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Computational modelling of frost heave induced soil–pipeline interaction
I. Modelling of frost heave

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Abstract

This research focuses on the development of a computational approach to the study of soil–pipeline interaction due to the development of discontinuous frost heave within a frozen soil region. The modelling of frost heave development within the soil is an important aspect in the computational treatment of the interaction problem. This paper deals with the calibration of a three-dimensional computational approach for the study of frost heave development, which takes into consideration the coupled effects of heat conduction and moisture migration. The results of one-dimensional frost heave tests are used to calibrate the computational approach. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

The development of frost action in soils is an important consideration in connection with the analysis and design of civil engineering components such as structural foundations, buried pipelines and culverts, highway pavements, retaining walls and other earth structures located in cold regions (see, e.g., Andersland and Anderson, 1978; Morgenstern, 1981; Anderson et al., 1984; National Academy Press, 1984; Phukan, 1985, 1993; Andersland and Ladanyi, 1994). The development of frost action in soils is also an important consideration for ground freezing techniques used in underground construction (see, e.g., Jessburger, 1979; Kinosita and Fukuda, 1985; Jones and Holden, 1988; Andersland and Ladanyi, 1994; Knutsson, 1997). An important aspect in the study of ground freezing pertains to the estimation of heave that accompanies the frost action. Traditionally, frost heave modelling has involved one-dimen-

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sional treatments (see, e.g., Anderson et al., 1984; Konrad and Morgenstern, 1984; Holden et al., 1985; Kay and Perfect, 1988; Lewis and Sze, 1988; Saarelainen, 1992). The subject matter is, however, continually being extended to include continuum models with three-dimensional forms (see, e.g., Blanchard and Fremond, 1985; Fremond and Mikkola, 1991; Hartikainen and Mikkola, 1997; Talamucci, 1997). The adaptability and reliability of these developments, for practical application to problems such as frost heave induced mechanics of civil engineering structures is as yet unproven. A comprehensive continuum model of frost heave mechanics should take into consideration of a variety of complex hydro-thermo-mechanical processes. These could include (i) coupled processes of heat conduction and moisture transport within the frozen and unfrozen soils, (ii) the mechanical behaviour of the frozen and unfrozen soils, (iii) moving boundary problems associated with the growth of a freezing front and (iv) the nucleation and growth of ice lenses in an anisotropic fashion. Currently, there appears to be no satisfactory model which can accommodate all of the above aspects in a comprehensive fashion. Furthermore, the simultaneous consideration of all such time- and temperature-dependent non-linear, hydraulic, mechanical and phase transformation processes is a difficult task. The difficulties arise from several aspects; firstly, the mathematical modelling of the fundamental processes governing all coupled thermo-mechanical processes can be defined only under highly idealized conditions; secondly, the modelling of three-dimensional effects involving the processes (i) to (iv) requires an inordinate amount of computing resources; and finally, the accurate in situ determination of the non-linear hydro-thermo-mechanical properties of soils encountered at even a specific location is a difficult task.

A prudent approach to the modelling of frost heave generation is to consider simplifications where the heat conduction and moisture movement and frost heave generation can be considered as coupled processes which are independent of the mechanical processes. Within the context of such a simplification the evolution of frost heave can be obtained by a separate analysis. Examples of such developments include the two-dimensional ''geothermal simulator'' approach proposed by Nixon (1987a; b)) which relies on the ''segregation potential'' approach proposed by Konrad and Morgenstern (1984) for the study of frost heave generation around buried pipes. The primary limitation of this model is that it is not a continuum theory where the governing equations are posed in complete tensorial form. Such generalized formulations are essential from the point of view of studies involving three-dimensional forms of frost heave generation. The approach, however, has considerable merit as a useful first approximation and has the distinct advantage that the constitutive parameters governing the frost heave process can be determined relatively conveniently (Konrad, 1987).

