illustrated in Fig. 1. The surface of the damage susceptible poroelastic medium is assumed to be free draining. The interface between the rigid rock socket and the poroelastic medium can possess pore pressure boundary conditions that correspond to either fully drained or impervious conditions. The displacements of the poroelastic medium are continuous across the rock socket–geomaterial interface. The dominant displacement of the embedded rock socket corresponds to lateral displacement at the point of application of the load $P(t)$. This load is represented by a time dependency in the form of a Heaviside step function. The time-dependent consolidation response of the rock socket is assessed in relation to its displacement at the point of application of $P(t)$.

Governing Equations

The basic investigation associated with this paper relates to the incorporation of damage mechanics within the context of the classical theory of poroelasticity proposed by Biot (1941). The evolution of stress-state-dependent isotropic damage will influence the alteration of the basic material parameters associated with the poroelastic model, in terms of the elastic modulus and the hydraulic conductivity. For completeness, we shall record here the basic partial differential equations governing the mechanics of a porous elastic solid saturated with an incompressible pore fluid. The dependent variables associated with the formulation are the displacement components $u_i(x_j,t)$ and the pore fluid pressure $p(x_j,t)$, where x_i are spatial coordinates and t is time. The complete development of the governing equations from various perspectives including continuum theories of mixtures can be found in the articles and texts by Rice and Cleary (1976), Desai and Christian (1977), Detournay and Cheng (1993), Lewis and Schrefler (1998), and Coussy (1995). Considering Hookean elastic behavior, Darcy's law, and the relevant conservation equations, we obtain the governing partial differential equations as follows:

$$\mu\left(\nabla^2 u_i + \frac{1}{(1-2\nu)}\frac{\partial \varepsilon_v}{\partial x_i}\right) = \frac{\partial p}{\partial x_i} \quad (1a)$$

$$C_V \nabla^2 p = \frac{\partial p}{\partial t} - \bar{K}\frac{\partial \varepsilon_v}{\partial t} \quad (1b)$$

where $\varepsilon_{ij}=(u_{i,j}+u_{j,i})/2$=skeletal strain tensor; $\varepsilon_v=\partial u_k/\partial x_k$=volumetric strain in the porous skeleton; μ and ν=respectively, linear elastic shear modulus and Poisson's ratio of the porous fabric; $C_V=2k\mu(1+\nu)/[3\gamma_w(1-2\nu)]$, $\bar{K}=2\mu(1+\nu)/[(3(1-2\nu))]$; k=hydraulic conductivity of the porous medium; and γ_w=unit weight of water.

To complete the mathematical formulation of an initial boundary value problem, it is necessary and sufficient to prescribe boundary conditions and initial conditions applicable to the dependent variables u_i and p. The boundary conditions can be posed in terms of the conventional Dirichlet, Neumann, or Robin type boundary conditions (Selvadurai 2000a, b) applicable to u_i and p.

Continuum Damage Modeling

The theory of continuum damage mechanics has been widely applied to examine the mechanics of a variety of natural and artificial materials including metals, concrete, composites, ice, bone, and other geomaterials. The concept of damage mechanics as originally introduced by Kachanov (1958) accounts for the evolution of micromechanical defects including microcracks and microvoids in an originally "intact" material. These defects are assumed to occur without violating the continuum character of the overall behavior of the material. The local manifestation of damage results in the creation of new surfaces, which then alters the load transfer at the internal surfaces. In a general sense, the stress-induced damage will have preferred orientations that are associated with the stress state. Such a general description of the theory of continuum damage mechanics is now firmly established and advances and references to further studies are given by Krajcinovic (1984), Lemaitre and Chaboche (1990), Katti and Desai (1995), Voyiadjis et al. (1998), Wohua and Valliapan (1998a,b), Shao and Desai (2000), and Desai (2000). In general, the more elaborate the definition of damage, the larger the number of constitutive parameters associated with the resulting mathematical description will be. In the context of geomaterials and particularly naturally occurring geomaterials, there are advantages to maintaining the description of damage in a simple form. This allows for better estimation of parameter sensitivity and material variability particularly associated with geomaterials. Isotropic damage is a suitable model for geomaterials without any directional dependence in the mechanical properties and subjected to stress levels well below those that can lead to anisotropic damage. In the isotropic damage model, the single damage variable is D, which is related to the reduced net area of a surface $\bar{A}$ of original area A_0 according to $D=(A_0-\bar{A})/A_0$. For the undamaged material, $D=0$ and in the limit case of the material $D\rightarrow D_C$ that corre-

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sponds to rupture and irreversible deformations within the porous skeleton. In the limit when $D \to D_C$, the material response is bound to deviate from the elastic model that is adopted in the current study. It is therefore desirable to limit the range of applicability of the damage to a critical value D_C, which can be used as a normalizing parameter, against which levels of damage can be compared. In a geomaterial that experiences isotropic damage, the net stress tensor σ_{ij}^n is related to the stress tensor σ_{ij} in the undamaged state by

$$\sigma_{ij}^n = \frac{\sigma_{ij}}{1-D} \tag{2}$$

The deformability parameters applicable to an initially isotropic elastic material, which experiences isotropic damage, can be updated by adjusting the linear elastic shear modulus by its equivalent applicable to the damaged state, i.e.

