The Fluid-filled Spherical Cavity in a Damage-susceptible Poroelastic Medium

A. P. S. SELVADURAI* AND A. SHIRAZI
Department of Civil Engineering and Applied Mechanics
McGill University, 817 Sherbrooke Street
West Montreal, QC, Canada H3A 2K6

ABSTRACT: This paper examines the problem of fluid pressure development in a fluid-filled spherical cavity located in an extended fluid-saturated poroelastic medium susceptible to damage. The evolution of damage introduces alterations in both the hydraulic conductivity and skeletal elasticity properties of the poroelastic solid. The paper examines the fluid-filled spherical cavity problem with a view to establishing the influence of the stress state-dependent damage on the amplification and decay of the fluid pressure in the spherical cavity. Sufficient computational results are available to aid the development of certain generalized conclusions relating to the influence of damage on the behaviour of encapsulated fluid domains.

KEY WORDS: damage mechanics, poroelastic damage, Biot consolidation, hydraulic conductivity alteration, spherical cavity problems, stress state-dependent damage.

INTRODUCTION

THE MECHANICS OF porous elastic materials saturated with deformable fluids can be described by classical poroelasticity. In the classical theory proposed by Biot (Biot, 1941), the mechanical behaviour of the porous skeleton is assumed to be linearly elastic and Darcy's law describes fluid transport through the pore space. Detailed accounts of the applications of classical poroelasticity and its extensions, to both geomechanics and biomechanics, are given by Schiffman, 1984; Detournay and Cheng, 1993; Coussy, 1995; Selvadurai, 1996, 2001; Lewis and Schrefler, 1998; Thimus et al., 1998; de Boer, 2000; Auriault et al., 2002. The assumption of linear elastic behaviour of the porous skeleton is a limitation when considering the applicability of the classical theory of poroelasticity to a wider class of

*Author to whom correspondence should be addressed. E-mail: patrick.selvadurai@mcgill.ca

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geomaterials that possess non-linear phenomena, particularly in the constitutive behaviour of the porous skeleton. A modification of the porous skeletal response to include elasto-plastic constitutive behaviour is one such approach that has been employed for the modelling of the poroelastic behaviour of geomaterials such as soft clays and other saturated soils. In such situations, the geomaterials display distinct attributes of failure that can be described by the criteria for initiation of failure, and constitutive laws that govern the post-failure behaviour (see e.g. Lewis and Schrefler, 1998; Davis and Selvadurai, 2002). Alternatively, in geomaterials such as soft rocks and heavily over-consolidated clays, the tendency is to display brittle elastic behaviour leading to degradation in the elastic stiffness of the porous skeleton. Stiffness degradations can manifest at stress levels well in advance of the stress states required to initiate brittle failure of the porous skeleton, as evidenced by the occurrence of discontinuities such as shear bands or failure planes. Such degradation can occur as a consequence of the development of micro-cracks and micro-voids in the porous fabric of the geomaterial while it continues to maintain its elastic character. This type of geomaterial behaviour lends itself to modelling by appeal to continuum damage mechanics. It is assumed that damage evolution will maintain the applicability of both the linear elastic constitutive behaviour and Darcy's law, but a damage evolution law, formulated through either micro-mechanical considerations or obtained through experimental means, will now govern the evolution of the parameters that describe the elasticity and the fluid transport characteristics. The description of poroelastic behaviour of geomaterials by appeal to damage mechanics is of some relevance to the modelling of the brittle characteristics of geomaterials such as granite, sandstone, claystone, limestone, concrete, etc., that tend to exhibit elastic stiffness reductions, and hydraulic conductivity increases well in advance of the pre-peak stress state. Experimental investigations of Cook, 1965; Bieniawski et al., 1967; Paterson, 1978; Martin and Chandler,1994; Chau and Wong, 1997 and others point to the load-induced degradation of elastic moduli of rocks. Results of related interest with applicability to unreinforced concrete are also discussed by Spooner and Dougill (1975), Suhawardy and Pecknold (1978) (see also Elfgren, 1989; Shah and Swartz, 1989).

Micro-crack and micro-void-induced evolution of hydraulic conductivity of saturated geomaterials has been observed by Zoback and Byerlee (1975), and their results, derived from tests conducted on granite, indicate a four-fold increase in the magnitude of the permeability. Shiping et al. (1994) report the results of tests conducted on sandstone, which indicate that for all combinations of stress states employed in their tests, the permeability increased by an order of magnitude. Kiyama et al. (1996) also present results of triaxial tests on anisotropic granite, pointing to an increase in the permeability characteristics. The results of experiments on rocks and clay stone, presented by Coste et al. (2002), indicate an increase in the permeability of up to two-orders of magnitude, with increased levels of deviator stresses. A discussion of experimental results pertaining to stress-induced hydraulic conductivity alterations in brittle geomaterials are given in the article by Selvadurai (2004). It should also be noted that not all the experimental studies focus on the evaluation of the hydraulic conductivity changes at stress levels well below the peak values. At the range of stress levels close to failure, geomaterials tend to display features such as shear bands, localization zones and/or dilatancy zones that will invariably alter the hydraulic conductivity of the porous medium in an inhomogeneous fashion. The objective of this study is to extend the classical theory of poroelasticity to include the effects of micro-void generation through the use of a theory such as continuum damage mechanics. By adopting a damage mechanics approach, the influences of micro-void generation in the porous medium are accounted for in a phenomenological sense. Such a theory is considered to be suitable for describing the mechanical behaviour of brittle elastic solids well in advance of the development of macro-cracks (e.g. fractures) or other irreversible phenomena (e.g. plasticity effects). In this model damage is phenomenological, resulting from the reduction in the elastic stiffness due to the generation of micro-voids and other micro-defects. Cheng and Dusseault (1993) developed an anisotropic damage model to examine the poroelastic behavior of saturated geomaterials, in the absence of hydraulic conductivity alteration during the damage process. The studies by Mahyari and Selvadurai (1998) and Shirazi and Selvadurai (2002) have considered the computational modelling of an axisymmetric poroelastic contact problem where both elastic stiffness reduction and hydraulic conductivity alteration during damage evolution are considered. The former study deals with the purely axisymmetric formulation and the latter deals with a completely three-dimensional formulation of the axisymmetric contact problem.