An alternative approach is to examine the development of frost action by considering a model which accommodates the coupled processes of heat conduction and moisture transport in saturated/unsaturated partially frozen soils. Examples of such developments in the literature are documented in the articles by (Anderson et al., 1984; Kay and Perfect, 1988; Saarelainen, 1992; Knutsson, 1997; Kujala, 1997). Of particular interest from the point of view of plausible engineering applications of the development of three-dimensional frost heave effects are the studies by Shen and Ladanyi (1987; 1991). The focus of this paper is to develop a three-dimensional computational formulation of the modified hydrodynamic model of frost action in soils proposed by Shen and Ladanyi (1987) and to calibrate the performance of the model in relation to available experimental results of one-dimensional frost heave tests. The paper develops the computational approach and presents both results of calibration exercises and results for idealized situations involving three-dimensional frost heave development in cuboidal elements.

2. Fundamental equations

The model proposed by Shen and Ladanyi (1987) is a continuum representation of heat and moisture flow in a freezing soil. The coupling occurs as a result of the influence of the heat conduction process on the moisture movement within both frozen and unfrozen zones. The primary mode of heat transfer in the medium is assumed to be that of heat conduction. The velocities associated with the flow of water and ice within the pore structure are assumed to be small enough to neglect the convection term associated with heat transfer. For heat conduction, we assume that the medium is thermally isotropic such that

$$C\frac{\partial T}{\partial t} = \lambda \nabla^2 T + L\rho_{(i)}\frac{\partial \theta_{(i)}}{\partial t} \tag{1}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{2}$$

is Laplace operator referred to the three-dimensional rectangular Cartesian coordinates (x, y, z); T is the temperature at a point within the continuum (units: °C); $\theta_{(i)}$ is the volume fraction of the ice (units: m^3/m^3) which is related to the gravimetric ice context ($m_{(i)}$) by $\theta_{(i)} = \rho_d m_{(i)}/\rho_{(i)}$. The thermal and physical parameters of medium are as follows: λ — thermal conductivity of soil [W m^{-1} (°C^{-1})]; C — heat capacity of soil [J m^{-3} (°C^{-1})]; L — latent heat of fusion [J/kg]; $\rho_{(i)}$ — density of ice [kg/m^3]; ρ_d — dry density of soil [kg/m^3]; $m_{(i)}$ — ice content by weight (normalized with respect to the weight of solids) [kg/kg].

It is evident that, in the absence of the moisture component ($\theta_{(i)} = 0$), the modified heat conduction equation (Eq. (1)) reduces to the classical Fourier equation for heat conduction.

In the frozen fringe in a fine grained soil, not all the water within the soil pores freezes at 0°C. In some clays up to 50% of the moisture may exist in a liquid state even at −2°C (Konrad, 1984; Konrad and Duquennois, 1993). This unfrozen water is mobile and can migrate under the action of a suction gradient. Again, assuming isotropy in relation to the moisture flow processes, we obtain the equation governing moisture flow as

$$\frac{\partial}{\partial t}\left[\theta_{(w)} + \frac{\rho_{(i)}}{\rho_{(w)}}\theta_{(i)}\right] = \frac{k}{\gamma_{(w)}}\nabla^2 P_{(w)} \tag{3}$$

where $P_{(w)}$ is the cryogenic pressure in the mobile water and $\theta_{(w)}$ is the volume fraction of water. The hydraulic and physical parameters associated with moisture flow are as follows: $\gamma_{(w)}$ = unit weight of water [kN/m^3]; $\rho_{(w)}$ = density of water [kg/m^3]; k = hydraulic conductivity of the soil [m/s].

The pressure developed in the frozen fringe consists of water pressure ($P_{(w)}$) and the ice pressure ($P_{(i)}$). The water pressure and the ice pressure can be related by the Clausius–Clapeyron equation. By assuming that ice pressure (i) reduces to zero at the frost front (0°C isotherm); (ii) is equal to the local mean stress at the coldest side of the freezing fringe and (iii) has a linear variation within the frozen fringe (Shen and Ladanyi, 1987), the water pressure can be obtained from the relationship;

$$\frac{P_{(w)}}{\rho_{(w)}} - \frac{P_{(i)}}{\rho_{(i)}} = L\ln\left(\frac{T_k}{T_0}\right) \tag{4}$$

where T_k is the absolute temperature in the soil. Assuming that $\theta_{(w)} = \theta_{(w)}(T)$, and using Eqs. (1), (2) and (4), we obtain the equation governing coupled heat and moisture flow as

$$\bar{C}\frac{\partial T}{\partial t} = \bar{\lambda}\nabla^2 T + L\rho_{(w)}k\nabla^2 P_{(i)} \tag{5}$$