$$\mu^d = (1-D)\mu \tag{3}$$

In the following, we examine the mechanics of poroelastic materials that exhibit isotropic damage through the introduction of a poroelastic model with a constitutive relationship of the form

$$\sigma_{ij} = 2(1-D)\mu\varepsilon_{ij} + \frac{2(1-D)\mu\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij} + p\delta_{ij} \tag{4}$$

where $\varepsilon_{ij}=(u_{i,j}+u_{j,i})/2$=infinitesimal strain tensor; $\varepsilon_{kk}=u_{i,i}$; μ=linear elastic shear modulus; ν=Poisson's ratio; and δ_{ij}=Kronecker's delta function. Implicit in Eq. (4) is the assumption that Poisson's ratio for the material experiencing damage is unaltered from its value applicable to the undamaged material. This assumption was put forward by Lemaitre (1984) in the strain equivalence hypothesis. In addition to the constitutive behavior defined by Eq. (4), it is also necessary to prescribe a damage evolution criterion that can be based on either micromechanical considerations or determined through experimentation. Experimental data on measurement of damage evolution are scarce; the limited data on sandstone were examined and they propose a damage evolution criterion that is defined by

$$\frac{\partial D}{\partial \xi_d} = \eta \frac{\alpha \xi_d}{1+\gamma\alpha_d}\left(1-\frac{D}{D_C}\right) \tag{5}$$

where ξ_d=equivalent, shear strain defined by

$$\xi_d = (e_{ij}e_{ij})^{1/2}, \quad e_{ij} = \varepsilon_{ij} - \tfrac{1}{3}\varepsilon_{kk}\delta_{ij} \tag{6}$$

and η,α=positive material constants. In this formulation, the normalizing damage measure is the critical damage D_C, which is associated with the damage corresponding to a limit value of the strength of the soft rock under uniaxial compression such that the deformations cannot be treated as a reversible elastic response and should be modeled by appeal to plasticity. It should be noted that the damage evolution function defined by Eq. (5) satisfies the second law of thermodynamics. The evolution of the damage variable can be obtained by the integration of Eq. (5) between limits D_0 and D, where D_0 is the initial value of the damage variable corresponding to the intact state (e.g., zero for materials in a virgin state). Integrating Eq. (5) between the limits, the evolution of D can be prescribed as follows:

$$D = D_C - (D_C - D_0)(1+\alpha\xi_d)^{\eta/\alpha D_C} \cdot \exp(-\eta\xi_d/D_C) \tag{7}$$

The development of damage criteria that can account for alterations in the hydraulic conductivity during evolution of damage in saturated geomaterials, is necessary for the modeling of such phenomena in poroelastic media. Literature on the coupling between microcrack developments and permeability evolution in saturated geomaterials is primarily restricted to the experimental evaluation of the alteration in permeability of geomaterials that are subjected to triaxial stress states. Zoback and Byerlee (1975) have documented results of experiments conducted on granite and Shiping et al. (1994) give similar results for tests conducted on sandstone (Fig. 2). These studies illustrate that the fluid transport characteristics of geomaterials can be increased due to evolution of damage in porous fabric. Kiyama et al. (1996) observed similar results for the permeability evolution in granites subjected to triaxial stress states, which suggests that localization phenomena and fluid pressure-induced microfracturing could result in significant changes in the permeability in the localization zones.

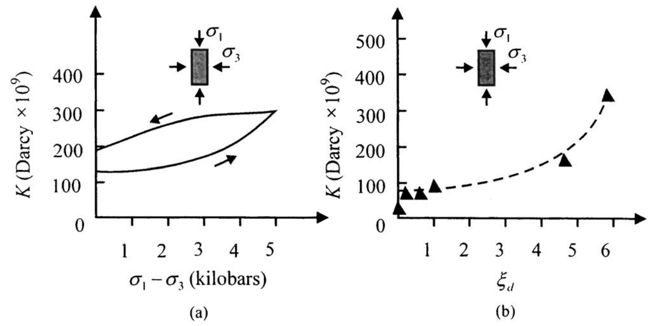

Fig. 2. Permeability evolution in saturated geomaterials: (a) after Zoback and Byerlee (1975); and (b) after Shiping et al. (1994)

Based on the experimental studies conducted by Shiping et al. (1994), which examine the damage-induced increase in the hydraulic conductivity of sandstone due to the applied shear strains, Mahyari and Selvadurai (1998) have obtained the following relationship for the dependency of hydraulic conductivity on the equivalent shear strain ξ_d, which takes the form

$$k^d = (1+\beta\xi_d^2)k^0 \tag{8}$$

where k^d=hydraulic conductivity applicable to the damaged material; k^0=hydraulic conductivity of the undamaged material; and β=material constant. Through Eq. (7), the dependency of the hydraulic conductivity on damage evolution is implied. A documentation on available experimental studies related to hydraulic conductivity evolution in fluid saturated geological porous media resulting from development of microdefects is given by Selvadurai (2004).

Computational Approach

Methods for the solution of the governing partial differential Eqs. (1) are diverse and these include analytical, finite difference, finite element, and boundary element methods. Advances in this area include the seminal articles by Sandhu and Wilson (1969), Ghaboussi and Wilson (1973), and summarized by Desai and Christian (1977), Schiffman (1985), Dominguez (1992), Detournay and Cheng (1993), Coussy (1995), Selvadurai (1996, 2001), Lewis and Schrefler (1998), and Smith and Griffiths (1998). The analytical, finite difference, and boundary element techniques are less flexible for the solution of the poroelasticity problems that incorporate influences of damage mechanics. Due to the incremental nature of damage development, it is convenient to employ the finite element method for the analysis of damage evolution in poroelastic materials. The development of finite element tech-

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niques for the study of the poroelasticity is now well established and details of these advances can be found in the references cited above. The basic Galerkin procedure can be applied to convert the governing partial differential equations to their matrix equivalents applicable to a finite domain. The resulting matrix equations take the form

$$\begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^T & \{-\gamma\Delta t\mathbf{H}+\mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t+\Delta t} \\ p_{t+\Delta t} \end{Bmatrix} = \begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^T & \{(1-\gamma)\Delta t\mathbf{H}+\mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_t \\ p_t \end{Bmatrix} + \{\mathbf{F}\} \tag{9}$$

where $\mathbf{K}$=stiffness matrix of the solid skeleton; $\mathbf{C}$=stiffness matrix due to interaction between the solid skeleton and the pore fluid; $\mathbf{E}$=compressibility matrix of the pore fluid; $\mathbf{H}$=permeability matrix; $\mathbf{F}$=force vector that includes external traction, body forces and flows; $\mathbf{u}_t$ and p_t, respectively, nodal displacements and pore pressure at time t; and Δt=time increment. The time-integration constant γ varies between 0 and 1. The criteria governing stability of the integration scheme given by Booker and Small (1975) requires that $\gamma \geqslant 1/2$. According to Selvadurai and Nguyen (1995) and Lewis and Schrefler (1998), the stability of the solution can be achieved by selecting values of γ close to unity.

