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An elliptical disc anchor in a damage-susceptible poroelastic medium

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SUMMARY

A flat rigid elliptical anchorage located in a damage-susceptible fluid-saturated poroelastic medium is subjected to an in-plane load, which induces a pure translation in the plane of the anchor. This paper develops computational estimates for the time-dependent displacement of the disc anchor for the classical problem that involves Biot consolidation and compares the results with situations where the porous skeleton can experience micro-mechanical damage that leads to an alteration in both its elasticity and fluid transport characteristics. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: damage mechanics; poroelastic damage; elliptical disc inclusion; Biot consolidation; hydraulic conductivity alteration; elastic stiffness alteration; in-plane indentation; degree of consolidation

1. INTRODUCTION

Anchorages are used to enhance the stability of earth retaining structures and slopes, to provide restraint against uplift forces acting on guyed towers and tall structures and for improving the stability of foundations and pipelines subjected to flotation, wave action and earth movements. The geotechnical study of anchors requires the estimation of both their ultimate load carrying capacity and their time-independent and time-dependent displacements under sustained loads particularly in the working range. In this paper, we are primarily concerned with the study

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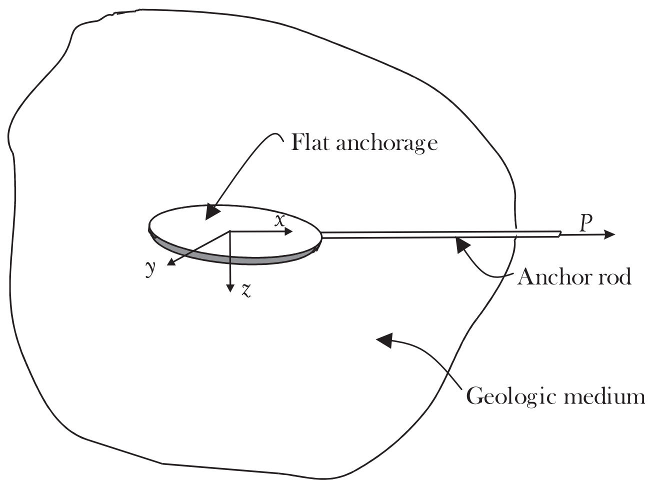

Figure 1. A rigid flat anchorage located in a geologic medium.

of the mechanics of *flat anchors* that are embedded in fluid-saturated poroelastic media that are susceptible to damage evolution during their loading in a sustained manner. A flat anchor is an idealized concept of an anchorage that can be created in a section of a borehole in a geologic medium by the pressurized injection of a cementitious fluid. The pressurization creates a hydraulic fracture zone, which allows for migration of the cementitious fluid and, upon setting, will serve as the anchorage. The orientation of the anchorage zone can be arbitrary and will depend on the *in situ* geostatic state of stress and the fracture characteristics of the geomaterial. The desirable flat anchorage is one that is oriented along the line of application of the anchor rod force (Figure 1).

Elastostatic flat anchor problems have been extensively studied in connection with the assessment of their time-independent behaviour in the working load range. Although these elastic solutions are not directly related to the topic of poroelastic geomaterials susceptible to micro-mechanical damage, the results can be used to examine the accuracy of computational schemes, including those developed for unbounded domains. It should also be noted that in a generalized elastic solution for an anchor problem, the case involving $v = 0.50$ (where v is Poisson's ratio) corresponds to the consolidation response as $t \to 0$ and the elastic solution with $v \to v'$ (where v' is Poisson's ratio for the porous skeleton) corresponds to the poroelastic response at the end of consolidation as $t \to \infty$. The earliest mathematical study of a disc embedded in an elastic infinite space is due to Collins [1], who presented an exact closed form solution to the problem of the axial loading of an embedded rigid circular disc. Keer [2] examined the problem related to the in-plane loading of a rigid circular disc located in an isotropic elastic infinite space. The study by Kassir and Sih [3] dealt with the problem of a rigid elliptical anchor plate or an inclusion embedded in an isotropic elastic medium and subjected to generalized force resultants at its centroid. More general approaches to the study of disc-shaped anchor problems were presented by Kanwal and Sharma [4] and Selvadurai [5], who examined problems related to spheroidal anchor regions embedded in elastic media, using singularity methods and spheroidal function techniques, respectively. From these solutions, the results for disc anchor problems and elongated needle-shaped anchor problems can be recovered as special cases. These studies have also been extended by Zurieck [6] to include the problem of a spheroidal anchor region located in a transversely isotropic elastic medium. Selvadurai [7] has given an extensive account of the disc anchor problem related to elastic media of infinite and semi-infinite extent. These studies also include considerations of transverse isotropy of the elastic medium, bi-material regions, elliptic geometry of the disc anchor, influence of boundary surfaces, etc.

The modern theory of poroelasticity commences with the work of Biot [8], which is based on the assumptions of Hooke's law for the elastic behaviour of the porous skeleton, and Darcy's law, for describing fluid flow through the porous skeleton. The linear theory has been applied to the study of time-dependent transient phenomena encountered in a wide range of natural and synthetic materials, including geomaterials and biomaterials [9–16]. The assumption of Hookean elastic behaviour and Darcy flow response of the porous skeleton, which will remain unaltered during loading of a poroelastic material, is recognized as a limitation of the classical theory. An alternative is to introduce concepts such as elasto-plasticity to account for irreversible effects in the behaviour of the porous skeleton. Such extensions, however, do not accurately model materials that maintain their poroelastic character but essentially give rise to alterations in the elasticity and fluid flow behaviour as a result of micro-mechanical damage-type processes mainly in the form of generation of micro-cracks and micro-voids in the porous fabric. The experimental basis for this assumption is provided in a number of investigations, admittedly without the advantages of identifying a direct relationship to a measure of damage. For example, experimental investigations by Cook [17] and Bieniawski *et al.* [18] all point to the load-induced degradation of elastic moduli of rocks. Further investigations related to sandstone and granite, are discussed by Cheng and Dusseault [19], Martin and Chandler [20] and Chau and Wong [21]. Of related interest are the fundamental studies conducted by Spooner and Dougill [22] who investigate, experimentally, the evolution of damage in concrete. Turning to fluid transport aspects, micro-crack-induced alterations of the hydraulic conductivity of saturated granite have been documented by Zoback and Byerlee [23], who observed increases of up to a factor of four in the magnitude of the permeability. Tests on sandstone conducted by Shiping *et al.* [24] indicate that for all combinations of the stress states employed in their tests, the permeability increased by an order of magnitude. Kiyama *et al.* [25] reported the results of triaxial tests on anisotropic granite, which also indicate increases in the permeability characteristics. Coste *et al.* [26] discuss the results of experiments involving rocks and clay stone and their conclusions support the observations concerning an increase in the permeability, of up to two-orders of magnitude, with an increase in deviator stresses. Souley *et al.* [27] describe the excavation damage-induced alterations in the permeability of granite of the Canadian Shield, where an approximately four-orders of magnitude increase in the permeability in the excavation damage zone is observed. It must be remarked that some of these measurements are applicable to stress states where there can be substantial deviations from the elastic response of the material because of generation of localized shear zones and foliation-type extensive brittle fracture. In a recent study, Bossart *et al.* [28] observed an alteration of the permeability characteristics of an argillaceous material around deep excavations, a phenomenon attributed to the alteration of the properties of the material in the excavation damage zone.

Using a damage mechanics approach, the influences of the deterioration of the elasticity characteristics and enhancement of the hydraulic conductivity characteristics in the porous medium can be accounted for in a phenomenological sense. Such a theory is considered suitable for describing the mechanical behaviour of brittle elastic solids well in advance of the development of macro-cracks (e.g. fractures) or other irreversible phenomena (e.g. plasticity effects). Experimental studies by Samaha and Hover [29] indicate an increase in the permeability of concrete subjected to compression and the work of Gawin *et al.* [30] examines the thermo-mechanically induced damage of concrete at high temperatures, and propose empirical relationships to describe the alterations in the fluid transport characteristics as a function of temperature and damage, the dominant agency responsible for the alterations is thermally induced generation of micro-cracks and fissures. Bary *et al.* [31] also present experimental results that point to the evolution of permeability of concrete subjected to axial stresses; these studies were conducted in connection with the modelling of concrete gravity dams that are subjected to fluid pressures.

It should be noted that the poroelastic material itself has a pore space; the damage-induced micro-voids and micro-cracks are assumed to be sparse and substantially larger than the characteristic dimension associated with the pore space in the undamaged material. Furthermore, the material alterations resulting from the damage process are likely to be highly anisotropic and could conceivably be restricted to localized zones. In this study, however, damage is treated as a phenomenological process, resulting from the reduction in the elastic stiffness and enhancement of the fluid conductivity characteristics due to generation of micro-voids and other micro-defects. Cheng and Dusseault [19] developed an anisotropic damage model to examine the poroelastic behaviour of saturated geomaterials. Mahyari and Selvadurai [32] considered a computational modelling of an axisymmetric poroelastic contact problem where both elastic stiffness reduction and linear and quadratic variations hydraulic conductivity alteration during damage evolution are considered. Recently, Selvadurai and Shirazi [33] and Shirazi and Selvadurai [34] have extended the procedures proposed in Reference [32] to examine both axi symmetric and three-dimensional problems related, respectively, to fluid inclusions embedded in a damage-susceptible poroelastic medium and the problem of the indentation of a poroelastic half-space by a rigid cylindrical punch with a smooth flat base. Selvadurai [35] has recently introduced the concept of *stationary damage*, which could be used as a simpler alternative to the modelling of damage-induced consolidation of poroelastic media, where the elastic damage that initially occurs during the commencement of application of the external loads persists throughout the ensuing transient poroelastic phase. The work of Bary *et al.* [31] also examines the coupling effects of damage on hydro-fracturing of concrete and the studies by Gawin *et al.* [30, 36] investigate the influence of thermal effects and the coupling effect to permeability alterations, with specific references to high temperature loading of partially saturated concrete.