where

$$\bar{C} = C + L\rho_{(w)}\frac{\partial \theta_{(w)}}{\partial T}$$

$$\bar{\lambda} = \lambda + \frac{k\rho_{(w)}^2 L^2}{\gamma_{(w)}T_k}. \tag{6}$$

If the effect of ice pressure on the heat conduction can be neglected, Eq. (5) can be reduced to the form

$$\bar{C}\frac{\partial T}{\partial t} = \bar{\lambda}\nabla^2 T \tag{7}$$

Eqs. (3), (4) and (7) serve as the governing equations for coupled heat conduction and moisture flow. In these equations, the relationship between the liquid water content in the frozen soil and the temperature ($\theta_{(w)} = f(T)$) must be determined experimentally.

The boundary conditions governing the field equations are related to T, $P_{(w)}$ and $\theta_{(w)}$. Considering a region Ω with boundary S, the essential boundary conditions for the temperature field can be written as

$$T = T^{*} \text{ on } (x,y,z) \in S_1 \tag{8}$$

where S_1 is a subset of S. For a boundary S_2 through which there is heat loss

$$\lambda \frac{\partial T}{\partial n} + a_T T + b_T = 0 \text{ on } (x,y,z) \in S_2 \tag{9}$$

where a_T and b_T are constants chosen to fit a particular boundary and n is the unit normal to S_2.

Similarly on a boundary with prescribed $P_{(w)}$ we have

$$P_{(w)} = P^{*}_{(w)} \text{ on } (x,y,z) \in S_3. \tag{10}$$

The boundary condition governing restricted water flow across a boundary can be written as

$$k \frac{\partial P_{(w)}}{\partial n} + c_{(w)} P_{(w)} + d_{(w)} = 0 \text{ on } (x,y,z) \in S_4 \tag{11}$$

where $c_{(w)}$ and $d_{(w)}$ are constants chosen to fit the particular boundary. The formulation of the initial boundary value problem will be complete when suitable, initial conditions are prescribed for T and $P_{(w)}$, i.e.,

$$T = \bar{T}^{0};\ P_{(w)} = \bar{P}^{0}_{(w)} \text{ on } (x,y,z) \in \Omega. \tag{12}$$

It must be remarked that in the Clausius–Clapyeron equation (Eq. (4)), both the pore water pressure and ice pressure are treated as scalar variables. While the pore water pressure can be treated as a scalar variable, the pressures in the ice will not be a scalar quantity. Due to the rigidity of the ice, the stresses in the ice will have a tensorial structure. This aspect is a limitation in the modelling but a useful and universally accepted result. The Clausius–Clapyeron equation can be generalized to include three-dimensional effects (see, e.g., Fremond and Mikkola, 1991). The adaptation of these results are, however, beyond the scope of the current study.

The freezing of pore water induces a volumetric strain. By virtue of ice lens formation, the volumetric expansion is generally anisotropic. In the current study, however, the frost heave is assumed to be distributed, isotropic and the associated incremental strains are given by

$$\mathrm{d}\varepsilon^{(h)}_{ij} = \mathrm{d}\varepsilon^{(h)}\delta_{ij} \tag{13}$$

where $\mathrm{d}\varepsilon^{(h)}$ is the volumetric expansion strain due to frost heave for a time interval $\mathrm{d}t$, i.e.,

$$\mathrm{d}\varepsilon^{(h)} = 0.09\mathrm{d}\theta_{(i)} + \mathrm{d}\theta_{(w)} \tag{14}$$

and $\mathrm{d}\theta_{(i)}$ and $\mathrm{d}\theta_{(w)}$ are, respectively, the incremental changes in the pore ice content and pore water at time $\mathrm{d}t$.

From the preceding discussions, it is clear that no allowance is made for mechanical deformations of the soil skeleton, the pore ice or the frozen soil. These aspects will be considered in the examination of the complete soil–pipeline interaction problem. The primary objective of this paper is to examine the basic response of the frost heave model in replicating observed trends in tests conducted with frost susceptible soils.