An incremental three-dimensional finite element procedure has been developed to accommodate the damage-induced evolution of the elastic shear modulus and hydraulic conductivity of the geomaterial. Two approaches to the damage evolution problem are adopted; the first permits damage-induced alterations of the elasticity and hydraulic conductivity to evolve according to the criteria Eqs. (7) and (8) without consideration of the dependency of damage on the sense of the stress state (i.e., compressive/tensile first invariant of the strain tensor). The scalar damage variables are first obtained at the Gauss points within the finite elements. The elastic shear modulus μ^d and the hydraulic conductivity k^d are updated at these locations, at each time step, to account for evolution of damage. The discretized governing equations are then solved to obtain the state of strain at each integration point using the updated values of μ^d and k^d. An iterative process, using a standard Newton–Raphson technique, solves the coupling between the state of strain and the state of damage at each step. Details of the iterative procedure and the associated numerical algorithm are along the lines proposed by Mahyari and Selvadurai (1998) for the axisymmetric case. In the second approach we assume that the damage evolution is dependent on the sense of the stress state as defined by the various combinations of the principal strains. We therefore assume that damage can be initiated only when the material experiences dilatational strains. A variety of criteria can be adopted for this purpose. The work of Schulze et al. (2001), based on research conducted on rock salt, suggests that isotropic damage evolves when

$$I_1 = \text{tr}\,\varepsilon_{ij} > 0 \tag{10}$$

where I_1=first invariant of strain and tensile strains are considered to be positive. The basic computational algorithm used in the numerical computations according to this second criterion is shown in Fig. 3.

(I) Compute $I_1 = \varepsilon_{ii}$

(II) Check the criteria

$I_1 < 0$

No: Damage growth. Go to (III)

Yes: No further damage evolution. Use poroelastic parameters E, k with no more damage-induced modification. Go to (V).

(III) Compute D at Gauss integration points

$D = D_C - (D_C - D_0)(1+\gamma\xi_d)^{\eta/\gamma D_C} \exp(-\eta\xi_d / D_C)$

$\xi_d = (e_{ij}e_{ij})^{1/2}$, $e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}$

(IV) Update the poroelastic parameters

$\mu^d = (1-D)\mu$

$k^d = (1+\beta\xi_d^{\,2})k^0$

(V) Solve the governing equations for u_t, p_t and calculate the strain tensor

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$

(VI) Check the criteria

$D \geq D_C$

No: Damage growth. Return to (I).

Yes: No further damage evolution. Exit.

Fig. 3. Computational algorithm for stress-state-dependent evolution of damage in poroelastic medium

Laterally Loaded Rigid Rock Socket Problem

We now apply the computational procedures summarized previously to examine the idealized problem of a rigid cylindrical rock socket (length L and diameter d) that is embedded in bonded contact with a damage susceptible poroelastic half space. Rock sockets are generally regarded as short rigid piles. As an example, the maximum L/d ratio investigated by Rowe and Armitage (1987) is 4.0; and Leong and Randolph (1994) modeled L/d ratios [Whitworth and Turner (1989)] that ranged from 1 to 5. At the interface between the rigid rock socket and the poroelastic half space, several types of interface (boundary conditions) can be prescribed. Since complete bonding conditions are assumed, there is no provision for separation to develop at the interface. Consequently, the interface conditions relate only to the displacements and pore fluid pressures. In the present analysis, it is assumed that the displacements are continuous at the rigid rock socket–poroelastic medium interface. The pore pressure boundary condition at the interface largely depends on the method of installation of the rigid rock socket and the hydraulic conductivity characteristics of the rock socket in relation to the poroelastic medium. This particular boundary condition cannot be prescribed with certainty. For this reason, in the computational modeling, we assume that the interface can exhibit conditions corresponding to either completely pervious (i.e., fully draining) or completely impervious (i.e., undrained) pore pressure boundary conditions. The boundary conditions associated with drainage conditions can be treated as two limiting cases of the possible real drainage condition associated with a rock socket embedded in a damage-susceptible poroelastic half space. The rock socket and the poroelastic region are both modeled by 20-noded isoparametric solid elements. In order to model the rock socket as a rigid cylindrical element the modulus of elasticity of the rock socket region is considered to be 10^3 times larger than that of the undamaged poroelastic region. This enables the modeling of a nearly rigid rock socket and the relative rotation between the head (z=0) and the toe (z=L) of the rock socket along its axis less than 10^{-4} rd. The translational displacement of nodes in the rock socket are also checked at any time step to ensure that all nodes rotate in the same value and consequently, the rock socket acts as a rigid

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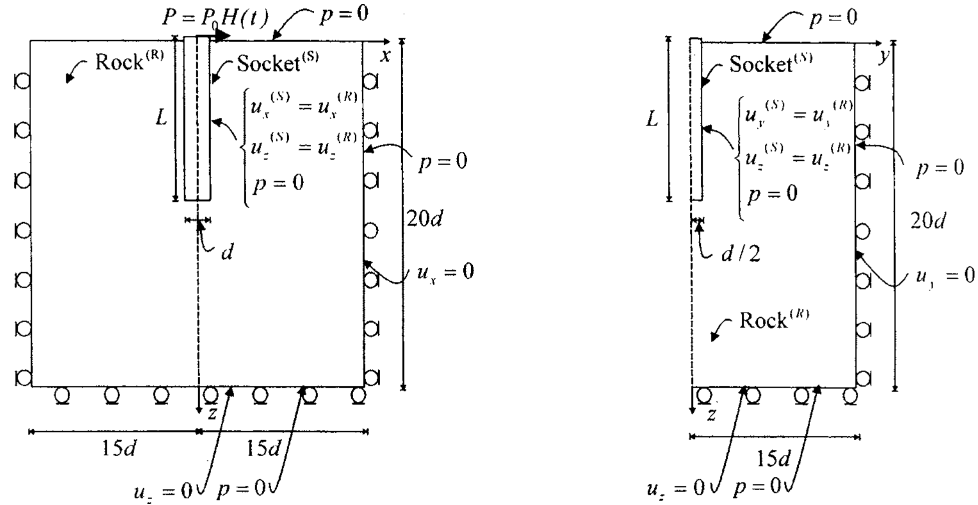