This paper examines the problem of an extended, damage-susceptible poroelastic medium, which is bounded internally by a fluid-filled spherical cavity. The damage susceptibility of the poroelastic medium takes into account both alterations in the elastic stiffness and the attendant alterations (usually an increase) in the hydraulic conductivity. The poroelastic medium is subjected to a triaxial stress state in the far field. The application of this stress induces pore pressures in the fluid within the spherical cavity. As time progresses, the pore pressure field in the cavity increases due to the compatible interactions between the incompressible pore fluid and the deformations of the porous solid skeleton. This phenomenon is the celebrated Mandel-Cryer effect that has been predicted theoretically and observed experimentally for ideal poroelastic solids. Mandel (1950, 1953) was the first to demonstrate, mathematically, the existence of the effect by considering the poroelastic response of a cubical element loaded under plane strain conditions. Cryer (1963) also observed this response in connection with the problem of a poroelastic solid sphere that was loaded by an external radial stress field with provision for complete fluid drainage at the boundary. Experimental observations of the Mandel-Cryer effect are also cited by Gibson et al. (1963) and Verruijt (1965). A physical explanation for the phenomenon is related to the observation that in the initial stages of the poroelastic response, the volume changes associated with consolidation will invariably occur at regions close to surfaces that allow drainage. The reduction in volume in these regions due to consolidation will force a compression of the interior regions and such stressing action will lead to the development of additional fluid pressures in the interior regions. The proof of existence of the Mandel-Cryer effect in fluid inclusions in poroelastic media is therefore an outcome of these observations. Further references to studies in this area can be found in review articles (Schiffman, 1984; Detournay and Cheng, 1993). Cryer (1963) also suggests that the amplification and subsequent decay in the pore pressure will depend on the elastic modulus and hydraulic conductivity characteristics of the poroelastic medium. It is of interest to examine the extent to which the Mandel-Cryer effect can materialize in poroelastic media that, with damage of the porous skeleton, gives rise to alterations in both the elastic stiffness of the porous skeleton and its hydraulic conductivity. The objective of this paper is to address this question through the computational modelling of the problem of an extended medium that is bounded internally by a fluid-filled spherical inclusion (Figure 1). A preliminary investigation of the fluid-filled spherical cavity was discussed by Selvadurai and Shirazi (2002) and the problem is now extended to introduce important new features, including the tri-axial states of the external stress field and the stress space dependency of the damage evolution based on the second invariant of the strain deviator tensor. The accommodation of these effects can be accomplished only through a computational scheme; as such, the presentation of numerical results is restricted to specific situations that are sufficient to demonstrate key facets of the influence of damage evolution within the elastic range on the generation and decay of pressure within the fluid-filled cavity located in a damage-susceptible poroelastic medium.

CLASSICAL POROELASTICITY THEORY

The equations of classical poroelasticity relate to the quasi-static response of a poroelastic medium consisting of a porous isotropic linear elastic soil

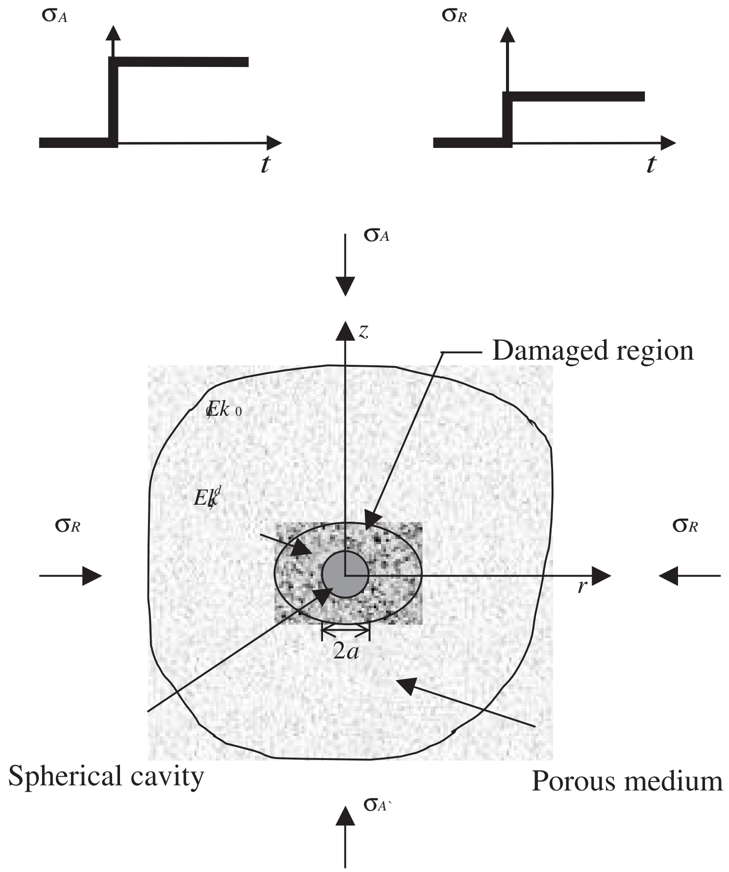

Figure 1. *Fluid filled spherical cavity in an extended poroelastic medium.*

skeleton saturated with an incompressible pore fluid. The governing partial differential equations resulting from Hookean elastic behaviour of the fluid filled poroelastic medium and Darcy flow in the porous space take the forms

$$G\left(\nabla^2 u_i - \frac{1}{(1-2\nu')}\frac{\partial \varepsilon_v}{\partial x_i}\right) = \frac{\partial p}{\partial x_i} \tag{1}$$

$$\frac{\partial \varepsilon_v}{\partial t} = C_v \nabla^2 \varepsilon_v \tag{2}$$

$$\frac{2G(1+\nu')}{3k(1-2\nu')}\nabla^2 \varepsilon_v = -\nabla^2 p \tag{3}$$

where σ_{ij} is the total stress tensor, p is the pore fluid pressure, ε_v is the volumetric strain in the porous skeleton; ν' and G are the "drained" values of Poisson's ratio and the linear elastic shear modulus applicable to the porous fabric and k is the hydraulic conductivity of the porous medium. Also in these equations, the soil skeletal strains ε_{ij} are given by

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right); \quad \varepsilon_v = \frac{\partial u_k}{\partial x_k} \tag{4}$$

where u_i are the displacement components and the coefficient of consolidation C_v is given by

$$C_v = \frac{2kG(1+\nu')}{3\gamma_w(1-2\nu')} \tag{5}$$

To complete the mathematical formulation of the initial boundary value problem it is necessary to prescribe boundary conditions and initial conditions applicable to the dependent variables u_i and p and/or their derivatives. The boundary conditions can be interpreted in terms of the conventional Dirichlet, Neumann or Robin boundary conditions. The systems of partial differential equations governing quasi-static poroelasticity are of the elliptic-parabolic type and appropriate uniqueness theorems are available in the literature.