2. POROELASTICITY WITH DAMAGE EVOLUTION

A fundamental assumption of the classical theory of poroelasticity as proposed by Biot [8], is that the porous structure remains intact during time-dependent deformations of the poroelastic skeleton. This assumption is satisfied in many instances where the stress levels in the porous skeleton do not induce appreciable alterations in its constitutive responses. Extreme examples of alterations in the poroelastic behaviour can include the development of irreversible plastic deformations in regions of the porous soil skeleton to the development of discrete fractures, shear bands and discontinuities. These material alterations can introduce highly localized non-linear phenomena, which can be accompanied by significant alterations in the fluid transport characteristics of the medium, particularly in the failure zones. In this paper, we deal primarily with geomaterials that maintain their *predominantly elastic behaviour*, but experience some degree of *micro-mechanical damage*. A result of such damage can be the inhomogeneous alteration of the elastic behaviour of the porous skeleton and the attendant alteration in the material properties of the medium. Alterations of material properties can occur in zones of a poroelastic medium that experience continuum damage according to a prescribed damage evolution law. The mechanism that is most likely to lead to alterations in both the deformability characteristics and the permeability characteristics of a poroelastic medium, and at the same time maintain the elastic nature of the geomaterial skeletal response, is the creation of voids or cavities within the skeleton. The process of cavity generation in porous media is a phenomenological concept introduced to simplify a rather complicated set of processes associated with cavity nucleation, cavity expansion, cavity coalescence, etc. The creation of cavities is also largely governed by the stress state in the geomaterial skeleton, and the defects themselves can exhibit a directional dependence. The experimental evidence to date is insufficiently refined to support the development of any sophisticated theory to account for *directional dependency* in the defect evolution in a porous geomaterial fabric. Furthermore, the evolution of defects will occur in a porous medium that already has a void component. The meaningful introduction of a directional dependency in the voids created by damage can therefore be accomplished only by considering the characterization of the initial anisotropy of the pore space. The experimental verification of the pore space anisotropy is a difficult and non-routine procedure. The simplest approach for introducing the alterations in the elasticity and hydraulic conductivity of an initially porous geomaterial is to resort to the theory of *continuum damage mechanics*. The introduction of fundamental concepts of continuum damage mechanics is attributed to Kachanov [37], and over the past four decades these concepts have been widely applied to model degradation phenomena in a wider class of engineering materials. The coupling of elasticity and damage modelling has been discussed by a number of researchers including, Bazant [38], Simo and Ju [39], Chow and Wang [40], Lemaitre and Chaboche [41], Selvadurai and Hu [42], Wohua and Valliappan [43, 44], Krajcinovic [45] and Valliappan *et al.* [46, 47]. While these studies develop and apply the theory of damage mechanics in a general sense, for the purposes of the paper and owing to limited data available on the experimental characterization of anisotropic damage evolution in geomaterials, attention is focused on *isotropic damage* defined by a scalar damage variable $D(=1-\{\bar{A}/A_0\})$, where $\bar{A}$ is the surface area of a damaged region, the original surface area of which was A_0. Theoretically, the damage variable can vary between zero and unity. In terms of application to the poroelastic media it is necessary to identify the damage level D_c, which is the critical value corresponding to the development of fracture of the material. Restricting the damage development in the poroelastic medium to satisfy the constraint $D \ll D_c$, ensures that the assumption of poroelastic behaviour is satisfied. The critical damage parameter can therefore be viewed as a normalizing parameter against which damage evolution can be estimated. Admittedly, this requirement will be violated in locations where the stress fields in the poroelastic skeleton are singular. The justification for maintaining the poroelastic behaviour of the damaged material with local singular stress states is in the same spirit as the assumption of classical poroelasticity or even classical elasticity in the presence of singular stress fields. For isotropic damage, the net stress dyadic $\boldsymbol{\sigma}^n$ is related to the stress dyadic in the undamaged state $\boldsymbol{\sigma}$ according to

$$\boldsymbol{\sigma}^n = \frac{\boldsymbol{\sigma}}{(1-D)} \tag{1}$$

When considering the constitutive behaviour of a damaged poroelastic material that experiences isotropic damage, it is necessary to consider the damage-induced evolution of both elastic constants that describe the resulting isotropic material. In this paper, however, we shall restrict the discussion to situations where Poisson's ratio for the material remains at the initial value during isotropic damage evolution. This is an approximation resulting from the '*strain equivalence hypothesis*' proposed by Lemaitre [48]. There are alternatives to the strain equivalency hypothesis and an example of such a development, which makes use of energetic arguments, is given by Wohua and Valliappan [43, 44] and Valliappan *et al.* [46, 47]. Considering the strain equivalent hypothesis, the constitutive equation for the poroelastic medium with an isotropically damaged fabric can be written as

$$\boldsymbol{\sigma} = 2(1-D)\mu\boldsymbol{\varepsilon} + \frac{2(1-D)\mu v}{(1-2v)}(\nabla\cdot\mathbf{u})\mathbf{I} + \alpha(D)p\mathbf{I} \tag{2}$$

where $\boldsymbol{\sigma}$ is the total stress dyadic, p is the pore fluid pressure, ζ_v is the volumetric strain in the compressible pore fluid; v and μ are the 'drained' values of Poisson's ratio and the linear elastic shear modulus applicable to the porous fabric prior to damage evolution, $\mathbf{I}(=\mathbf{ii}+\mathbf{jj}+\mathbf{kk})$ is the unit dyadic. Also in (2), $\boldsymbol{\varepsilon}$ is the soil skeletal strain dyadic, defined by

$$\boldsymbol{\varepsilon} = \tfrac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla) \tag{3}$$

where $\mathbf{u}$ is the displacement vector and ∇ is the gradient operator and the parameter $\alpha(D)$, which defines the compressibility of the pore fluid, is given by

$$\alpha(D) = \frac{3(v_u - v)}{\tilde{B}(D)(1-2v)(1+v_u)} \tag{4}$$

and $\tilde{B}(D)$ is the pore pressure parameter introduced by Skempton [49], which now depends on the level of damage, because of the dependency of the compressibility of the porous skeleton on the shear modulus of the porous skeleton.

In the absence of body forces, the quasi-static equations of equilibrium for the complete fluid-saturated porous medium take the form

$$\nabla\cdot\boldsymbol{\sigma} = \mathbf{0} \tag{5}$$

The velocity of fluid transport within the pores of the medium is governed by Darcy's law

$$\mathbf{v} = -\kappa(D)\nabla p \tag{6}$$

where $\mathbf{v}$ is the vector of fluid velocity and $\kappa(D)$ $(=k(D)/\gamma_{\mathrm{w}})$ is the permeability (expressed in m^2), which is assumed to be dependent on the level of isotropic damage, and is related to the hydraulic conductivity $k(D)$ and the unit weight of the pore fluid γ_{w} as indicated above. The mass conservation equation for the pore fluid can be stated as an equation of continuity, which, for quasi-static flows can be stated in the form

$$\frac{\partial\zeta_v}{\partial t} + \nabla\cdot\mathbf{v} = 0 \tag{7}$$

Considering the thermodynamic requirements for a positive definite strain energy potential (see, e.g. Reference [50]) it can be shown that the material parameters should satisfy the following constraints:

$$\mu > 0; \quad 0 \leqslant \tilde{B}_0 \leqslant 1; \quad -1 < v < v_u \leqslant 0.5; \quad \kappa_0 > 0 \tag{8}$$

where, $\tilde{B}_0$ and κ_0 are, respectively, Skempton's pore pressure parameter and permeability for the undamaged poroelastic material. Since in the damaged state $(1-D) > 0$, these thermodynamic constraints also extend to the response of the damaged poroelastic medium at levels of damage where $D < 1$. The governing equations for the displacement vector $\mathbf{u}$ and the scalar pore fluid pressure p at any level of isotropic damage can be reduced to the following forms:

$$\mu(1-D)\nabla^2\mathbf{u} + \frac{\mu(1-D)}{(1-2v)}\nabla(\nabla\cdot\mathbf{u}) + \alpha(D)\nabla p = \mathbf{0} \tag{9}$$

and

$$\kappa(D)\beta(D)\nabla^2 p - \frac{\partial p}{\partial t} + \alpha(D)\beta(D)\frac{\partial}{\partial t}(\nabla\cdot\mathbf{u}) = 0 \tag{10}$$

where

$$\beta(D) = \frac{2\mu(1-D)(1-2v)(1+v_u)^2}{9(v_u - v)(1-2v_u)} \tag{11}$$

The mathematical formulation of the initial boundary value problem posed by (9) and (10), is made complete by prescribing boundary conditions and initial conditions applicable to the dependent variables $\mathbf{u}$ and p. The boundary conditions can be interpreted in terms of the conventional Dirichlet, Neumann or Robin boundary conditions. At any level of damage, $D < 1$, systems of partial differential equations governing quasi-static poroelasticity are of the elliptic-parabolic type. The availability of a uniqueness of theorem [51] for the classical poroelasticity problem, ensures that the initial boundary value problem posed by (9) and (10) is also *well-posed* in a Hadamard sense [52].