3. Computational modelling of frost heave development

The Galerkin finite element technique has been used quite successfully to examine transient problems involving coupled processes (see, e.g., Bathe, 1982; Lewis and Schrefler, 1987; Zienkiewicz and Taylor, 1989; Selvadurai and Nguyen, 1995; Selvadurai, 1996). By introducing an arbitrary weighting function δT for the temperature field, in general, we have

$$\int_{\Omega} \left\{ \bar{\lambda} \nabla \delta T (\nabla T) - \bar{C} \delta T \frac{\partial \boldsymbol{T}}{\partial t} \right\} \mathrm{d}\Omega + \int_{S_2} \delta T \{ a_T T + b_T \} \mathrm{d}S = 0. \tag{15}$$

Introducing the shape function [N], it can be shown that (see, e.g., Eq. (7))

$$\int_{\Omega}[\mathbf{N}]\left[\bar{\lambda}\nabla^{2}T\right]\mathrm{d}\Omega=\int_{\Omega}[\mathbf{N}]\bar{C}\frac{\partial T}{\partial t}\mathrm{d}\Omega. \tag{16}$$

Considering the natural boundary condition, $(\partial T)/(\partial n)=0$ and using the result

$$\{\mathbf{T}\}=[\mathbf{N}]\{\mathbf{T}_{\mathrm{i}}\} \tag{17}$$

Eq. (16) can be rewritten as

$$[\mathbf{K}]\{\mathbf{T}\}-[\mathbf{K}_{c}]\frac{\partial\{\mathbf{T}\}}{\partial t}=\{\mathbf{F}\} \tag{18}$$

where

$$[\mathbf{K}]=\int_{\Omega}\bar{\lambda}[\mathbf{B}]^{T^{*}}[\mathbf{B}]\mathrm{d}\Omega$$

$$[\mathbf{K}_{c}]=\int_{\Omega}\bar{C}[\mathbf{N}]^{T^{*}}[\mathbf{N}]\mathrm{d}\Omega$$

$$[\mathbf{B}]=\nabla[\mathbf{N}] \tag{19}$$

$[\mathbf{B}]^{T*}$ denotes the transpose of $[\mathbf{B}]$ and $\{\mathbf{F}\}$ is a column matrix which is determined by the internal heat source and boundary conditions.

The time integration (Eq. (18)) is performed by employing a Crank–Nicholson method. For the jth and $(j+1)$th time increment

$$\left[[\mathbf{K}]+\frac{2}{\Delta t}[\mathbf{K}_{c}]\right]\{\mathbf{T}\}^{j+1}=\left[\frac{2}{\Delta t}[\mathbf{K}]_{c}-[\mathbf{K}]\right]\{\mathbf{T}\}^{j}+2\{\mathbf{F}\} \tag{20}$$

The temperature field at any time t can be computed by summation of all temperatures computed by Eq. (20) for each time step. When the temperature at some points are below the freezing temperature, the cryogenic suction or pore water pressure $P_{(\mathrm{w})}$ at these locations can be calculated by using the Clausius–Clapeyron equation (Eq. (4)). The moisture movement can be defined by the coupling between the volumetric water content $\theta_{(\mathrm{w})}$ and pore water pressure $P_{(\mathrm{w})}$. The governing equation (Eq. (3)) can be solved by using an explicit finite difference scheme. We have