DAMAGE MODELLING IN POROELASTIC MEDIA

Classical poroelasticity assumes that the porous structure remains unaltered during deformations of the elastic skeleton of the medium. This is a convenient assumption for many situations where the associated stress levels do not induce appreciable alterations in the poroelastic constitutive responses. As discussed previously, more extreme examples of alterations in the poroelastic behaviour can range from the development of irreversible plastic deformations in regions of the porous soil skeleton to the development of discrete fractures, or shear bands that can introduce highly localized non-linear phenomena, accompanied by drastic alterations in the fluid transport characteristics of the medium as a whole. While such processes can occur in geomaterials that are subjected to stress levels approaching the failure stress states (Chen, 1975; Desai and Siriwardane, 1984; Davis and Selvadurai, 2002), we restrict attention to geomaterials that maintain their predominantly elastic behaviour but suffer some degree of micro-mechanical damage. Alterations in the material characteristics can be introduced in zones of a poroelastic medium that experience continuum damage according to some prescribed damage evolution law. Phenomenologically, the mechanism that will most likely lead to the alterations in both the deformability characteristics and the hydraulic conductivity characteristics of a poroelastic medium, and at the same time maintain the elastic nature of the geomaterial skeletal response, is the creation of voids or cavities within the skeleton. The process of cavity generation in porous media is an idealization introduced to simplify a rather complex set of processes associated with cavity nucleation, fracture generation at cavity boundaries, cavity coalescence, etc. The creation of cavities is also largely governed by the stress state in the geomaterial skeleton, and the defects themselves can exhibit preferred orientations. The available experimental results are insufficient to warrant the development of any sophisticated theory to account for directional dependency in the defect evolution in a porous geomaterial fabric. Furthermore, the evolution of defects will occur in a medium that already has a void component. A meaningful introduction of a directional dependency in the voids can be accomplished only by considering the initial characterization of the pore space. Therefore the simplest approach for introducing the alterations in the elasticity and hydraulic conductivity of an initially porous geomaterial is to adopt a theory based on *continuum damage mechanics*. The introduction of continuum damage mechanics is attributed to Kachanov (1958) and over the past four decades it has become a well-researched area. The coupling of elasticity and damage modelling has been discussed by a number of researchers (Bazant, 1986; Chow and Wang, 1987; Simo and Ju, 1987; Ju, 1990; Lemaitre and Chaboche, 1990; Krajcinovic, 1996; Mahyari and Selvadurai, 1998; Voyiadjis et al., 1998). Attention is focused on the type of behaviour where the geomaterial experiences isotropic damage defined by a scalar damage variable D, which can be related to the initial area A_0 and the reduced area $\bar{A}$ through the relationship

$$D = \frac{A_0 - \bar{A}}{A_0} \tag{6}$$

The damage variable ranges between 0 and D_c, which is the critical value corresponding to the development of fracture of the material. (The critical damage parameter can be viewed as a normalizing parameter against which damage evolution can be estimated.) For isotropic damage, the net stress dyadic σ_{ij}^n is related to the stress dyadic in the undamaged state σ_{ij} according to

$$\sigma_{ij}^n = \frac{\sigma_{ij}}{(1 - D)} \tag{7}$$

Considering the 'strain equivalence hypothesis' proposed by Lemaitre (1992), the constitutive equation for the damaged skeleton of the poroelastic medium can be written as

$$\sigma_{ij} = 2(1-D)\mu\varepsilon_{ij} + \frac{2(1-D)G\nu'}{(1-2\nu')}\varepsilon_v\delta_{ij} + p\delta_{ij} \tag{8}$$

which implies that Poisson's ratio remains constant, which is an added constraint when considering the material behaviour in three-dimensions. In addition to specifying the constitutive relations for the mechanical response of the damaged geomaterial skeleton, it is also necessary to prescribe damage evolution criteria; these can be postulated either by appeal to micro-mechanical considerations or determined through experimentation. For example, based on a review of the results of experiments conducted on rocks it has been shown (Cheng and Dusseault, 1993) that

$$\frac{\partial D}{\partial \xi_d} = \eta \frac{\gamma \xi_d}{(1+\xi_d)}\left(1 - \frac{D}{D_c}\right) \tag{9}$$

where η and γ are positive material constants and ξ_d is related to the second invariant of the deviator strain dyadic, and given in terms of the strains as

$$\xi_d = \frac{1}{2}\left[(\mathrm{tr}\varepsilon_{ij})^2 - \mathrm{tr}(\varepsilon_{ij})^2\right] \tag{10}$$

The evolution of the damage variable can be obtained through an integration of (9) between limits D_0 and D, where D_0 is the initial value of the damage variable corresponding to the intact state. (e.g. D_0 is zero for materials in a virgin state.) The deformability parameters applicable to an initially isotropic elastic material that experiences isotropic damage can be updated by adjusting the elastic constants in the elasticity matrix C_{ijkl} by their equivalent applicable to the damaged state C^D_{ijkl} but maintaining Poisson's ratio constant. Using the result (9), the evolution of D can be prescribed as follows:

$$D = D_c - (D_c - D_0)(1+\gamma\xi_d)^{\eta/\gamma D_c}\exp(-\eta\xi_d/D_c) \tag{11}$$

With the creation of defects such as micro-cracks and micro-voids, it is plausible that these defects can also lead to alterations of the permeability characteristics of poroelastic media. The limited literature on permeability evolution in porous media is more focused on the experimental evaluation

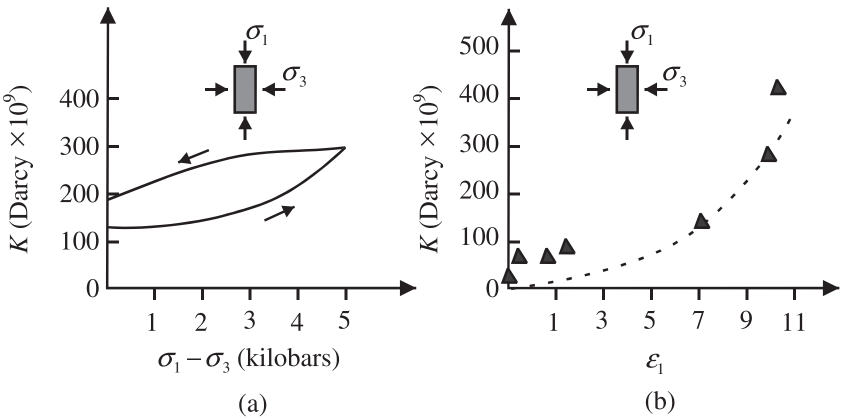

Figure 2. *Permeability evolution in saturated geomaterials: (a) after Zoback and Byerlee (1975) and (b) after Shiping et al. (1994).*

of the alteration in permeability of geomaterials that are subjected to triaxial stress states. Zoback and Byerlee (1975) have documented results of experiments conducted on granite and Shiping et al. (1994) give similar results for tests conducted on sandstone (Figure 2). Kiyama et al. (1996) documented the results of triaxial tests conducted on granite. While the general trends indicate alterations in the permeability characteristics with increase in stress/strain, it is not entirely clear whether such alterations are restricted to zones where strain localization is present. In relation to the studies conducted to examine the indentation of a damage-susceptible poroelastic halfspace by a rigid cylinder with a flat base, Mahyari and Selvadurai (1998) proposed empirical results for the evolution of the permeability κ (expressed in m^2) as a function of the parameter ξ_d; these can be directly applied to describe the evolution of hydraulic conductivity of the poroelastic material: i.e.

$$k^D = \left(1 + \beta \xi_d^2\right) k^0 \tag{12}$$

where β is a constant and k^0 is the hydraulic conductivity of the undamaged material. The quadratic-dependency of the permeability on ξ_d is a plausible estimate, which is adopted strictly for purposes of the ensuing computational modelling.