To complete the description of the poroelastic medium with damage evolution, it is necessary to specify the nature of evolution of the two material parameters, namely, the shear modulus and the permeability of the porous skeleton with the level of damage. In this regard, two approaches are usually advocated. The first approach attempts to relate the evolution of the new surface area corresponding to a level of damage through considerations of the micro-mechanical processes including void generation, void expansion and void coalescence. The second approach is phenomenological and considers the development of the functional dependence through examination and analysis of experimental data. The first is a more complex approach, which requires knowledge of the precise distribution of the pore space in the intact material, the specification of criteria for void nucleation and at most their expansion. This approach is perhaps best suited for fabricated materials where the micro-level criteria can be determined by recourse to further experimentation. In the opinion of the authors, this approach has limitations with regard to its application to natural geologic media where the determination of these secondary level criteria presents considerable experimental difficulties. For this reason, the focus of the application of damage mechanics concepts to fluid-saturated porous media has largely relied on the determination of the functional dependence of both the shear modulus and permeability on damage through consideration of experimental data. For example, based on a review of the results of experiments conducted on rocks, it has been shown by

Cheng and Dusseault [19] that isotropic damage evolution, in terms of the dependency of the damage parameter on the strain in the porous skeleton, can be expressed through a relationship of the form

$$\frac{\partial D}{\partial \xi_d} = \eta \frac{\psi \xi_d}{(1+\xi_d)} \left(1 - \frac{D}{D_c}\right) \tag{12}$$

where η and ψ are positive material constants and ξ_d is related to the second invariant of the deviator strain dyadic and given by

$$\xi_d = \tfrac{1}{2} [(\mathrm{tr}\,\mathbf{e})^2 - \mathrm{tr}\,\mathbf{e}^2], \quad \mathbf{e} = \boldsymbol{\varepsilon} - \tfrac{1}{3}\,\mathrm{tr}\,\boldsymbol{\varepsilon}\mathbf{I} \tag{13}$$

and D_c is a critical damage parameter, used for the purposes of normalization. The evolution of the damage variable can be obtained through an integration of (12) between limits D_0 and D where D_0 is the initial value of the damage variable corresponding to the intact state. (e.g. D_0 is zero for materials in a virgin state.) Using the result (12), the evolution of D can be prescribed as follows:

$$D = D_c - (D_c - D_0)(1 + \psi \xi_d)^{\eta/\psi D_c} \exp(-\eta \xi_d / D_c) \tag{14}$$

The constants η, ψ defined in (12) describing damage evolution can also be obtained by correlating the damage variable D obtained from the stress–strain curve for a brittle geomaterial with respect to the equivalent shear strain.

When damage in poroelastic media results in the creation of defects such as micro-cracks and micro-voids, it is likely that these defects will also lead to alterations in the permeability characteristics of the poroelastic medium. The literature on permeability evolution in porous media tends to focus on experimental evaluation of the alteration in the permeability of geomaterials subjected to conventional triaxial stress states. Results of permeability experiments conducted on granite and sandstone are documented by Zoback and Byerlee [23] and Shiping *et al.* [24], respectively. The experimental results given by Kiyama *et al.* [25] also indicate permeability alterations in granite with an increase in stress/strain; however, it is not clear whether such observations could be attributed to alterations occurring predominantly in strain localizations zones. In the studies conducted by Mahyari and Selvadurai [32] to examine the indentation of a damage-susceptible poroelastic half-space by a rigid cylindrical indenter with a flat base, the following expression was proposed for the evolution of permeability κ as a function of the parameter ξ_d;

$$\kappa(D) = (1 + \chi \xi_d^2)\kappa_0 \tag{15}$$

where χ is a constant and κ_0 is the permeability of the undamaged material. The quadratic dependency of the permeability on ξ_d is a plausible relationship that can be substantiated particularly through the results for sandstone given by Shiping *et al.* [24]. The result (15) is adopted here strictly for purposes of conducting the exercise in computational modelling.

3. COMPUTATIONAL MODELLING

The finite element method has been widely applied for the analysis of problems in the classical theory of poroelasticity and complete discussions of the computational aspects of poroelasticity are given by Sandhu and Wilson [53], Ghaboussi and Wilson [54], Booker and Small [55], Lewis and Schrefler [13] and Selvadurai [10]. The Galerkin approximation technique is applied to transform the partial differential equations into a discretized matrix form giving rise to the following time-incremental forms of the matrix equations for the partial differential equations governing poroelastic media:

$$\begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \{-\gamma\Delta t\mathbf{H}+\mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t+\Delta t} \\ \tilde{p}_{t+\Delta t} \end{Bmatrix} = \begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \{(1-\gamma)\Delta t\mathbf{H}+\mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t} \\ \tilde{p}_{t} \end{Bmatrix} + \{\mathbf{F}\} \tag{16}$$

where $\mathbf{K}$ = stiffness matrix of the porous skeleton; $\mathbf{C}$ = stiffness matrix due to interaction between porous skeleton and pore fluid; $\mathbf{E}$ = compressibility matrix of the pore fluid; $\mathbf{H}$ = hydraulic conductivity matrix; $\mathbf{F}$ = force vectors due to external traction; $\mathbf{u}_t$, $\tilde{p}_t$ = nodal displacements and pore pressures at time t; Δt is a time increment, γ is a time-integration constant and $(\,)^{\mathrm{T}}$ denotes the transpose.

The components of the coupling matrix [$\mathbf{C}$] due to the interaction between the porous skeleton and the pore fluid are given by the following:

$$[\mathbf{C}] = \alpha \int_R \frac{\partial N_I^u}{\partial x_i} N_K^p \,\mathrm{d}R \tag{17}$$

where N_I^u and N_K^p are the appropriate shape functions. The stiffness matrix [$\mathbf{K}$] applicable to the porous skeleton takes the form:

$$[\mathbf{K}] = \int_R [\mathbf{B}]^{\mathrm{T}}[\mathbf{D}][\mathbf{B}]\,\mathrm{d}R \tag{18}$$

where [$\mathbf{D}$] is the stiffness matrix corresponding to isotropic elastic behaviour of the porous skeleton. For a material that exhibits isotropic linear elastic response, the stiffness matrix [$\mathbf{D}$] depends only on the elastic constants μ and v. The strains are related to the nodal displacements through the matrix [$\mathbf{B}$], which depends on the shape functions N_I^u. The matrix [$\mathbf{H}$] takes the form:

$$[\mathbf{H}] = \int_R \frac{\partial N_I^p}{\partial x_i} \frac{k_{ij}}{\gamma_w} \frac{\partial N_K^p}{\partial x_j}\,\mathrm{d}R \tag{19}$$

The compressibility matrix for the pore fluid [$\mathbf{E}$] takes the form

$$[\mathbf{E}] = \int_R N_I^p \frac{1}{\beta} N_K^p \,\mathrm{d}R \tag{20}$$

and β is defined by (11). The time-integration constant γ varies between zero and one. The criterion governing stability of the integration scheme given by Booker and Small [55] requires that $\gamma \geqslant 1/2$. Investigations documented by Lewis and Schrefler [13] and Selvadurai and Nguyen [56] suggest that the stability of the solution can be achieved by selecting values of γ close to unity.

To analyse the poroelasticity problem that incorporates influences of damage evolution, an incremental finite element procedure needs to be developed. An example of such a development is given by Mahyari and Selvadurai [32]. Their procedure accounts for the alterations in both the linear elastic shear modulus $\mu(D)$ and the permeability $\kappa(D)$. Although formal similarities between the hypoelastic non-linear approach [57, 58] and the damage-induced degradation of the elasticity properties can be identified, the procedures used here are within the context of a *transient coupled fluid flow-mechanical deformation* problem, which requires the evaluation of both stress- and time-dependent evolution of both the elasticity and fluid transport characteristics of the medium as a function of a single scalar damage variable. In addition to the iterative procedures identified by Mahyari and Selvadurai [32], we also identify two approaches for accommodating the influences of the stress state on damage evolution. The first considers the damage-induced evolution of the elasticity and hydraulic conductivity to be *independent of the* '*sense*' of the stress state as defined by the compressive or tensile character of the principal stresses, and governed by the damage and permeability evolution criteria described previously by (14) and (15), respectively. At each time step, the constitutive matrix $\mathbf{C}$ and the permeability κ in (16) are updated at the integration points to account for the evolution of damage. The discretized forms of the governing equations are then solved to obtain the state of strain at each integration point, using these updated values, denoted by $\mathbf{C}(D)$ and $\kappa(D)$, respectively. The iterative procedure uses a standard Newton–Raphson technique to solve the coupling between the state of strain and the extent of damage at each increment. The convergence criterion adopted in the analysis is based on the norm of the evolution of the damage variable in relation to a prescribed tolerance, as suggested by Simo and Ju [39]. Details of the iterative procedure and the associated numerical algorithm are summarized by Mahyari and Selvadurai [32]. In the second approach, we assume that the damage evolution will be governed by *the sense of stress state* as defined by the various combinations of the principal strains. For example, it is foreseeable that the process of void and defect development in the poroelastic fabric is enhanced when the triaxial stress state is tensile and that such effects can be suppressed when the stress state is compressive. Other combinations of principal stresses, involving tensile and compressive stresses can lead to different levels of damage evolution. As with the development of yield criteria for geomaterials, the characterization of stress-state-dependent damage requires the experimental determination of the material responses to differing stress or strain paths. Experimental verifications of the stress-state-dependent damage evolution in geomaterials are scarce, in the sense that the database is insufficient to develop a comprehensive theory applicable to the class of brittle geomaterials of particular interest to this paper. The limited data suggest that damage can increase when the material experiences a volume expansion [59]. A possible approximation is to assume that damage will initiate only when the strain state satisfies the criterion

$$I_1 = \operatorname{tr} \boldsymbol{\varepsilon} > 0 \tag{21}$$

where tensile strains are considered positive. These two procedures can be regarded as two limiting responses associated with the stress-state-dependent damage evolution processes.