$$\begin{aligned}(\theta_{(\mathrm{i})})_{lmn}^{j+1}={}&(\theta_{(\mathrm{i})})_{lmn}^{j}+\frac{\rho_{(\mathrm{w})}}{\rho_{(\mathrm{i})}}\Big[a_{l}\bar{\lambda}_{l-\frac{1}{2}}^{j+1}(P_{(\mathrm{w})})_{l-1mn}^{j+1}\\&-\Big(a_{l}\bar{\lambda}_{l-\frac{1}{2}}^{j+1}+b_{l}\bar{\lambda}_{l+\frac{1}{2}}^{j+1}\Big)(P_{(\mathrm{w})})_{lmn}^{j+1}\\&+b_{l}\bar{\lambda}_{l+\frac{1}{2}}^{j+1}(P_{(\mathrm{w})})_{l+1mn}^{j+1}\Big]+\frac{\rho_{(\mathrm{w})}}{\rho_{(\mathrm{i})}}\\&\times\Big[a_{m}\bar{\lambda}_{m-\frac{1}{2}}^{j+1}(P_{(\mathrm{w})})_{lm-1n}^{j+1}\\&-\Big(a_{m}\bar{\lambda}_{m-\frac{1}{2}}^{j+1}+b_{m}\bar{\lambda}_{m+\frac{1}{2}}^{j+1}\Big)(P_{(\mathrm{w})})_{lmn}^{j+1}\\&+b_{m}\bar{\lambda}_{m+\frac{1}{2}}^{j+1}(P_{(\mathrm{w})})_{lm+1n}^{j+1}\Big]+\frac{\rho_{(\mathrm{w})}}{\rho_{(\mathrm{i})}}\\&\times\Big[a_{n}\bar{\lambda}_{n-\frac{1}{2}}^{j+1}(P_{(\mathrm{w})})_{lmn-1}^{j+1}-\Big(a_{n}\bar{\lambda}_{n-\frac{1}{2}}^{j+1}\\&+b_{n}\bar{\lambda}_{n+\frac{1}{2}}^{j+1}\Big)(P_{(\mathrm{w})})_{lmn}^{j+1}+b_{n}\bar{\lambda}_{n+\frac{1}{2}}^{j+1}\\&\times(P_{(\mathrm{w})})_{lmn+1}^{j+1}\Big]-\frac{\rho_{(\mathrm{w})}}{\rho_{(\mathrm{i})}}\Big[(\theta_{(\mathrm{w})lmn})^{j+1}\\&-(\theta_{(\mathrm{w})lmn})^{j}\Big]\end{aligned} \tag{21}$$

with

$$a_{l}=\frac{\Delta t}{\Delta x_{l}(\Delta x_{l}+\Delta x_{l+1})}$$

$$b_{l}=\frac{\delta t}{\Delta x_{l+1}(\Delta x_{l}+\Delta x_{l+1})} \tag{22}$$

l, m and n are the spatial nodes along x, y and z directions, respectively.

The ultimate objective of the coupled heat–moisture flow algorithm is to utilize the procedure to compute the time dependent evolution of frost heave strains within the medium. Using the incremental initial strain method, the equivalent nodal forces increment vector $\mathrm{d}\{\boldsymbol{R}\}$ due to volumetric expansion in soil can be written as

$$\mathrm{d}\{\boldsymbol{R}\}=\int_{\Omega}[\mathbf{B}]^{T^{*}}[\mathbf{D}]\delta\{\mathrm{e}^{(h)}\}\mathrm{d}\Omega. \tag{23}$$

When examining one-dimensional problems involving frost heave development, the constraining of the deformations in two orthogonal directions will influence the extent of frost heave development in the unconstrained direction. Therefore, for the modelling of constrained one-dimensional deformations it is necessary to consider the deformability characteristics of the frozen and unfrozen soils. This aspect of the modelling will be considered in greater detail in the second part of this research dealing with the soil–pipeline interaction problem. For the present purposes, we shall assume that in the modelling of one-dimensional effects, the mechanical response of the frozen soil will be modelled by an isotropic elastic response. More complex non-linear models of the mechanical behaviour will be considered in the companion paper.

4. Numerical simulation of frost heave development

The general three-dimensional finite element procedure developed in the preceding section is applied to examine three specific problems involving frost heave generation in a cuboidal region with an edge length of 10 cm. The hydrothermal properties governing the frost heave generation are the heat capacity, the thermal conductivity and the hydraulic conductivity of the frost susceptible soil. For the purpose of modelling the frost heave, the frost susceptible soil is treated as an isotropic elastic medium. A variety of frost susceptible soils can be modelled; here, we restrict attention to the category of frost susceptible soils in which the hydrothermal properties are as defined by the following (see, e.g., Kay et al., 1977).

(i) The heat capacity of the frozen soil is defined by

$$C = C_s \theta_s + C_w \theta_w + C_i \theta_i \qquad (24)$$

where C_s, C_w and C_i are the heat capacities of the soil grains, water and ice and θ_s, θ_w and θ_i are the respective volume fractions.

(ii) The thermal conductivity of the frozen soil is defined by

$$\lambda = \lambda_s^{\theta_s} \lambda_w^{\theta_w} \lambda_i^{\theta_i} \qquad (25)$$

where λ_s, λ_w and λ_i are, respectively, the thermal conductivities of the solid particles, the water and ice.