Fig. 4. Boundary conditions for pervious interface between rock socket and geomaterial

block. In the finite element modeling, the interface between the rock socket and the poroelastic region does not correspond to a precise cylindrical surface; the mesh division adopted ensures that the interface corresponds, approximately, to a cylindrical shape. The head of the rock socket is subjected to a lateral load directed along the x axis (Fig. 1) with a time dependency in the form of a Heaviside step function, i.e.

$$P(t) = P_0 H(t) \tag{11}$$

Since the poroelastic region is homogeneous, the problem exhibits a state of symmetry about the plane containing the line of action of the horizontal force $P(t)$. Attention is therefore restricted to modeling of the region $-15d \leqslant x \leqslant 15d$; $0 \leqslant y \leqslant 15d$; and $0 \leqslant z \leqslant 20d$, where d is the diameter of the rigid rock socket. The boundary conditions at the outer surfaces of the region correspond to the conventional zero normal displacement and zero shear traction conditions applicable to the porous skeletal response, and the pore fluid pressure boundary conditions are prescribed to be zero. The finite element discretization and the associated boundary conditions applicable to the two classes of interface conditions are shown in Figs. 4 and 5 The variables in the rigid rock socket problem examined in this paper include the following:

1. Geometry of the rigid rock socket (L/d);
2. Variability in the hydraulic conductivity evolution (k, β);
3. Elasticity parameters (E, ν);
4. Damage parameters (α, η, D_C);
5. Pore pressure boundary conditions at the rock socket–poroelastic medium interface (pervious/impervious); and
6. Damage governed by the sense of stress state (independent/dependent).

Admittedly, the consideration of the influence of all these parameters through a computational modeling exercise needs a significant number of separate numerical simulations. For this reason attention is restricted to the development of a specific set of results that will illustrate the influence of a pertinent number of the variables on the poroelastic response of the embedded rigid rock socket. We consider the following categories of problems related to a rigid cylindrical rock socket that is embedded in a poroelastic medium susceptible to damage. These include:

1. Purely poroelastic response of the medium (i.e., no elastic damage or hydraulic conductivity evolution);
2. Poroelastic response with damage evolution but with no al-

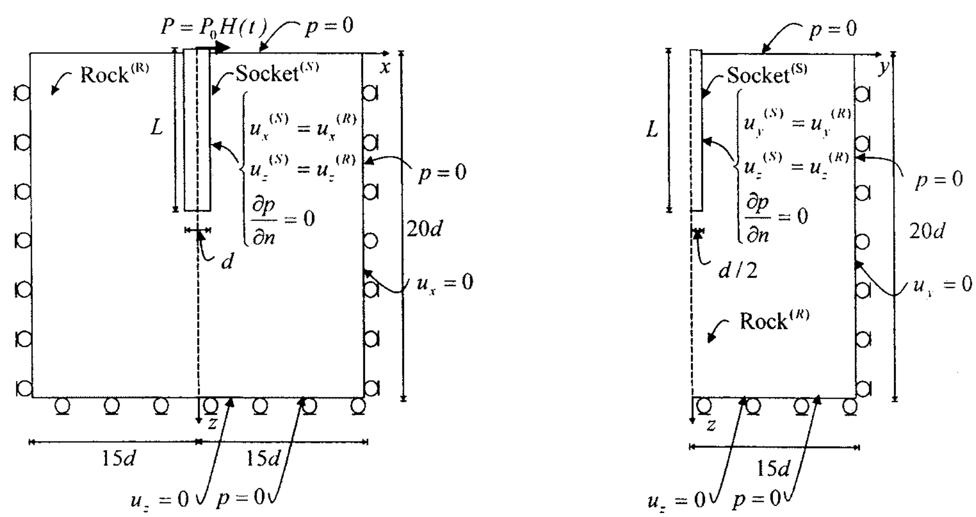

Fig. 5. Boundary conditions for impervious interface between rock socket and geomaterial

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Table 1. Comparison of Translational Stiffness of Rigid Rock Socket Subjected to Lateral Load and Elastic Behavior of Medium

	$2P_0/\mu d\Delta_h$		
Method of analysis	L/d=1.0	L/d=2.0	L/d=4.0
Present study	6.28	9.71	11.26
Selvadurai and Rajapakse (1985)	7.03	9.8	11.44

teration in the hydraulic conductivity characteristics;

3. Poroelastic response with both damage evolution and hydraulic conductivity alteration; and
4. Stress-state dependency of poroelastic response with both damage evolution and hydraulic conductivity alteration.

Numerical Results and Discussion

The idealized problem corresponds to a rigid rock socket that is embedded at the surface of a half space. In the computational modeling, however, the domain is restricted to a finite region. Therefore it is necessary to evaluate the accuracy of the finite domain used in the computational modeling in representing, approximately, a half-space region. To aid this evaluation we first examine the problem of a rigid rock socket that is embedded in an elastic half-space region. This problem has been examined by a number of investigators and a comprehensive mathematical treatment of the problem is given by Selvadurai and Rajapakse (1985). The results obtained by Selvadurai and Rajapakse (1985) for the rigid rock socket were compared with equivalent results obtained through a finite element modeling of the domain of finite extent. The results obtained, through the two schemes, for $2P_0/\mu d\Delta_h$, (where P_0 is the lateral load; μ is the linear elastic shear modulus; d is the rock socket diameter; and Δ_h is translational displacement of the head of the rock socket along the lateral load direction) for those different values of L/d are shown in Table 1. The results show reasonable agreement between the analytical and the computational estimates.