4. THE ELLIPTICAL FLAT ANCHOR PROBLEM

We consider the problem of a rigid flat anchor with an elliptical plan form, which is embedded in bonded contact with a fluid-saturated poroelastic medium the porous skeleton of which can experience isotropic micro-mechanical damage. The anchor is subjected to an in-plane force along one of its axes of symmetry, which causes a rigid body translation along the line of action of the force. The applied force has a time dependency in the form of a Heaviside step function.

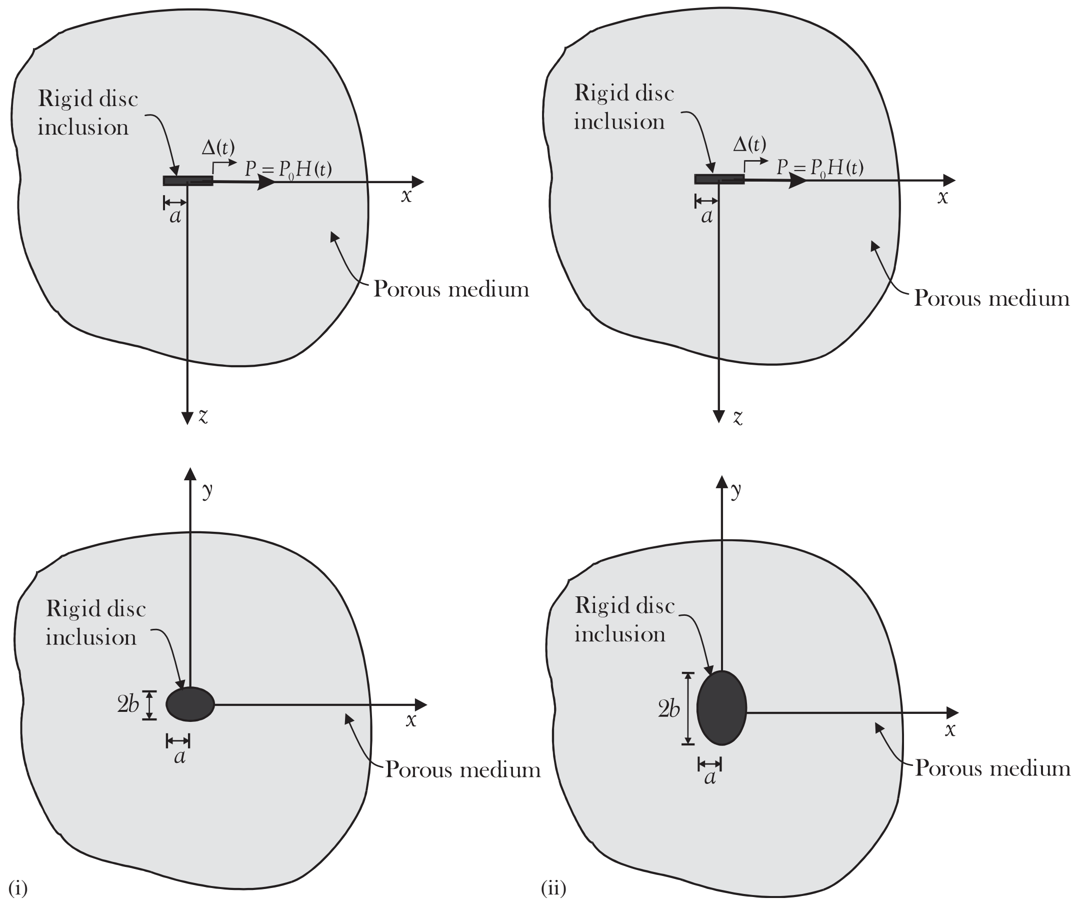

Figure 2. The geometry and loading of the rigid elliptical flat anchor embedded in a damage-susceptible poroelastic medium: (i) $b/a<1$; and (ii) $b/a>1$.

The elliptical plan form is chosen such that a wide range of anchor geometries of practical interest can be examined through the computations (Figure 2). Owing to the symmetry associated with the loading of the rigid elliptical disc anchor, the computational modelling can be restricted to a sub-domain of the damage-susceptible poroelastic medium where appropriate displacement, shear traction and pore pressure boundary conditions are imposed on the planes of symmetry. The flat anchor region is modelled as a purely elastic medium with sufficiently high elastic modulus to ensure that the anchor displacements correspond to those of a rigid domain. In the computations, the maximum strain within the anchor region of thickness $0.05a$ is approximately 50% of the maximum strain in the damage-susceptible poroelastic medium. The interface between the poroelastic medium and the flat anchor region is considered impervious. The pore fluid pressures at the external boundary of the domain are specified to be zero and the boundary conditions on the exterior surfaces correspond to zero normal displacements and zero shear tractions. The computational modelling is intended to simulate the time-dependent response of

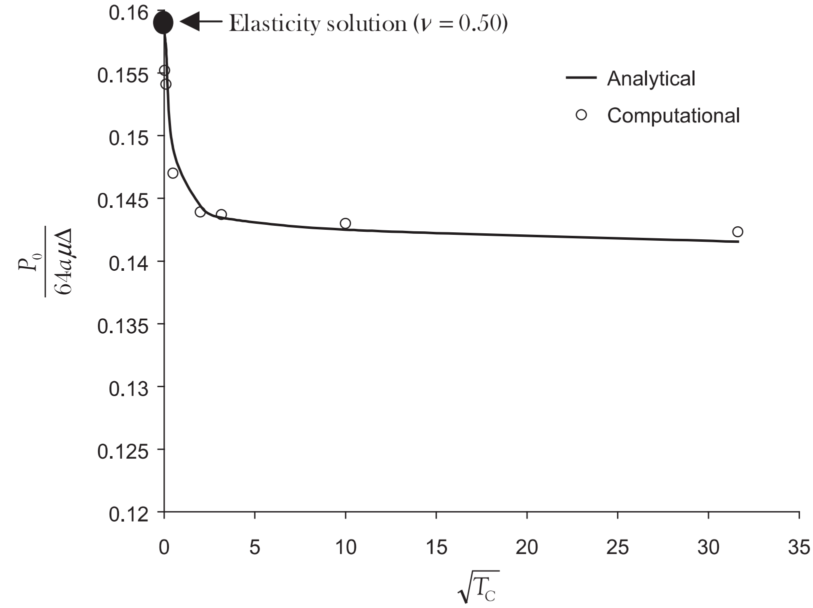

Figure 3. A comparison between the analytical results given by Yue and Selvadurai [62] for an impermeable circular anchorage and the computational results.

the anchor that is embedded in an extended poroelastic medium. In the computational treatment, however, there is no provision for infinite elements, which would model more accurately the far-field response in terms of the decay in the displacements and pore pressure fields. Details of such developments of specific interest to poroelastic media are given by Schrefler and Simoni [60] and Selvadurai and Karpurapu [61]. In this study, the location of the outer boundary, in terms of its ability to model an extended poroelastic domain, is established by comparing the computational results for the time-dependent consolidation displacements of a circular disc anchor located in a poroelastic medium with the corresponding analytical results derived by Yue and Selvadurai [62]. These analytical results are also obtained via the numerical solution of a set of coupled Fredholm-type integral equations associated with the mixed boundary value problem describing the mechanics of a rigid disc inclusion embedded in a poroelastic infinite space. Figure 3 presents a comparison of the time-dependent behaviour of the in-plane displacement of the rigid circular flat anchor derived through the analytical scheme with equivalent results derived from the computational scheme. The results are presented in terms of the normalized parameters $P_0/64a\mu\Delta$ and $\sqrt{T_C}$, where

$$T_C = \frac{2\mu(1-v)k_0 t}{(1-2v)a^2} \tag{22}$$

and for the case where $v = 0.195$. Also in (22), k_0 is the hydraulic conductivity of the undamaged poroelastic medium (i.e. $k_0 = \kappa_0\gamma_w$). (The hydraulic conductivity in (22) is taken as that of the undamaged material in order to identify the influences arising from alteration in the hydraulic conductivity of the porous skeleton, which occurs during the damage evolution process.) The results shown in Figure 3 clearly indicate that the domain selected for the computational modelling can adequately represent the overall response of an extended poroelastic solid, in terms of establishing the time-dependent displacement response of the flat anchor.