(iii) The hydraulic conductivity of the frozen soil is modelled as a temperature dependent property. An example of such a variation is given by Horiguchi and Miller (1983).

$$k = \begin{cases} 3.072 \times 10^{-11} e^{13.438T}; & -0.3°C < T < T_f; \\ 5.453 \times 10^{-13}; & T \leq -0.3°C \end{cases} \text{m/s} \qquad (26)$$

where T is the absolute temperature and T_f is the absolute freezing temperature.

(iv) The deformability response of the frost susceptible soils is modelled as a linear elastic isotropic solid with constant values of elastic modulus (E) and Poisson's ratio (υ). The specific values used in the numerical computation are as follows:

$$C_s = 2.20 \times 10^6 \text{ J m}^{-3} (°C)^{-1};$$

$$C_w = 4.14 \times 10^6 \text{ J m}^{-3} (°C)^{-1};$$

$$C_i = 1.93 \times 10^6 \text{ J m}^{-3} (°C)^{-1} \qquad (27)$$

$$\lambda_s = 1.950 \text{ W m}^{-1} (°C)^{-1};$$

$$\lambda_w = 0.602 \text{ W m}^{-1} (°C)^{-1};$$

$$\lambda_i = 2.220 \text{ W m}^{-1} (°C)^{-1}; \qquad (28)$$

$$E = 12{,}000 \text{ kPa}; \quad \nu = 0.3. \qquad (29)$$

The self weight of the soil is taken as 19 kN/m^3.

4.1. A one-dimensional problem of frost heave development

We focus attention on the one-dimensional frost heave test conducted by Penner (1986). In this test, a cylindrical sample of saturated soil measuring 10 cm in diameter and 10 cm in length is subjected to freezing. The freezing was conducted in an open system by ramping the temperature at the two ends of the cylindrical sample at 0.02°C/day. The initial temperature at the top of the sample was 0.55°C and the temperature at the base of the sample was specified at −0.35°C. The resulting time-dependent frost heave and the frost penetration were recorded. The finite element procedure was used to examine this one-dimensional problem. Fig. 1 shows the three-dimensional finite element configuration used to model the one-dimensional frost heave problem. Although the configuration of the finite element discretization is three-dimensional, the boundary conditions applicable to heat conduction, moisture transport and deformations are organized in such a way that the hydro-thermo-mechanical processes result in a one-dimensional problem. The analysis is therefore applicable to the one-dimensional experimental configuration discussed by Penner (1986).