COMPUTATIONAL MODELLING

Computational schemes based on the finite element method have been widely applied for the analysis of problems in the classical theory of poroelasticity and complete discussions of the computational aspects of poroelasticity are given by Lewis and Schrefler, 1998; Booker and Small, 1975; Selvadurai and Nguyen, 1995. The Galerkin variational technique is applied to transform the partial differential equations into a discretized matrix form, giving rise to the following incremental forms for the equations governing poroelastic media:

$$\begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \{-\gamma\Delta t\mathbf{H}+\mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t+\Delta t} \\ p_{t+\Delta t} \end{Bmatrix} = \begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \{(1-\gamma)\Delta t\mathbf{H}+\mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t} \\ \overline{p_{t}} \end{Bmatrix} + \{\mathbf{F}\} \quad (13)$$

where $\mathbf{K}$ is the stiffness matrix of the solid skeleton; $\mathbf{C}$ is the stiffness matrix due to interaction between the solid skeleton and the pore fluid; $\mathbf{E}$ is the compressibility matrix of the pore fluid; $\mathbf{H}$ is the permeability matrix; $\mathbf{F}$ is the force vectors due to external tractions, body forces and flows; $\mathbf{u}_t$ and p_t are, respectively, the nodal displacements and the pore pressure at time t; and Δt is the time increment. The time integration constant γ varies between 0 and 1. The criteria governing stability of the integration scheme given by Booker and Small (1975) requires that $\gamma \geq 1/2$. According to Selvadurai and Nguyen (1995) and Lewis and Schrefler (1998), the stability of the solution can be achieved by selecting values of γ close to unity. In order to analyze the poroelasticity problem that incorporates influences of damage evolution, an incremental finite element procedure needs to be developed. Such a procedure was outlined by Mahyari and Selvadurai (1998). We adopt two approaches to the damage evolution problem; the first permits damage-induced alteration of the elasticity and hydraulic conductivity to evolve *without consideration* of the dependency of damage on the stress space, but with the use of the damage and permeability evolution criteria described previously by Equations (11) and (12) respectively. The scalar damage variables are first obtained at the nine Gauss points within the finite elements. The constitutive matrix C^D_{ijkl} and the permeability $k^D$are updated at these locations, at each time step, to account for the evolution of damage. The discretized governing equations are then solved to obtain the state of strain at each integration point using the updated values of $\mathbf{C}^d$and κ^d. An iterative process using a standard Newton–Raphson technique solves the coupling between the state of strain and the state of damage in each step. The convergence criterion adopted in the analysis is based on the norm of the evolution of the damage variable in relation to a prescribed tolerance (Simo and Ju, 1987). Details of the iterative procedure and the associated numerical algorithm are summarized by Mahyari and Selvadurai (1998). In the second approach we assume that the damage evolution is *dependent* on the stress state as defined by the various combinations of the principal strains. It is reasonable to assume that the process of cavity and defect development in the poroelastic fabric is enhanced when the triaxial stress state is tensile and such effects can be suppressed when the stress state is compressive. Other combinations of principal stresses, involving tensile and compressive stresses can induce different magnitudes of damage evolution. As with the development of yield criteria for geomaterials, the characterization of stress-dependent damage requires the experimental determination of the material response to differing stress or strain paths. The experimental verifications of the stress-dependent damage evolution in geomaterials are relatively scarce; the limited data point to the observation that damage can increase when the material experiences volume expansion (Schulze et al., 2001). A plausible approximation is to assume that damage will be initiated when the strain state satisfies the criterion

$$I_1 = \mathrm{tr}\varepsilon_{ij} > 0 \tag{14}$$

where *tensile strains* are considered to be *positive*. There is also the possibility for the enhancement of the elasticity properties, which could occur as a result of void reduction or void closure during compressive loading of the poroelastic medium. Such phenomena have been observed during experiments conducted on materials such as granite (Zhu and Wong, 1997). The enhancement of the elastic properties are most likely to occur when the defects have elongated shapes such as cracks or flattened cavities; this will require the consideration of oriented defects that are beyond the scope of the present paper. Since attention is directed to the isotropic idealization of the damage modeling, we restrict attention to damage enhancement for all states of strain that satisfy (14) and assume that the porous fabric remains intact for any other state of strain. Since the objective of the study is to gain some insight into the loading-space dependency of damage evolution on the poroelastic response of the fluid inclusion, the assumption (14) as a criterion for stress space-dependent damage is justifiable. Also, since an objective of the paper is to conduct a comparison between the results for the pore pressure response of the fluid for the different types of damage development scenarios, attention is restricted to this relatively elementary representation of damage in the porous fabric of the poroelastic medium. The basic computational algorithm used in the numerical computations is shown in Figure 3. Again, the convergence criterion adopted in the termination of the computations is based on the norm of the evolution of the damage variable in relation to a prescribed tolerance.

(I) Compute $I_1 = \varepsilon_{ii}$

(II) Check the criteria

$$I_1 < 0$$

No: Damage growth. Go to (III)

Yes: No further damage evolution. Use poroelastic parameters E, k with no more damage-induced modification. Go to (V).

(III) Compute D at Gauss integration points

$$D = D_C - (D_C - D_0)(1 + \gamma\xi_d)^{\eta/\gamma D_C} \exp(-\eta\xi_d / D_C)$$

$$\xi_d = (e_{ij} e_{ij})^{1/2},\ e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij}$$

(IV) Update the poroelastic parameters

$$E^d = (1 - D)E$$

$$k^d = (c_1 + c_2\xi_d)k^0$$

or

$$k^d = (c_3 + c_4\xi_d{}^2)k^0$$

(V) Solve the governing equations for u_t, p_t and calculate the strain tensor

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

(VI) Check the criteria

$$D \geq D_C$$

No: Damage growth. Return to (I).

Yes: No further damage evolution. Exit.

Figure 3. *Computational algorithm for stress space-dependent evolution of damage in the poroelastic medium.*

THE FLUID-FILLED SPHERICAL CAVITY PROBLEM

Prior to the examination of the fluid cavity problem, we examine the poroelastic response of a one-dimensional element that is subjected to uniaxial total stress in the form of a Heaviside step function. The purpose of this analysis is to examine the accuracy of the computational algorithms developed for the treatment of both the general problem and the loading space-dependent damage evolution in the porous fabric of the poroelastic medium. The uniaxial problem is examined by considering a three-dimensional domain where appropriate boundary conditions are prescribed on the displacements, tractions and the pore pressure fields to simulate the one-dimensional problem (Figure 4). For the purposes of the computational modelling, we select the damage susceptible poroelastic material as sandstone, the properties of which have been documented by Cheng and Dusseault (1993) as follows:

Elasticity parameters: $E = 8300\,\text{MPa}$; $\nu = 0.195$; $\nu_u = 0.4999$

Failure parameters: $\sigma_C = 30\,\text{MPa}$; (compressive); $\sigma_T = 3\,\text{MPa}$ (tensile)