A further estimate of the accuracy and adequacy of the computational model in replicating the behaviour of an extended medium can be obtained by examining the elasticity problem for a rigid flat elliptical anchor that is embedded in bonded contact with an isotropic elastic medium and subjected to an in-plane force P_0, which induces an in-plane translation Δ. The analytical solution to the problem was first presented by Kassir and Sih [3]. An analytical solution for the in-plane translation of an elliptical anchor located at a bi-material elastic interface was given by Selvadurai and Au [63]. The latter authors presented an analytical solution for the in-plane translation of an elliptical anchor plate located at a bi-material elastic interface in the following form:

$$P_0 = \frac{8\pi a \Delta \mu_1 e_0^2}{[\{(3-4v_1)e_0^2+1\}K(e_0)-E(e_0)]}\{(1-v_1)+R(1-v_2)\chi\} \tag{23}$$

where

$$\chi = \left[\frac{\{(3-4v_1)e_0^2+1\}K(e_0)-E(e_0)}{\{(3-4v_2)e_0^2+1\}K(e_0)-E(e_0)}\right]; \quad e_0^2 = (a^2-b^2)/a^2; \quad R = \mu_1/\mu_2 \tag{24}$$

where a and b are the dimensions of the elliptic disc anchor; μ_1, μ_2 and v_1, v_2 are, respectively, elastic shear moduli and Poisson's ratios for the distinct geomaterials; and $K(e_0)$ and $E(e_0)$ are, respectively, the complete elliptic integrals of the first and second kind (see, e.g. Reference [64]). Recently, Selvadurai [65] has also developed an improved set of bounds for the in-plane translation of a circular disc anchor located at a bi-material elastic interface, which takes the form

$$\frac{(\varpi_1+\Gamma\varpi_2)}{2\varpi_1\{2+\beta_2+3\varpi_2\alpha_2\}+2\varpi_2\Gamma\{2+\beta_1+3\varpi_1\alpha_1\}} \leqslant \frac{P_0}{32a\Delta(\mu_1+\mu_2)} \leqslant \frac{(7-8v_1)(1-v_2)+\Gamma(7-8v_2)(1-v_1)}{(1+\Gamma)(7-8v_1)(7-8v_2)} \tag{25}$$

where

$$\alpha_i = \frac{1}{2\pi}\ln(3-4v_i); \quad \beta_i = \frac{(1-2v_i)}{\pi\alpha_i}; \quad \varpi_i = \frac{\alpha_i\beta_i}{(1+\alpha_i^2)}; \quad \Gamma = \mu_1/\mu_2 \tag{26}$$

Table I presents a comparison of the elastic solutions for the circular and elliptical anchors derived from the analytical results given by Kassir and Sih [3], Selvadurai and Au [63] and Selvadurai [65] and the computational estimates. The elasticity parameters used in the calculations are as follows: $E = 8300$ MPa; $v = 0.195$. The anchor dimension a is assumed to be 1.0 m and the load P_0 applied to the anchorage is 314 kN. Table I presents the analytical results for elliptical and circular disc anchors in the dimensionless form $P_0/64a\Delta\mu$, where P_0

Table I. Comparison of results for ideal elastic analysis ($\nu = 0.195$).

	$P_0/64a\Delta\mu$			
	Circular	Ellipse (1/2)	Ellipse (1/3)	Ellipse (1/5)
Present study	0.141	0.0485	0.0290	0.0104
Analytical solution	0.139	0.0480	0.0283	0.0100
	[3, 17, 65]	[3, 63]	[3, 63]	[3, 63]

is the in-plane load and a refers to either the radius of disc (for the case of circular anchorage) or the dimension of the elliptical anchor along the direction of application of the load and μ is the shear modulus of the surrounding elastic region. Also, in the case of a homogeneous elastic solid, the analytical result for the in-plane elastic stiffness of the rigid circular disc anchorage can be obtained in the exact closed form (see, e.g. References [6, 7]) as follows:

$$\frac{P_0}{64\mu a\Delta} = \frac{(1-\nu)}{(7-8\nu)} \tag{27}$$

The results for the in-plane-elastic stiffness of the rigid elliptical anchor plate derived via the computational scheme with a domain of finite extent agrees exceptionally well with the exact analytical results applicable for an infinite domain. This correlation is an important factor particularly as it relates to the computation of the elastic stiffness, since the absence of a satisfactory correlation usually results from the truncation of the domain in a finite manner.

The computational approach presented in this paper is now applied to examine the influence of poroelastic skeletal damage on the time-dependent in-plane translational displacement of the flat rigid anchor plate with an elliptical plan form. To aid the presentation of the results, we consider the following four categories of solution: (i) the behaviour of the poroelastic medium corresponds to a classical Biot-type poroelastic solid void of any damage development, (ii) the poroelastic medium exhibits damage evolution without any alteration of hydraulic conductivity characteristics of the porous skeleton, (iii) the poroelastic medium exhibits damage evolution with attendant alterations in both elasticity and hydraulic conductivity characteristics of the porous skeleton, (iv) the poroelastic medium exhibits stress-state-dependent evolution of damage with alterations in both the elasticity and hydraulic conductivity characteristics. Other combinations in the poroelastic skeletal responses are also possible; these four types of responses are sufficient for the purposes of illustrating the computational concepts outlined in the paper. Figure 4 illustrates the dimensions of the region adopted in the computational modelling and the associated boundary conditions derived from symmetry and far-field considerations.

For the purposes of the computational modelling, we select sandstone as the damage-susceptible poroelastic material. The material parameters relevant for the computational modelling can be obtained from the studies reported by Cheng and Dusseault [19] and Shiping *et al.* [24]. The constitutive parameters used are as follows:

Elasticity parameters: $E = 8300\,\text{MPa}$; $\nu = 0.195$; $\nu_u = 0.4999$

Fluid transport parameters: $k^0 = 10^{-6}\,\text{m/s}$

Failure parameters: $\sigma_C = 30\,\text{MPa}$ (compressive); $\sigma_T = 3\,\text{MPa}$ (tensile)

Damage parameters: $\gamma = \eta = 130$; $D_C = 0.75$, $\beta = 3.0 \times 10^5$

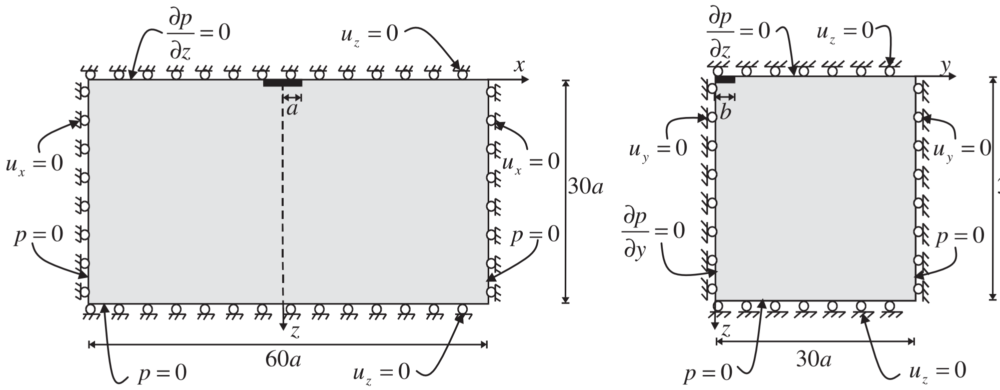

Figure 4. Boundary conditions for a rigid flat anchorage surrounded with a poroelastic medium.

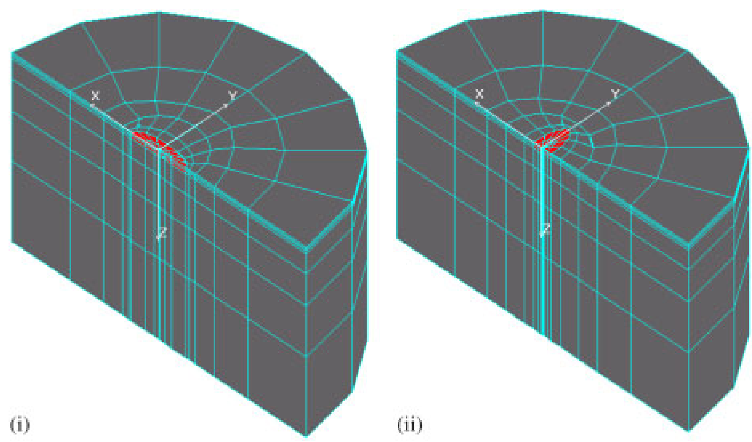

Figure 5. The near-field finite element discretization for a flat rigid anchorage for cases: (i) $b/a<1$; and (ii) $b/a>1$.

The anchor dimension a is taken as 1.0(m) and the time-independent constant load P_0 applied to the anchorage is 314 kN. Computational modelling is performed for different geometries of the elliptical anchor region defined by the aspect ratios (b/a) (Figure 2). The non-dimensional parameter that defines the time-dependent stiffness of the rigid anchor is set to $P/64\mu\Delta a$ (see, e.g. References [7, 65]) and the time factor is defined by (22). Figure 5 presents the mesh discretization employed in the computational modelling of the disc anchors with different plan forms.