The computational modeling of a rigid rock socket (Fig. 1) embedded in brittle poroelastic medium susceptible to damage was conducted through the iterative finite element technique, the computational algorithm of which is shown in Fig. 3, with basic procedures that take into account damaged-induced alterations in both the elasticity, and the hydraulic conductivity characteristics. The porous skeletal material has a Poisson's ratio of ν and the pore fluid is assumed to be nearly incompressible, i.e., ν_u=0.499. We also assume that the damage evolution is well below the levels of damage corresponding the strain levels at the peak stress state. This excludes the necessity for consideration of any strain-softening effect. The theoretical basis of the computational scheme is therefore applicable to elastic states prior to the attainment of the peak stress (leading to failure) or the development of strain softening (postpeak). The material parameters used in the computations are those that are provided for sandstone by Shiping et al. (1994) and are as follows: E=8,300 MPa; ν=0.195; $\gamma=\eta=130$ (damage parameters); σ_C (compressive strength)=30 MPa; σ_T (tensile strength)=3 MPa; D_C=0.75 (critical damage variable); $k_0=10^{-6}$ m/s; and $\beta=3.0\times10^5$.

The finite element discretization of the three-dimensional domain containing the laterally loaded rigid rock socket is shown in Fig. 6. The computational modeling is performed for different length to diameter (L/d) ratios of the rigid rock socket. The nondimensional parameter, which is used to represent the transient

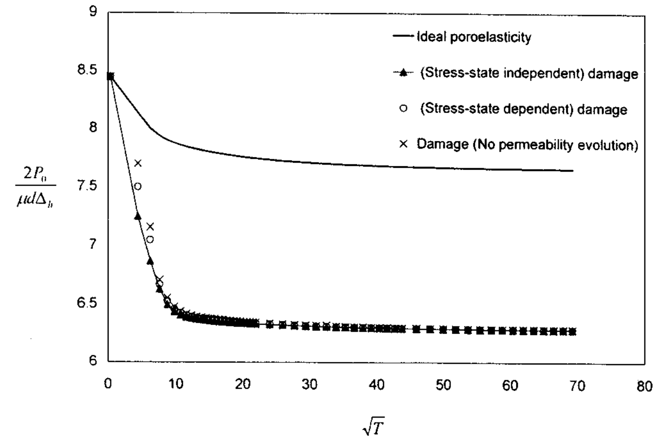

Fig. 7. Numerical results for transient translational displacement of rigid rock socket (L/d=1.0) embedded in brittle poroelastic half space (pervious interface)

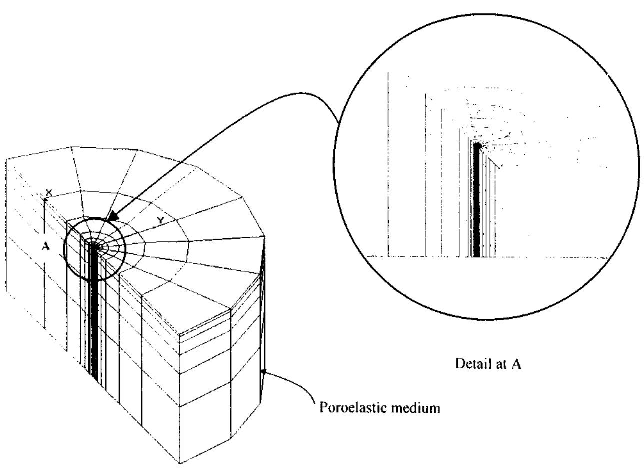

Fig. 6. Finite element discretization for rigid rock socket embedded in poroelastic half-space

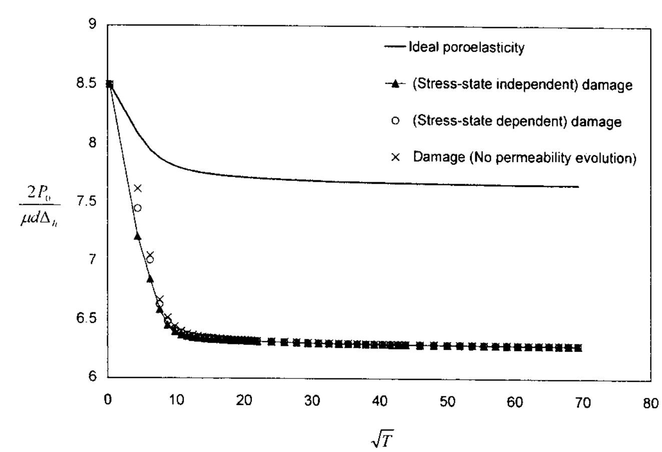

Fig. 8. Numerical results for transient translational displacement of rigid rock socket (L/d=1.0) embedded in brittle poroelastic half space (impervious interface)

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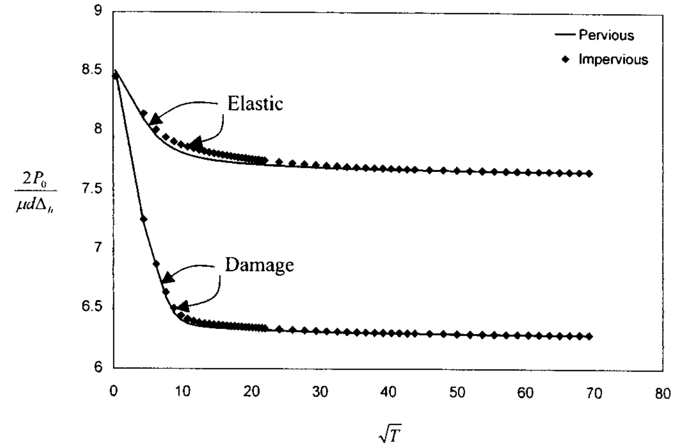

Fig. 9. Comparison of results for rigid rock socket with $(L/d=1.0)$ with either pervious or impervious interface between rock socket and poroelastic half space.