Damage parameters: $\gamma = \eta = 130$; $D_C = 0.75$; $\beta = 3.0 \times 10^5$

Fluid transport parameters: $k^0 = \kappa^0 \rho_w g / \mu_v = 10^{-6}\text{m/s}$

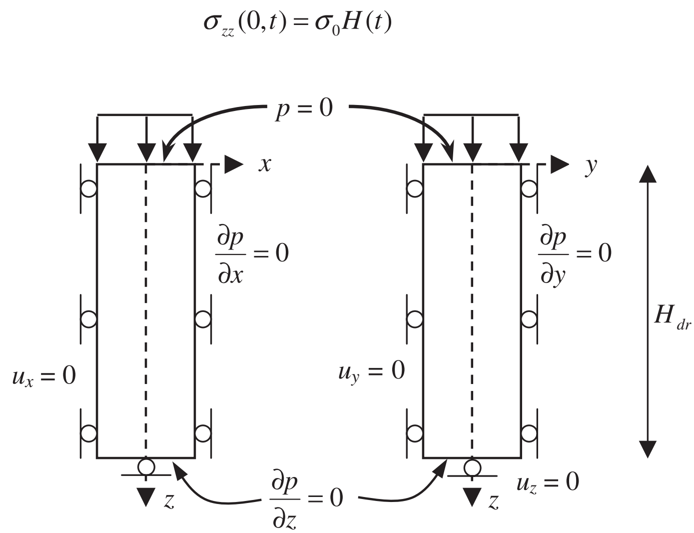

Figure 4. *Finite element discretization of the one-dimensional consolidation of a poroelastic medium; geometry and boundary conditions.*

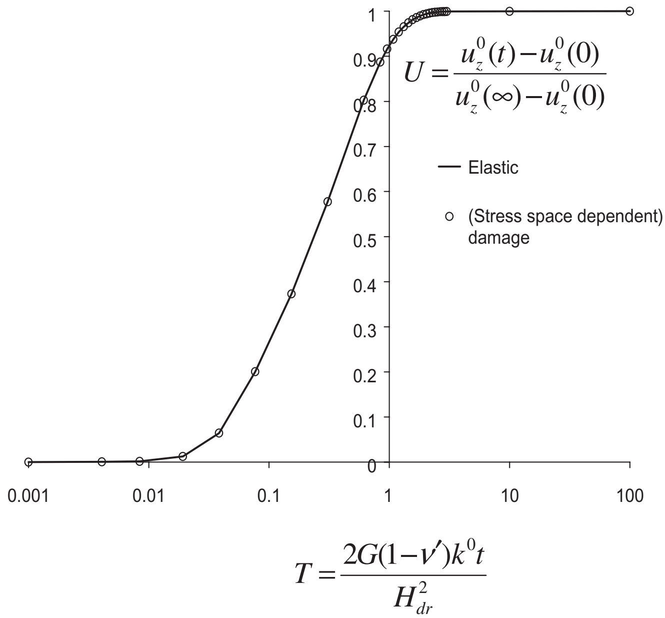

Figure 5. *One-dimensional consolidation response for a damage susceptible poroelastic medium: stress-independent damage evolution during compressive loading.*

where ρ_w is the mass density of water and μ_v is the dynamic viscosity of the fluid. Figure 5 illustrates the time-dependent variation in the degree of consolidation of the one-dimensional element, for the case where the step-function load corresponds to a *compressive stress*. In this instance, the poroelastic medium is uninfluenced by the damage process and the degree of consolidation corresponds to the classical result obtained by Terzaghi (1943). Figure 6 illustrates the time-dependent variation of the degree of consolidation for the case where the poroelastic material is subjected to a tensile load in the form of a Heaviside step function. In this case there is both damage-induced alteration of the *elastic stiffness* and *fluid transport* characteristics of the poroelastic medium. The results indicate the consolidation responses for various cases ranging from alterations in the elastic stiffness to alterations in the fluid transport characteristics.

We now focus our attention on the computational modelling of the problem of an extended, damage-sensitive poroelastic medium that is bounded internally by a fluid-filled spherical cavity and subjected to a far field triaxial stress state characterized by an axial stress σ_A and a radial stress σ_R, both of which have time-dependencies in the form of a Heaviside step function. The finite element discretization and the associated boundary conditions are shown in Figure 7. In the computational modelling, since the fluid within

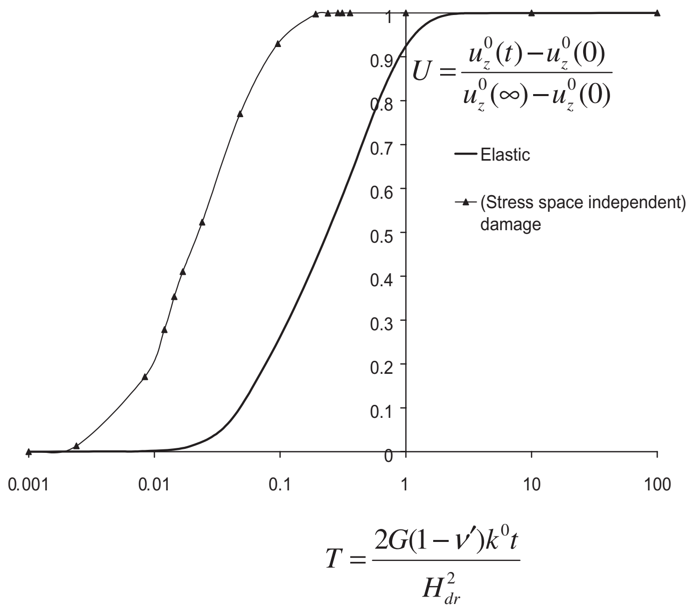

Figure 6. *One-dimensional consolidation response for a damage susceptible poroelastic medium: stress-independent damage evolution.*

the cavity is considered to be incompressible in comparison to the surrounding poroelastic medium, it is treated as a material with the following properties:

$$\text{Fluid deformability parameters: } G = 0;$$
$$\nu_f = 0.4999\,\text{(near incompressible fluid).}$$

This eliminates the need to model the fluid through the incorporation of special fluid elements. The computational modelling can be used to examine the pore pressure rise in the fluid-filled spherical cavity, for a variety of cases involving the following: (i) the alteration in the stiffness of the porous skeleton due to damage evolution, (ii) the alterations in the fluid transport characteristics due to damage evolution (quadratic variation defined by (13)) and (iii) the dependency of the damage-induced alterations of the deformability and fluid transport characteristics on the stress space. Figures 8 and 9 illustrate a wide range of results for the time-dependent evolution of the fluid pressure in the spherical cavity. The results indicate that permeability changes due to damage evolution have a greater influence on both the magnitude and decay rate in the pressures within the inclusion. Since the stress-dependency in the damage evolution is excluded from