Figure 6 illustrates the transient time-dependent horizontal displacement of a rigid circular disc anchor related to the four categories of poroelastic responses indicated previously. The results show that damage-induced alteration in the hydraulic conductivity of the brittle poroelastic medium has a greater influence on the time-dependent response of the disc anchor than

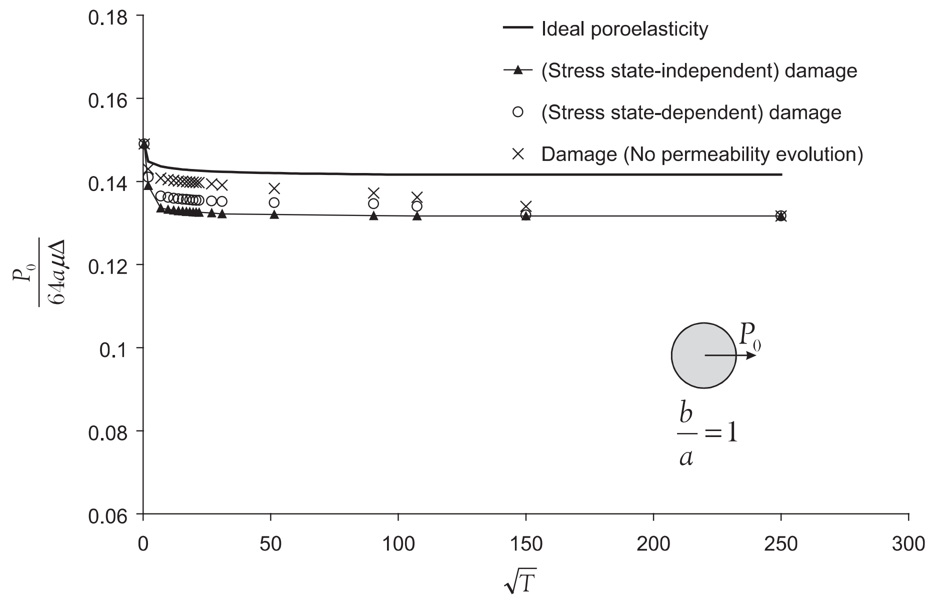

Figure 6. Numerical results for the time-dependent in-plane stiffness for a rigid flat circular anchor embedded in a poroelastic medium susceptible to damage.

in the case involving damage evolution, which leads only to a reduction in the elastic stiffness of the porous skeleton. The computational results for the case involving stress-state-dependent modelling of damage evolution results in a marginally lower difference in comparison with the result for the ideal poroelastic case involving damage-induced alterations in both the elasticity and hydraulic conductivity characteristics. This is most likely due to the development of a compressive state of stress within the damage-susceptible poroelastic medium, in certain regions of the poroelastic medium. According to the stress-state-dependent criteria for damage evolution, this region cannot experience damage-induced alterations in the poroelastic parameters. Figure 7 illustrates time-dependent horizontal displacement of an elliptical anchorage subjected to an in-plane load directed along its major axis ($b/a=1/2$). The results indicate similar trends for the elliptical anchor. The results, however, show a greater difference between the modelling involving ideal poroelasticity and that involving damage-induced alterations in the poroelasticity parameters. This is likely due to development of high locally-stressed zones in the poroelastic medium in the vicinity of the boundary of the anchor. Figures 8 and 9 illustrate similar results applicable to the rigid elliptical anchorage with $b/a=1/3$ and $b/a=1/5$. These results show a greater influence of damage-induced alterations in the hydraulic conductivity on the time-dependent in-plane displacement of the rigid anchorage, which could be attributed to the elongated shape of the anchorage in the direction of application of the force. Figure 10 illustrates the time-dependent horizontal displacement for an elliptical disc anchor, with an aspect ratio $b/a=2$ and subjected to an in-plane load directed along its minor axis. The results show the same trends for both the disc anchor and the elliptical anchor subjected to a lateral load directed along its major axis. In comparison to the results for the circular disc anchor,

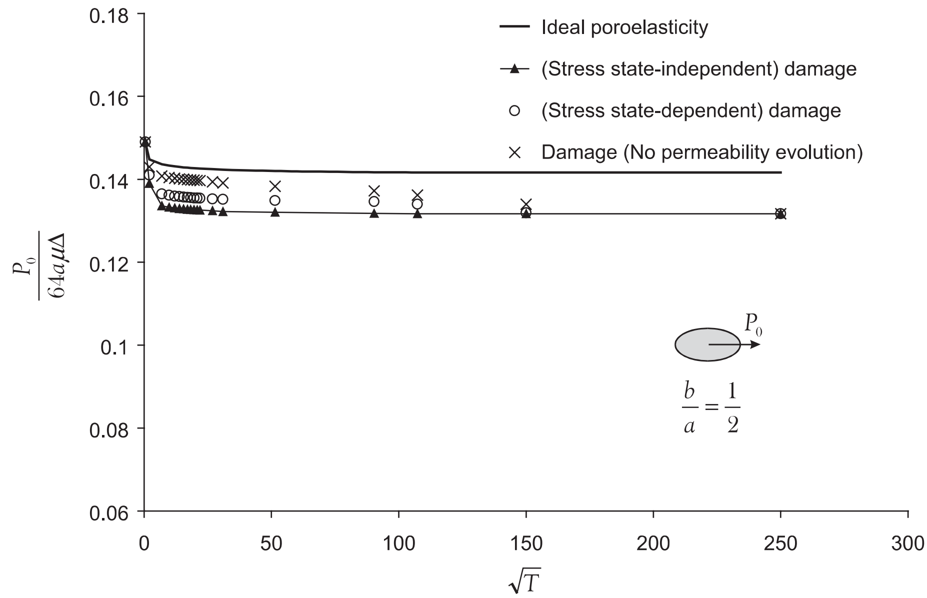

Figure 7. Numerical results for the time-dependent in-plane stiffness for a rigid flat anchor ($b/a = 1/2$) embedded in a poroelastic medium susceptible to damage.

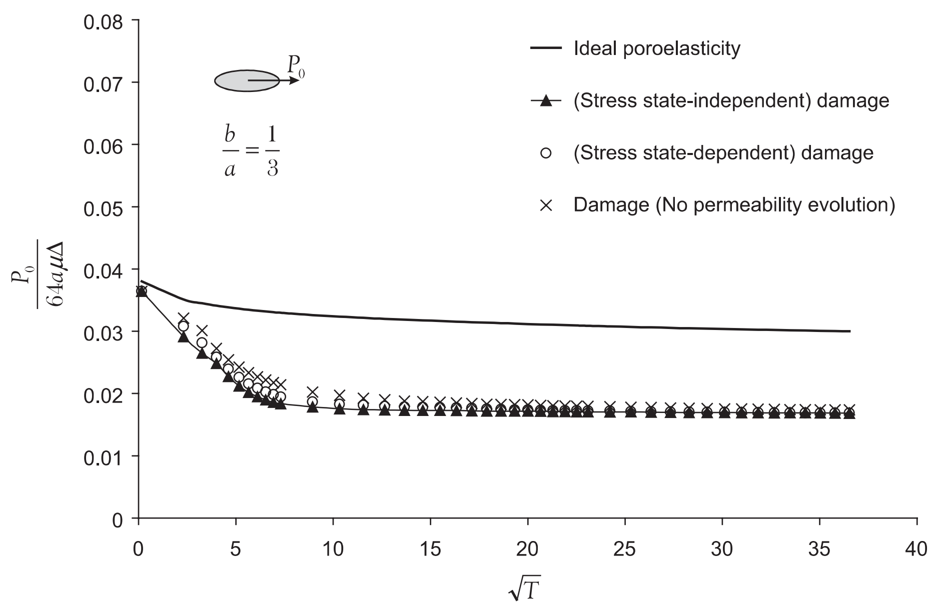

Figure 8. Numerical results for the time-dependent in-plane stiffness for a rigid flat anchor ($b/a = 1/3$) embedded in a poroelastic medium susceptible to damage.

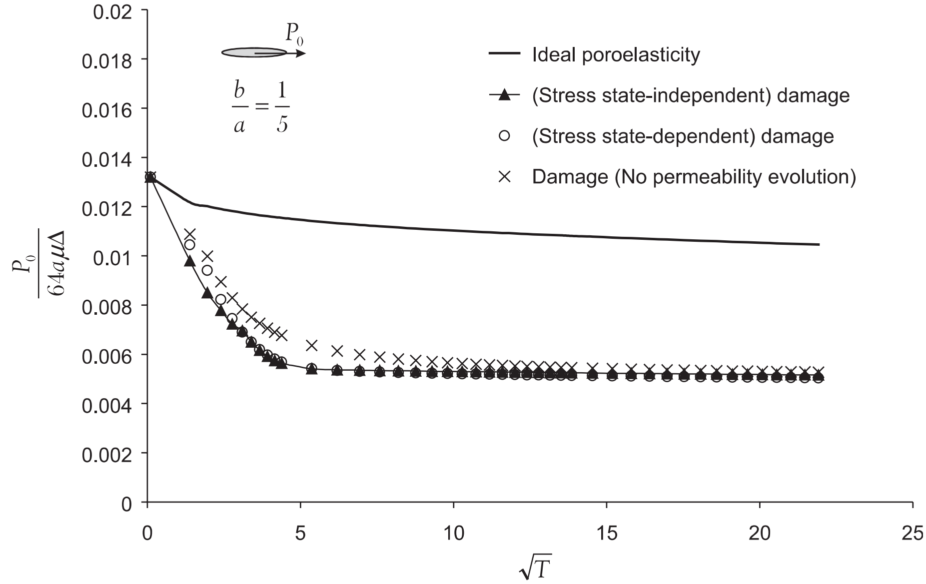

Figure 9. Numerical results for the time-dependent in-plane stiffness for a rigid flat anchor ($b/a = 1/5$) embedded in a poroelastic medium susceptible to damage.

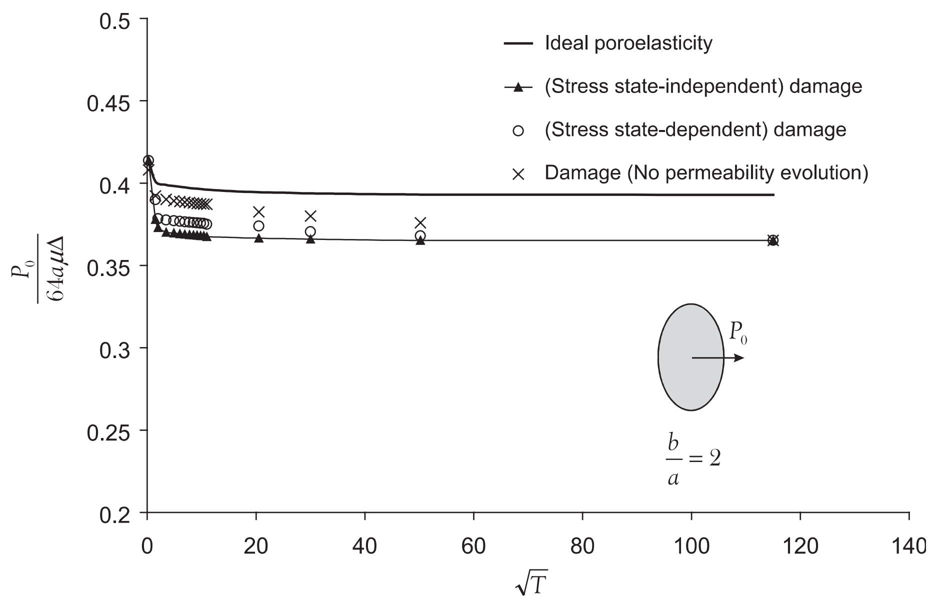

Figure 10. Numerical results for the time-dependent in-plane stiffness for a rigid flat anchor ($b/a = 2$) embedded in a poroelastic medium susceptible to damage.