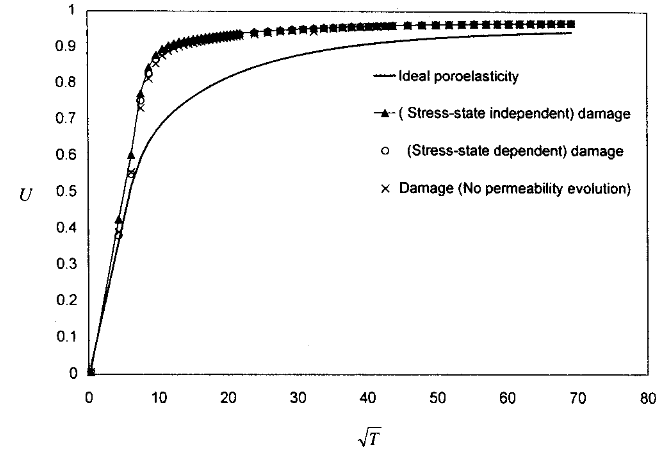

Fig. 11. Numerical results for degree of consolidation for rigid rock socket $(L/d=1.0)$ embedded in brittle poroelastic half space (impervious interface)

translational displacement of the rigid rock socket, is the same as that used by Selvadurai and Rajapakse (1985), and takes the form $2P_0/\mu d\Delta_h$. The time factor (T) is defined by

$$T=\frac{8\mu(1-\nu)k^0 t}{(1-2\nu)d^2} \tag{12}$$

The induced degree of consolidation of the rock socket as estimated from the rock socket head lateral displacement is given by

$$U=\frac{\Delta_h(t)-\Delta_h(0)}{\Delta_h(\infty)-\Delta_h(0)} \tag{13}$$

where $\Delta_h(t)$=translational displacement at the center of rock socket in the direction of applied load at any time t.

Fig. 7 illustrates the time-dependent translational displacement at the head of a rock socket with $L/d=1.0$, for the case where the interface between the rigid rock socket and the poroelastic medium is fully drained. The results presented in this figure relate to four categories of poroelastic response; namely, ideal poroelasticity that is void of any damage, poroelastic damage with only reduction in elasticity, stress-state-independent damage evolution with both alterations in elasticity and hydraulic conductivity characteristics, and stress-state-dependent damage evolution with alterations in both elasticity and hydraulic conductivity characteristics. The results show that damage-induced alterations in hydraulic conductivity of the brittle porous medium have a greater influence on the transient response of the rigid rock socket than the case of reduction in elastic stiffness without any alteration in the hydraulic conductivity. The numerical results for stress-state-dependent modeling of damage shows less of a difference between the ideal poroelastic case and the case that accounts for the damage-induced alterations in elasticity and hydraulic conductivity. This is most likely due to the development of a predominantly compressive state of stress within the porous medium, located at one side of the laterally loaded rigid rock socket, which restricts the development of damage according to the stress-state-dependent criteria for damage evolution defined by the constraint Eq. (10). Fig. 8 illustrates the time-dependent lateral displacement at the head of a rigid rock socket $(L/d=1.0)$, for the case where the interface between the rigid rock socket and the poroelastic medium is a fully draining interface. The results show the same trend for the impervious interface. Fig. 9 illustrates a comparison between the two cases of a pervious

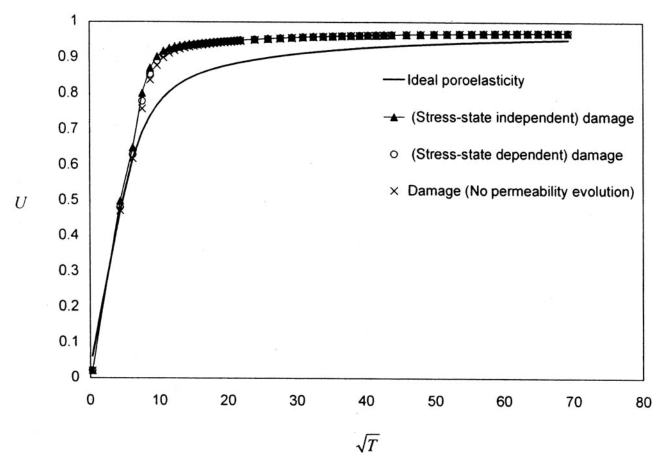

Fig. 10. Numerical results for degree of consolidation for rigid rock socket $(L/d=1.0)$ embedded in brittle poroelastic half space (pervious interface)

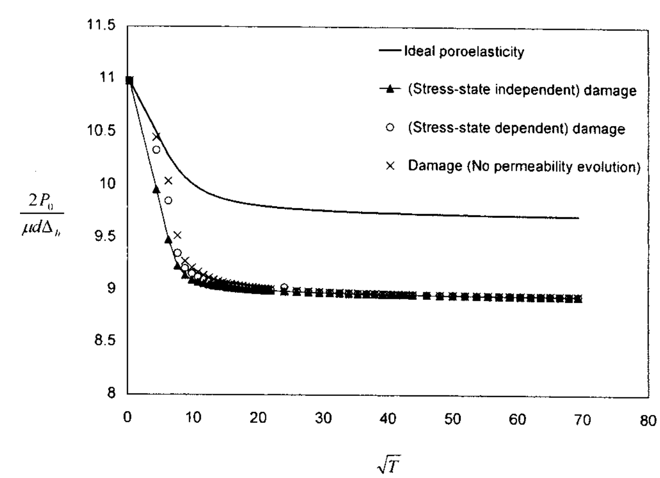

Fig. 12. Numerical results for transient translational displacement of rigid rock socket $(L/d=2.0)$ embedded in brittle poroelastic half space (pervious interface)

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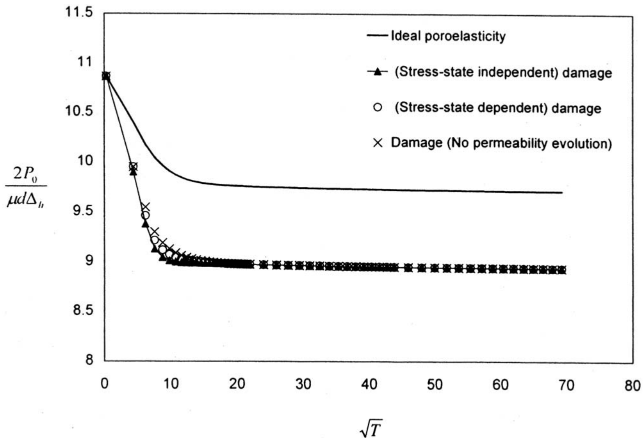