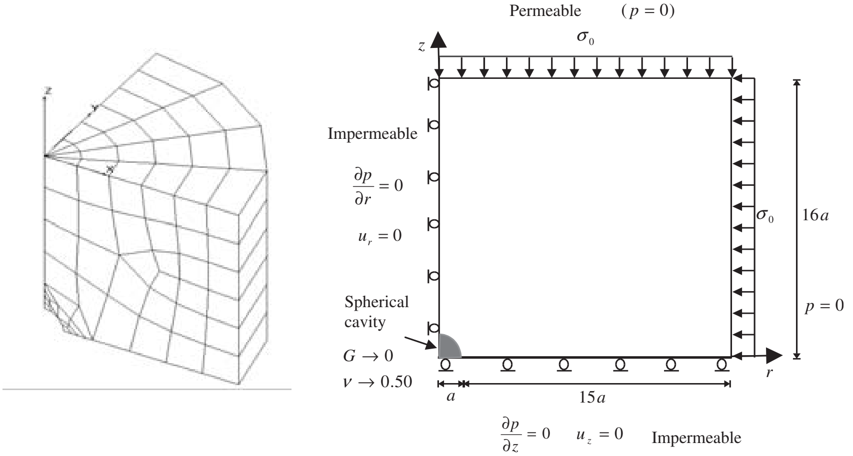

Figure 7. *Finite element discretization of damage susceptible poroelastic medium bounded internally by a spherical fluid inclusion: geometry and boundary conditions.*

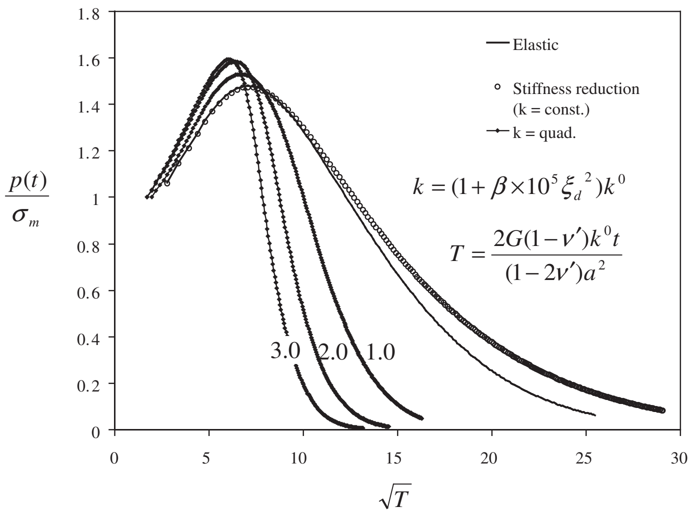

Figure 8. *Evolution of pressure in the fluid-filled spherical cavity: poroelastic medium subjected to an isotropic stress field (Stress space-independent damage evolution).*

the computations, the results are equally applicable to the case where the far-field applied stresses are tensile. In this case the induced fluid pressures in the inclusion should be interpreted in a tensile sense. The analysis of the fluid-filled spherical cavity problem is now extended to include the case where the extended medium is subjected to a far field triaxial stress state defined by an axial stress σ_A and a radial stress σ_R (Figure 1). Both these

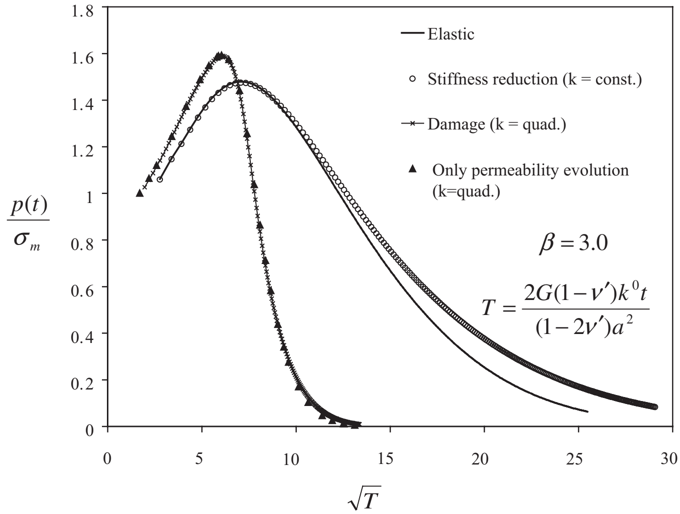

Figure 9. *Evolution of pressure in the fluid-filled spherical cavity: the poroelastic medium subjected to an isotropic stress field (Stress space-independent damage evolution).*

stresses are applied as time-dependent stresses defined by a Heaviside step function commencing at time $t = 0$. In this case the development of the damage zones will not display the spherical symmetry encountered in the previous example and the pressure evolution within the fluid inclusion has to be evaluated through computational considerations. The parameters influencing the poroelasticity problem include (i) the alteration in the stiffness of the porous skeleton due to damage evolution, (ii) the alterations in the fluid transport characteristics due to damage evolution (quadratic variation defined by (12)), and (iii) the ratio of the far field stresses. Typical results that illustrate the influence of the far-field stress ratio R on the development of pressures within the fluid-filled spherical cavity are shown in Figures 10–13. Again, the stress space-dependency in the damage evolution is not considered in the computations. For the range of values of the stress ratio examined, the differences between the poroelastic problem and the poroelastic problem that incorporates influence of damage appears to be marginal, particularly with respect to the development of the peak pressure. The decay rate is, however, influenced by the presence of material damage. Figure 14 presents a comparison between the pressure decay patterns for the fluid-filled spherical cavity in non-damaging and damage-susceptible poroelastic media and subjected to non-isotropic far field stress fields. These results indicate an appreciable influence of the anisotropy of the far-field stress states on the pore fluid pressure decay pattern. Figures 15–17 present the influence of stress space-dependent damage evolution for isotropic and

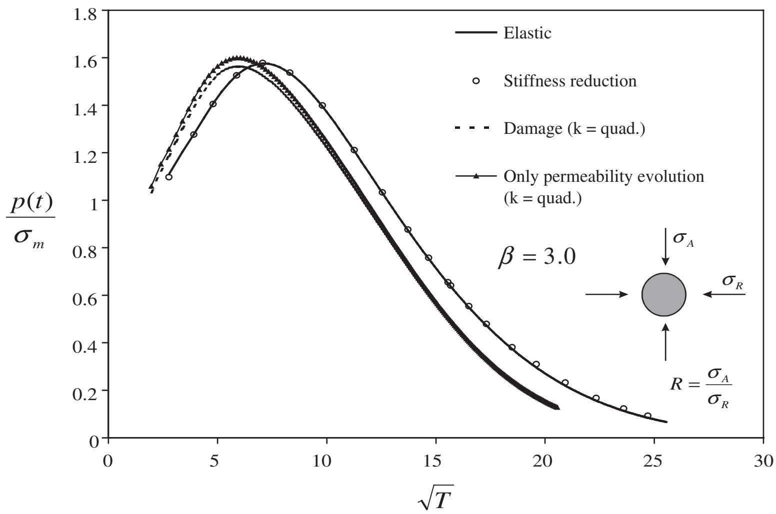

Figure 10. *Evolution of pressure in the fluid-filled spherical cavity: the poroelastic medium subjected to a non-isotropic stress field* ($R = 2$) *(Stress space-independent damage evolution).*