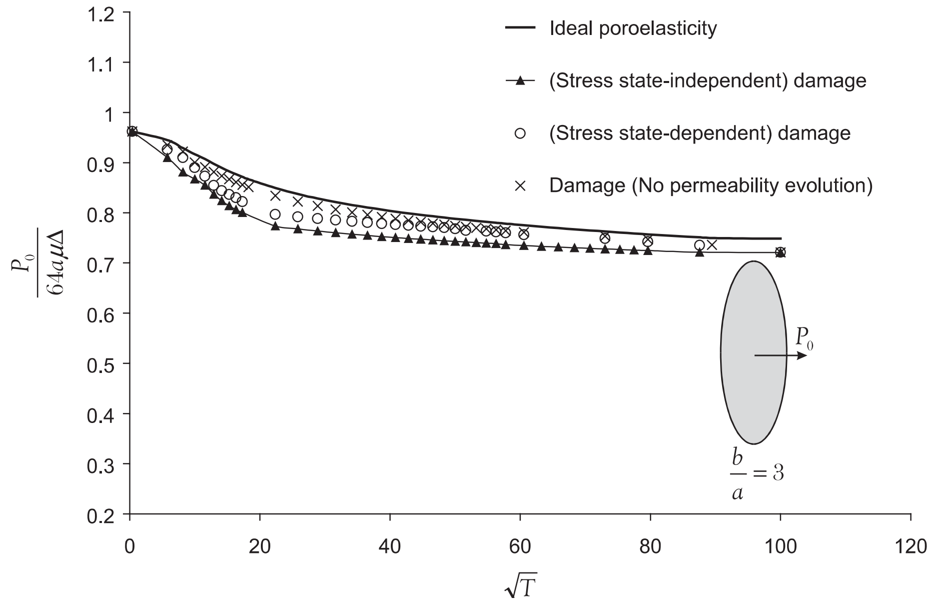

Figure 11. Numerical results for the time-dependent in-plane stiffness for a rigid flat anchor ($b/a=3$) embedded in a poroelastic medium susceptible to damage.

computational results for the case of an elliptical anchor subjected to an in-plane load directed along the minor axis show a greater difference between the ideal poroelastic case and the case that involves damage-induced alterations in the poroelastic parameters. Again, this is likely due to development of the zones of high stress at the tip of the anchor. Figures 11 and 12 illustrate similar results applicable to the rigid elliptical anchors with aspect ratios defined by $b/a=3$ and 5. In general, the computational results presented here indicate that the geometric shape of the disc anchor can increase the damage-induced alterations in the hydraulic conductivity and consequently influence the transient behaviour of the anchorage. The elongated anchorage can generate higher shear stresses in regions of the poroelastic solid at the edges of the anchorage and this results in the evolution of damage in zones surrounding the rigid anchor, which again influences the consolidation response.

5. CONCLUDING REMARKS

The concept of a damage-susceptible poroelastic medium is introduced as a representation of a fluid-saturated geomaterial that will maintain its essential elastic and fluid transport responses in the presence of distributed micro-mechanical damage. The isotropic form of damage and its evolution can be a suitable model for examining predominantly brittle behaviour of geomaterials which can experience micro-cracking and micro-void generation at stress levels substantially lower than the stresses required to cause failure of the material in terms of either discrete fracture development or plastic failure. The paper shows that the classical theory of poroelasticity can be adopted quite effectively to examine fluid-saturated media with a porous skeleton

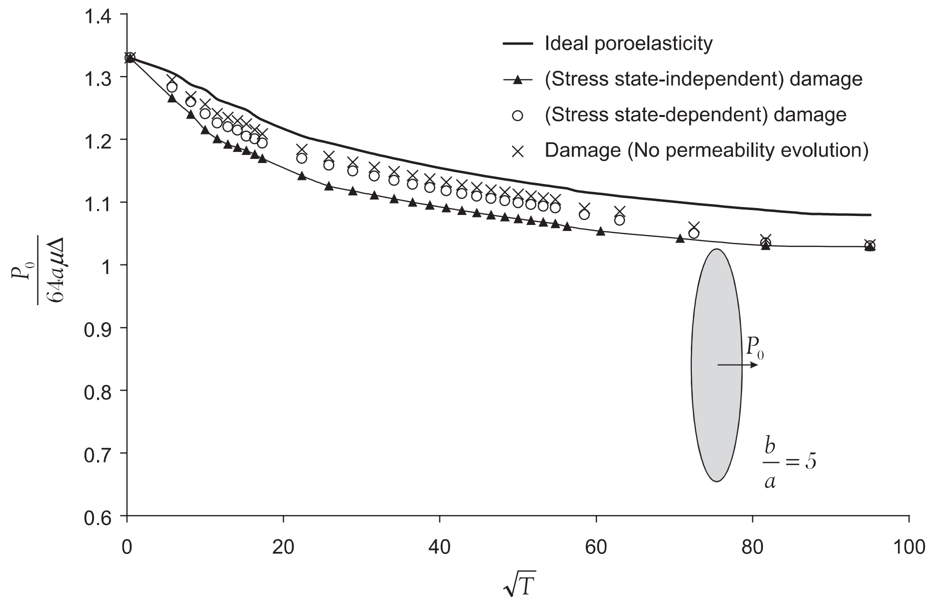

Figure 12. Numerical results for the time-dependent in-plane stiffness for a rigid flat anchor ($b/a = 5$) embedded in a poroelastic medium susceptible to damage.

susceptible to damage. The approach presented in the paper is of limited interest if the time dependency in the behaviour of the rigid plate anchor results from creep deformations of the surrounding geomaterial as opposed to consolidation. A computational approach is used to examine the flat anchor problem, which is of interest to geomechanics. The computational modelling shows that the problem of the in-plane loading of a flat anchor referred to an extended medium can be examined through consideration of a finite domain and without the requirement for incorporation of infinite elements. This conclusion is dictated by the mode of in-plane displacement of the rigid anchor, which subjects the poroelastic domain to predominantly shear stresses. Related computational studies indicate that when the mode of deformation corresponds an axial translation of the disc anchor, either the dimensions of the domain need to be increased or infinite elements need to be provided to accurately model an extended medium. Insofar as the poroelastic consolidation problem is concerned, the geometry of the flat plate anchor and the damage development can influence its time-dependent consolidation response. The overall effect of damage evolution, either in terms of the reduction in the elasticity properties or an increase in the permeability characteristics of the porous medium is to enhance the consolidation process. The degree to which such effects materialize will depend on the other aspects of the damage response including stress state dependency. The results given in the paper are for illustrative purposes only, but adequately demonstrate the role of a damage-susceptible poroelastic medium on the mechanics of the anchor.

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REFERENCES

1. Collins WD. Some axially symmetric stress distributions in elastic solids containing penny-shaped cracks I. Cracks in an infinite solid and a thick plate. *Proceedings of the Royal Society Series A* 1962; **203**: 359–386.
2. Keer LM. A note on the solution of two asymmetric boundary value problems. *International Journal of Solids and Structures* 1965; **1**:257–264.
3. Kassir MK, Sih GC. Some three-dimensional inclusions problems in elasticity theory. *International Journal of Solids and Structures* 1968; **4**:225–241.
4. Kanwal RP, Sharma DL. Singularity methods for elastostatics. *Journal of Elasticity* 1976; **6**:405–418.
5. Selvadurai APS. The load-deflexion characteristics of a deep rigid anchor in an elastic medium. *Géotechnique* 1976; **26**:603–612.
6. Zureick AK. Transversely isotropic medium with a rigid spheroidal inclusion under an axial pull. *Journal of Applied Mechanics* 1988; **55**:495–497.
7. Selvadurai APS. Analytical methods for embedded flat anchor problems in geomechanics. In *IACMAG '94*, *Proceedings of the 8th International Conference on Computational Methods and Advances in Geomechanics*, Siriwardane HJ, Zaman MM (eds), vol. 3, 1994; 305–321.
8. Biot MA. General theory of three-dimensional consolidation. *Journal of Applied Physics* 1941; **12**:155–164.
9. Detournay E, Cheng AH-D. Fundamentals of poroelasticity. *Comprehensive Rock Engineering*, vol. 2. Pergamon Press: Oxford, 1993; 113–171.
10. Selvadurai APS (ed.). *Mechanics of Poroelastic Media*. Kluwer Academic Publications: Dordrecht, The Netherlands, 1996.
11. Selvadurai APS. On some recent developments in poroelasticity. In *IACMAG 2001*, *Proceedings of the 10th International Conference on Computational Methods and Advances in Geomechanics*, Tucson, Arizona, Desai CS *et al.* (eds), vol. 2. A.A. Balkema: Rotterdam, The Netherlands, 2001; 1761–1769.
12. Cheng AH-D, Detournay E, Abousleiman Y (eds). Poroelasticity. Maurice A. Biot memorial issue. *International Journal of Solids and Structures* 1998; **5**:4513–5031.
13. Lewis RW, Schrefler BA. *The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media*. Wiley: New York, 1998.
14. de Boer R. *Theory of Porous Media*. Springer: Berlin, Germany, 2000.
15. Auriault J-L, Geindreau C, Royer P, Bloch J-F, Boutin C, Lewandowska J (eds). Poromechanics II. *Proceedings of the 2nd Biot Conference on Poromechanics*, Grenoble, France. A.A. Balkema: Rotterdam, The Netherlands, 2002.
16. Ehlers W, Bluhm J (eds). *Porous Media*: *Theory*, *Experiments and Numerical Applications*. Springer: Berlin, 2002.
17. Cook NGW. Fracture of rock. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts* 1965; **2**:389–403.
18. Bieniawski ZT, Denkhaus HG, Vogler UW. Failure of fractured rock. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts* 1967; **6**:323–346.
19. Cheng H, Dusseault MB. Deformation and diffusion behaviour in a solid experiencing damage: a continuous damage model and its numerical implementation. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts* 1993; **30**:1323–1331.
20. Martin CD, Chandler NA. The progressive fracture of Lac du Bonnet granite. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts* 1994; **31**:643–659.
21. Chau KT, Wong RHC. Effective moduli of micro-cracked rocks: theories and experimentation. *International Journal of Damage Mechanics* 1997; **6**:258–277.
22. Spooner DC, Dougill JW. A quantitative assessment of damage sustained in concrete during compressive loading. *Magazine of Concrete Research* 1975; **27**:151–160.
23. Zoback MD, Byerlee JD. The effect of micro-crack dilatancy on the permeability of Westerly Granite. *Journal of Geophysical Research* 1975; **80**:752–755.