Fig. 13. Numerical results for transient translational displacement of rigid rock socket ($L/d=2.0$) embedded in brittle poroelastic half space (impervious interface)

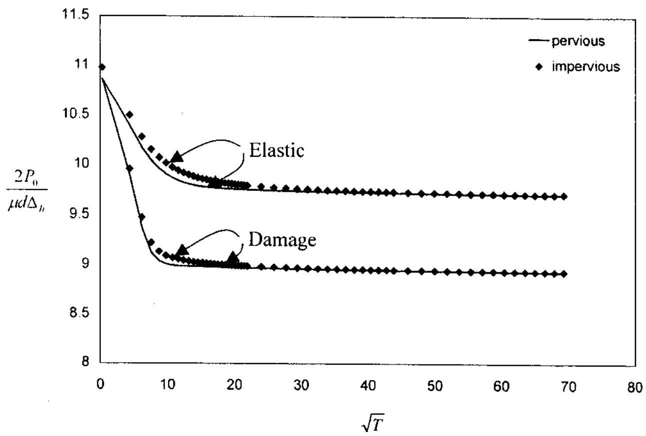

Fig. 14. Comparison of results for rigid rock socket with ($L/d=2.0$) with either pervious or impervious interface between rock socket and poroelastic half space

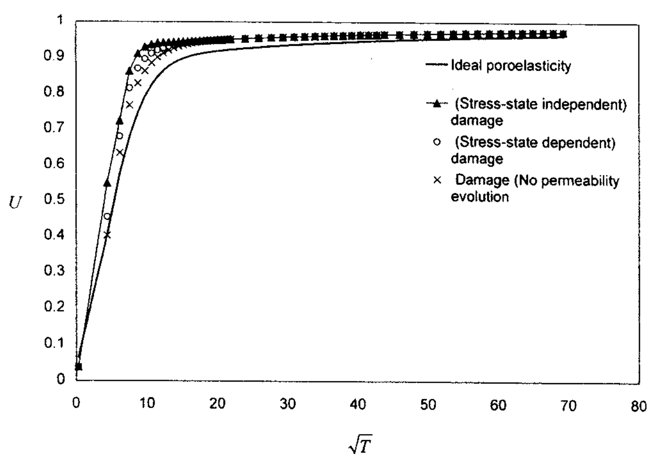

Fig. 15. Numerical results for degree of consolidation for rigid rock socket ($L/d=2.0$) embedded in brittle poroelastic half space (pervious interface)

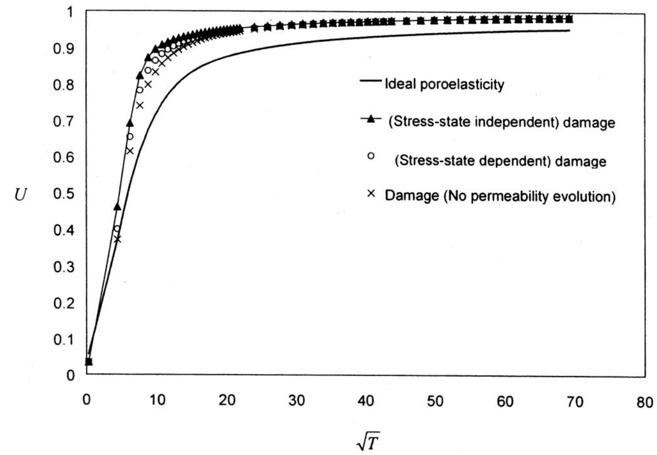

Fig. 16. Numerical results for degree of consolidation for rigid rock socket ($L/d=2.0$) embedded in brittle poroelastic half space (impervious interface)

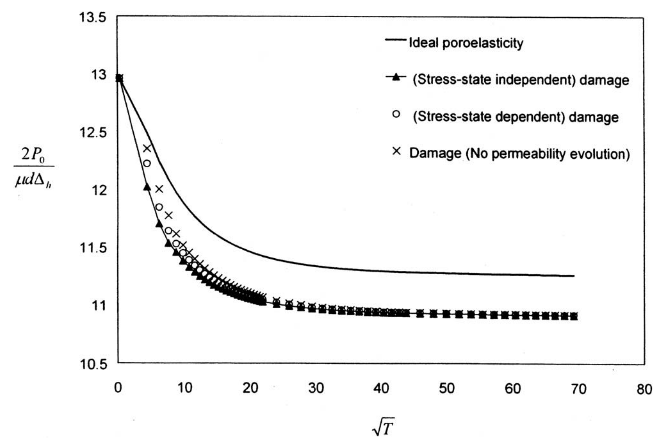

Fig. 17. Numerical results for transient translational displacement of rigid rock socket ($L/d=4.0$) embedded in brittle poroelastic half space (pervious interface)

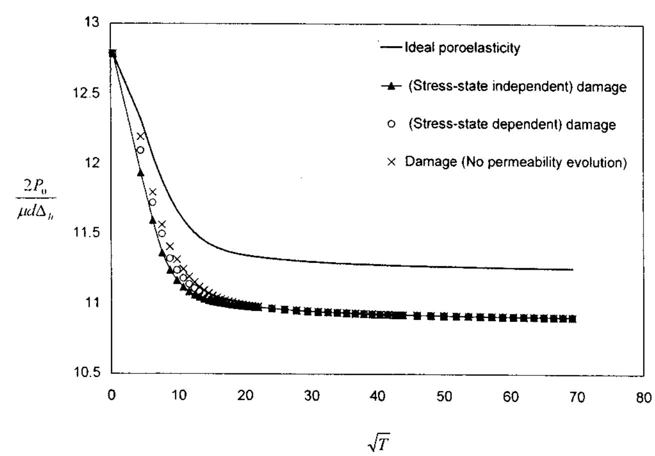

Fig. 18. Numerical results for transient translational displacement of rigid rock socket ($L/d=4.0$) embedded in brittle poroelastic half space (impervious interface)