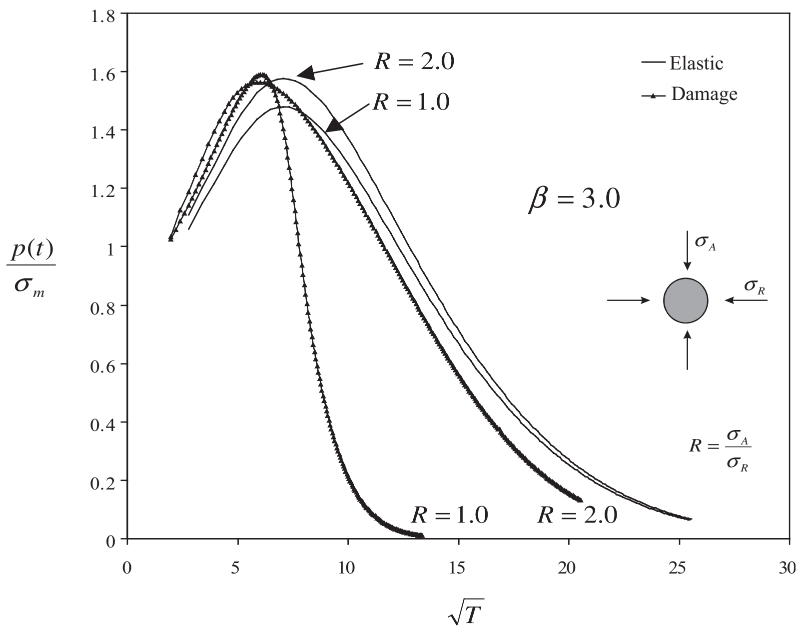

Figure 11. *Evolution of fluid pressure in the fluid-filled spherical cavity: poroelastic medium subjected to a non-isotropic stress field. Comparison of results for damaged and non-damaged material responses (Stress space-independent damage evolution).*

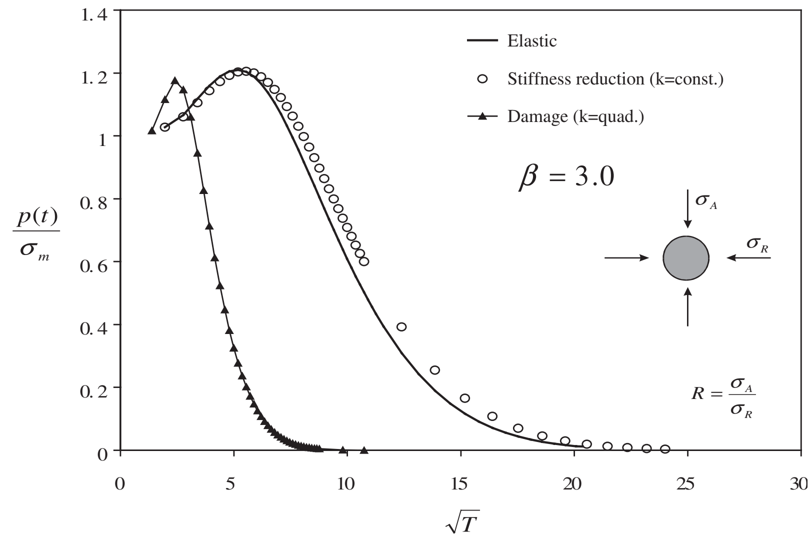

Figure 12. *Evolution of pressure in the fluid-filled spherical cavity: the poroelastic medium subjected to a non-isotropic stress field* ($R = 1/6$) *(Stress space-independent damage evolution).*

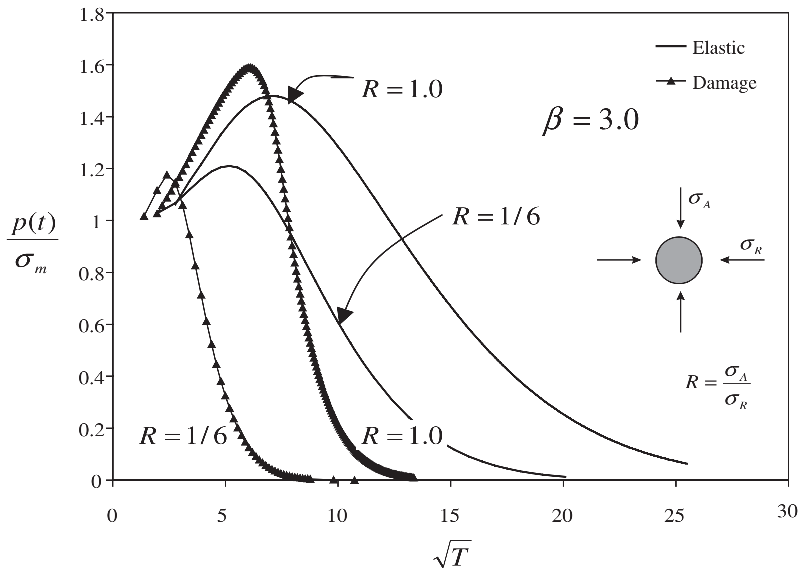

Figure 13. *Evolution of fluid pressure in the fluid-filled spherical cavity: poroelastic medium subjected to a non-isotropic stress field. Comparison of results for damaged and non-damaged material responses (Stress space-independent damage evolution).*

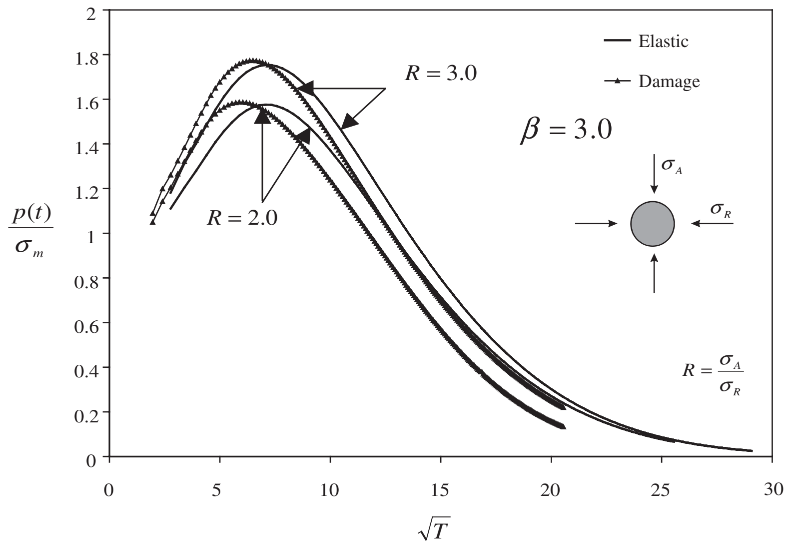

Figure 14. *Evolution of fluid pressure in the fluid inclusion: poroelastic medium subjected to a non-isotropic stress field. Comparison of results for the damaged and non-damaged material responses.*