24. Shiping L, Yushou L, Yi L, Zhenye W, Gang Z. Permeability-strain equations corresponding to the complete stress–strain path of Yinzhuang Sandstone. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts* 1994; **31**:383–391.
25. Kiyama T, Kita H, Ishijima Y, Yanagidani T, Akoi K, Sato T. Permeability in anisotropic granite under hydrostatic compression and tri-axial compression including post-failure region. *Proceedings of the 2nd North American Rock Mechanics Symposium*, 1996; 1643–1650.
26. Coste F, Bounenni A, Chanchole S, Su K. A method for measuring mechanical, hydraulic and hydromechanical properties during damaging in materials with low permeability. In *Hydromechanical and Thermohydromechanical Behaviour of Deep Argillaceous Rock*, Hoteit N, Su K, Tijani M, Shao J-F (eds). A.A. Balkema: Rotterdam, The Netherlands, 2002; 109–116.
27. Souley M, Homand F, Pepa S, Hoxha D. Damage-induced permeability changes in granite: a case example at the URL in Canada. *International Journal of Rock Mechanics and Mining Sciences* 2001; **38**:297–310.
28. Bossart P, Meier PM, Moeri A, Trick T, Mayor J-C. Geological and hydraulic characterization of the excavation damaged zone in the Opalinus clay of the Mont Terri Rock Laboratory. *Engineering Geology* 2002; **66**:19–38.
29. Samaha HR, Hover KC. Influence of micro-cracking on the mass transport properties of concrete. *ACI Materials Journal* 1992; **89**:416–424.
30. Gawin D, Pesavento F, Schrefler BA. Simulation of damage-permeability coupling in hydro-thermo-mechanical analysis of concrete at high temperature. *Communications in Numerical Methods in Engineering* 2002; **18**: 113–119.
31. Bary B, Bournazel J-P, Bourdarot E. Poro-damage approach to hydrofracture analysis of concrete. *Journal of Engineering Mechanics* (ASCE) 2000; **126**:937–943.
32. Mahyari AT, Selvadurai APS. Enhanced consolidation in brittle geomaterials susceptible to damage. *Mechanics of Cohesive-Frictional Materials* 1998; **3**:291–303.
33. Selvadurai APS, Shirazi A. Mandel–Cryer effects in fluid inclusions in damage-susceptible poroelastic geologic media. *Computers and Geotechnics* 2004; **31**:285–300.
34. Shirazi A, Selvadurai APS. Indentation of a poroelastic halfspace susceptible to damage. *Proceedings of the Canadian Conference on Civil Engineering*, *Paper No. GES 020*, Montreal, 2002; 10.
35. Selvadurai APS. Stationary damage modelling of poroelastic contact. *International Journal of Solids and Structures* 2004; **41**:2043–2064.
36. Gawin D, Marjorana CE, Schrefler BA. Numerical analysis of hydro-thermic behaviour and damage of concrete at high temperature. *Mechanics of Cohesive-Frictional Materials* 1999; **4**:37–74.
37. Kachanov LM. Time of rupture process under creep conditions. *Izvestiya Akademii Nauk SSR*, *Otelenie Tekhnicheskikh* 1958; **8**:26–31.
38. Bazant ZP. Mechanics of distributed cracking. *Applied Mechanics Reviews* 1986; **39**:675–705.
39. Simo JC, Ju JW. Strain- and stress-based continuum damage models-II. Computational aspects. *International Journal of Solids and Structures* 1987; **23**:841–869.
40. Chow CH, Wang J. An anisotropic theory of elasticity for continuum damage mechanics. *International Journal of Fracture* 1987; **33**:3–16.
41. Lemaitre J, Chaboche JL. *Mechanics of Solid Materials*. Cambridge University Press: Cambridge, England, 1990.
42. Selvadurai APS, Hu J. The axial loading of foundations embedded in frozen soils. *International Journal of Offshore and Polar Engineering* 1995; **6**:96–103.
43. Wohua Z, Valliappan S. Continuum damage mechanics theory and application. Part I—Theory. *International Journal of Damage Mechanics* 1998; **7**:250–273.
44. Wohua Z, Valliappan S. Continuum damage mechanics theory and application. Part II—Application. *International Journal of Damage Mechanics* 1998; **7**:274–297.
45. Krajcinovic D. *Damage Mechanics*. North-Holland: Amsterdam, The Netherlands, 1996.
46. Valliappan S, Yazdchi M, Khalili N. Earthquake analysis of gravity dams based on damage mechanics concept. *International Journal for Numerical and Analytical Methods in Geomechanics* 1996; **20**:725–752.
47. Valliappan S, Yazdchi M, Khalili N. Seismic analysis of arch dams—a continuum damage mechanics approach. *International Journal for Numerical Methods in Engineering* 1999; **45**:1695–1724.
48. Lemaitre J. How to use damage mechanics. *Nuclear Engineering and Design* 1984; **80**:233–245.
49. Skempton AW. The pore pressure coefficients A and B. *Géotechnique* 1954; **4**:143–147.
50. Rice JR, Cleary MP. Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents. *Reviews of Geophysics and Space Physics* 1976; **14**:227–241.

51. Altay GA, Dokmeci MC. A uniqueness theorem in Biot's poroelasticity theory. *Journal of Applied Mathematics and Physics* (ZAMP) 1998; **49**:838–847.
52. Selvadurai APS. *Partial Differential Equations in Mechanics*, vol. 2. Springer: Berlin, Germany, 2000.
53. Sandhu RS, Wilson EL. Finite element analysis of flow in saturated porous media. *Journal of Engineering Mechanics Division* (ASCE) 1969; **95**(EM3):641–652.
54. Ghaboussi J, Wilson EL. Flow of compressible fluids in porous elastic media. *International Journal for Numerical Methods in Engineering* 1973; **5**:419–442.
55. Booker JR, Small JC. An investigation of the stability of numerical solution of Biot's equations of consolidation. *International Journal of Solids and Structures* 1975; **11**:907–917.
56. Selvadurai APS, Nguyen TS. Computational modelling of isothermal consolidation of fractured porous media. *Computers and Geotechnics* 1995; **17**:39–73.
57. Desai CS, Siriwardane HJ. *Constitutive Laws for Engineering Materials with Emphasis on Geologic Materials*. Prentice-Hall Inc.: Englewood Cliffs, NJ, 1984.
58. Chen W-F, Saleeb AF. *Constitutive Equations for Engineering Materials*, vol. 1. Wiley: New York, 1982.
59. Schulze O, Popp T, Kern H. Development of damage and permeability in deforming rock salt. *Engineering Geology* 2001; **61**:163–180.
60. Schrefler BA, Simoni L. Non-isothermal consolidation of unbounded porous media using mapped infinite elements. *Communications in Applied Numerical Methods* 1987; **3**:445–452.
61. Selvadurai APS, Karpurapu R. A composite infinite element for modelling unbounded saturated soil media. *Journal of Geotechnical Engineering* (ASCE) 1989; **115**:1633–1646.
62. Yue ZQ, Selvadurai APS. On the mechanics of a rigid disc inclusion embedded in a fluid saturated poroelastic medium. *International Journal of Engineering Science* 1995; **33**:1633–1662.
63. Selvadurai APS, Au MC. Generalized displacements of a rigid elliptical anchor embedded at a bi-material geological interface. *International Journal for Numerical and Analytical Methods in Geomechanics* 1986; **10**:633–652.
64. Byrd PF, Friedman MD. *Handbook of Elliptic Integrals*. Springer: Berlin, 1971.
65. Selvadurai APS. On certain bounds for the in-plane translational stiffness of a disc inclusion at a bi-material elastic interface. *Mechanics Research Communications* 2003; **30**:227–234.