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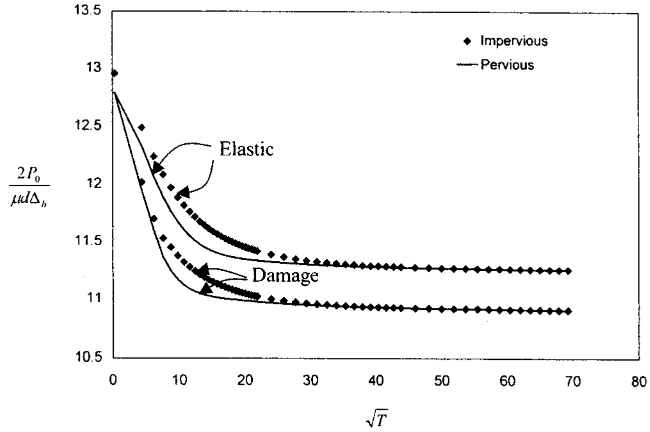

Fig. 19. Comparison of results for rigid rock socket with (L/d=4.0) with either pervious or impervious interface between rock socket and poroelastic half space

interface and an impervious interface for the rock socket geometry, L/d=1.0. The influence of damage-induced alterations in the case of the impervious interface is greater, but the overall change is not significant. This is due to the highly localized response between rock socket and poroelastic medium, which extends to a region substantially smaller in comparison to the surrounding poroelastic medium. As a result any changes in the pore pressure variations at the interface region have negligible effects in comparison to variations in the bulk of the poroelastic medium. The change can be attributed to slower rate of dissipation for the case of an impervious interface and any alteration in hydraulic conductivity characteristics influences the transient response at a greater rate. Figs. 10 and 11, respectively, illustrate the degree of consolidation for the pervious and impervious pore pressure boundary conditions at the interface of the rock socket and poroelastic half space. The rate of consolidation increases where the alterations in hydraulic conductivity characteristics are taken into consideration. In addition, for the case of stress-state-dependent damage evolution, less change has been observed. Figs. 12–21 illustrate identical results applicable to rock socket dimensions defined by L/d=2.0 and 4.0.

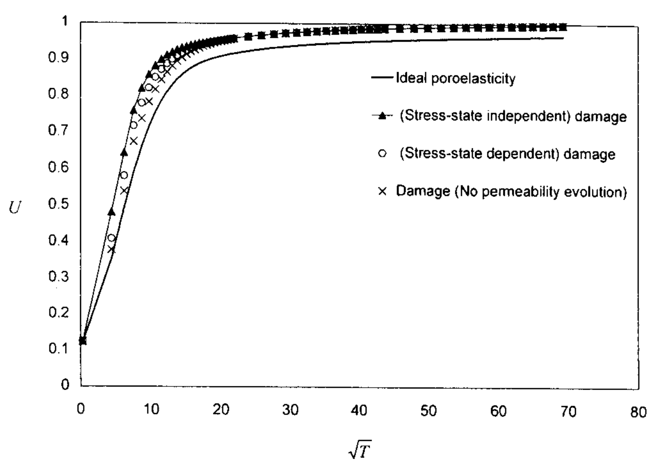

Fig. 20. Numerical results for degree of consolidation for rigid rock socket (L/d=4.0) embedded in brittle poroelastic half space (pervious interface)

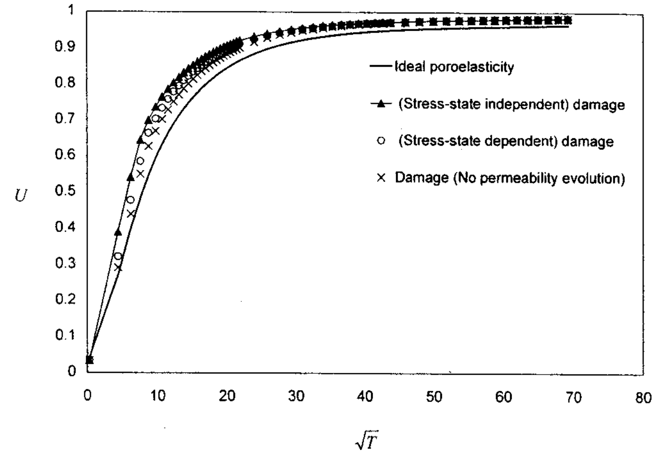

Fig. 21. Numerical results for degree of consolidation for rigid rock socket (L/d=4.0) embedded in brittle poroelastic half space (impervious interface)

Concluding Remarks

The classical theory of poroelasticity for a fluid saturated brittle geomaterial has been adopted to investigate the isotropic damage-induced alterations in both deformability and hydraulic conductivity parameters. An iterative finite element technique has been used to examine the influence of the isotropic damage-induced alterations in the hydraulic conductivity of the porous medium on the time-dependent response of a rigid rock socket, with different length to diameter ratios, embedded in a brittle poroelastic half space and subjected to a lateral load. Investigations have been carried out for cases involving both pervious and impervious pore pressure boundary conditions at the interface between rock socket and poroelastic medium. The numerical results presented in this paper examine both time-dependent transient translational displacement and the time-dependent degree of consolidation. The results of the computational modeling illustrate that the consideration of hydraulic conductivity alteration during the damage evolution process has a significant influence on the actual time-dependent translational displacement of the rock socket, whereas its influence on the degree of consolidation is marginal. This influence depends on the rock socket dimensions as defined by the length to diameter ratio. The larger this ratio, the greater the influence observed, and this also depends on the pore pressure boundary conditions at the interface of the rock socket and surrounding poroelastic half space. The effects are greater when the pore pressure boundary conditions correspond to an impervious interface. The dependency of the transient response on the stress state in the surrounding poroelastic half space also supports the above conclusions.

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