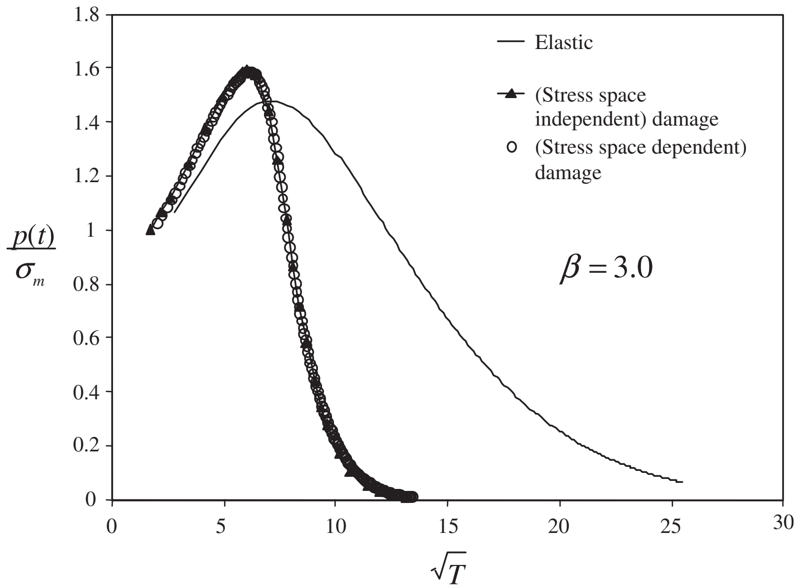

Figure 15. *Evolution of pressure in the fluid-filled spherical cavity: the poroelastic medium subjected to an isotropic stress field (Stress space-dependent and stress space-independent damage evolution).*

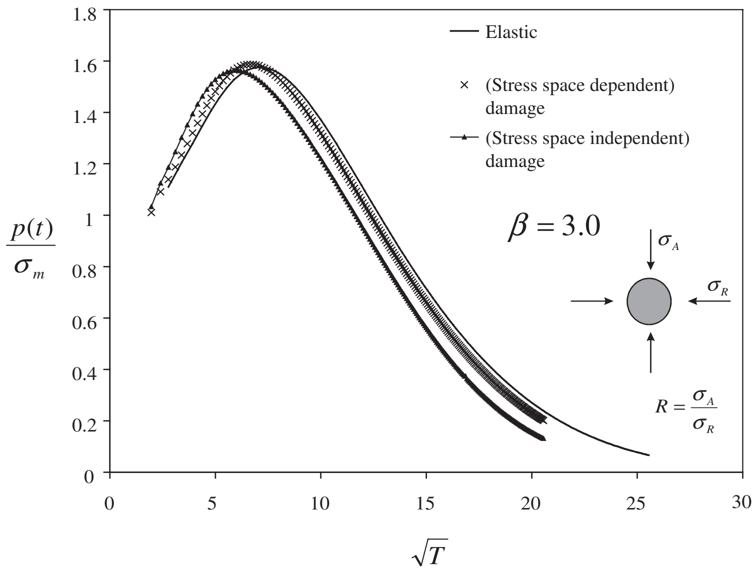

Figure 16. *Evolution of pressure in the fluid-filled spherical cavity: the poroelastic medium subjected to a non-isotropic stress field ($R = 2$). Comparison of results for stress space-independent and stress space-dependent damaged material responses.*

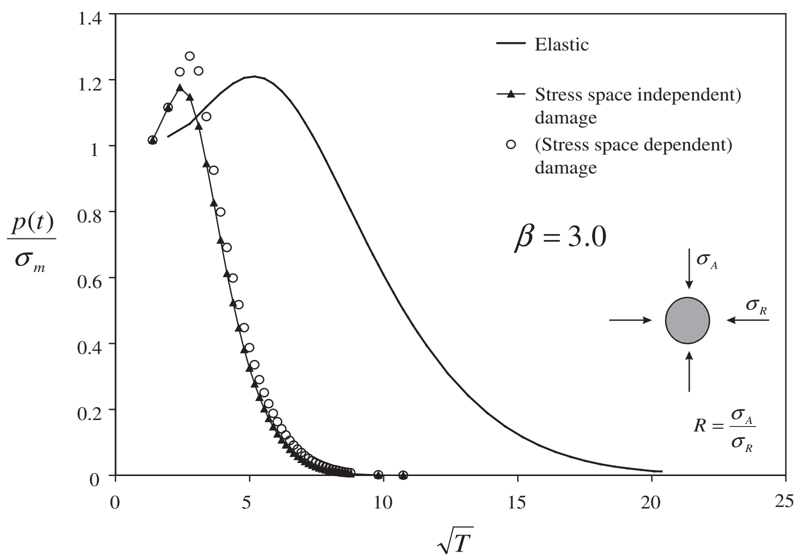

Figure 17. *Evolution of pressure in the fluid-filled spherical cavity: the poroelastic medium subjected to a non-isotropic stress field ($R = 1/6$). Comparison of results for stress space-independent and stress space-dependent damaged material responses.*

triaxial far-field stress states with different far-field stress ratio R on the development of pressures within the fluid-filled spherical cavity. These results indicate an appreciable influence of stress space-dependency for damage evolution on the pore fluid pressure decay pattern.

CONCLUDING REMARKS

The development of damage is conventionally identified with the reduction of the deformability characteristics of a continuum region. In the context of a fluid-saturated poroelastic medium, the development of damage can also introduce changes to its fluid transport characteristics. This latter effect can have a significant influence on the rate at which pore pressures decay within the poroelastic medium. This paper proposes an elementary approach for the computational modelling of the development of transient damage that is accompanied by reduction in the elasticity characteristics of the porous medium and an attendant increase in the hydraulic conductivity characteristics. The computational developments also examine the influence of the stress state on the damage-induced alterations in the poroelastic properties. The methodology is applied to examine the behaviour of a damage-susceptible poroelastic solid that is bounded internally by a fluid-filled spherical cavity. The rise and decay of the pressure in the fluid-filled cavity is taken as an indicator of the various influences of the damage-induced alterations in the elasticity and fluid transport characteristics of the poroelastic solid. It is shown that the damage-induced alterations in the hydraulic conductivity characteristics of the poroelastic medium have a significant influence on the magnitude of the amplification of the fluid pressure in the spherical cavity as well as its decay rate. The damage-induced alterations in the elasticity characteristics of the poroelastic medium appear to have a lesser influence on the fluid pressure transients in the fluid-filled spherical cavity. The response of the fluid pressure in the cavity due to non-isotropic triaxial far field stress states have also been investigated. The results of this study also point to the dominant influence of the hydraulic conductivity alterations on both the amplification and decay rate of the induced fluid pressure in the spherical cavity. The numerical results generally support the view that fluid pressure evolution and decay in poroelastic media, in terms of its amplification and decay, can be strongly influenced by both the damage-dependent evolution of fluid transport and stiffness characteristics.

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