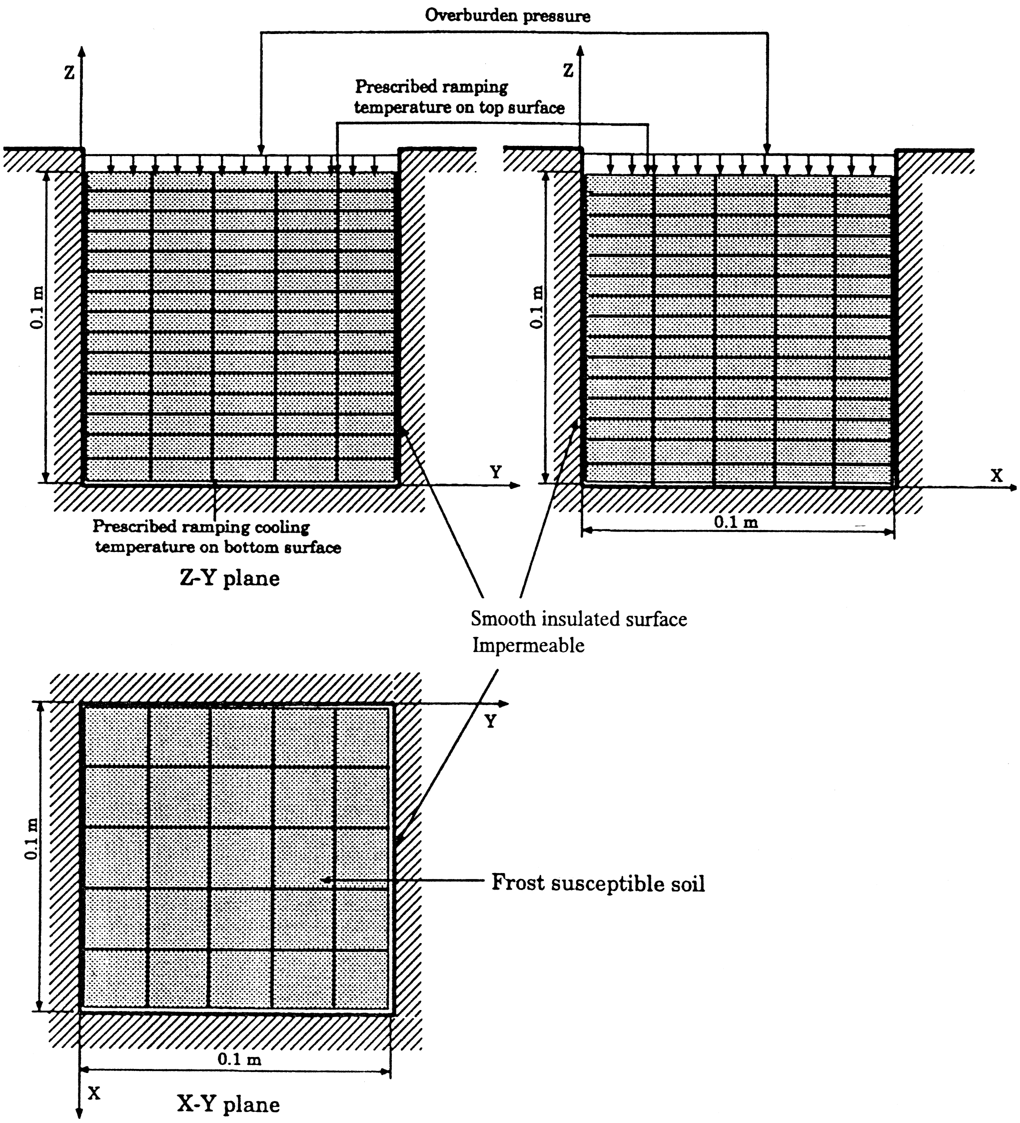

Fig. 1. Finite element discretization of the cuboidal element.

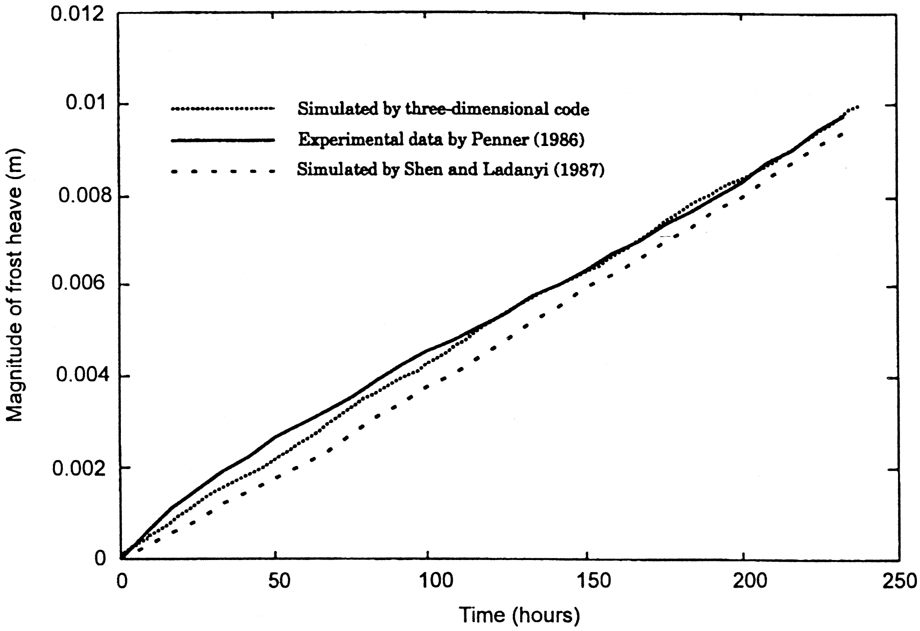

Fig. 2. Frost heave generation in a sample of silty clay.

The paper by Penner (1986) records the development of frost heave in an experiment which lasted approximately 250 h. Unfortunately, there is no record of the hydro-thermo-mechanical parameters

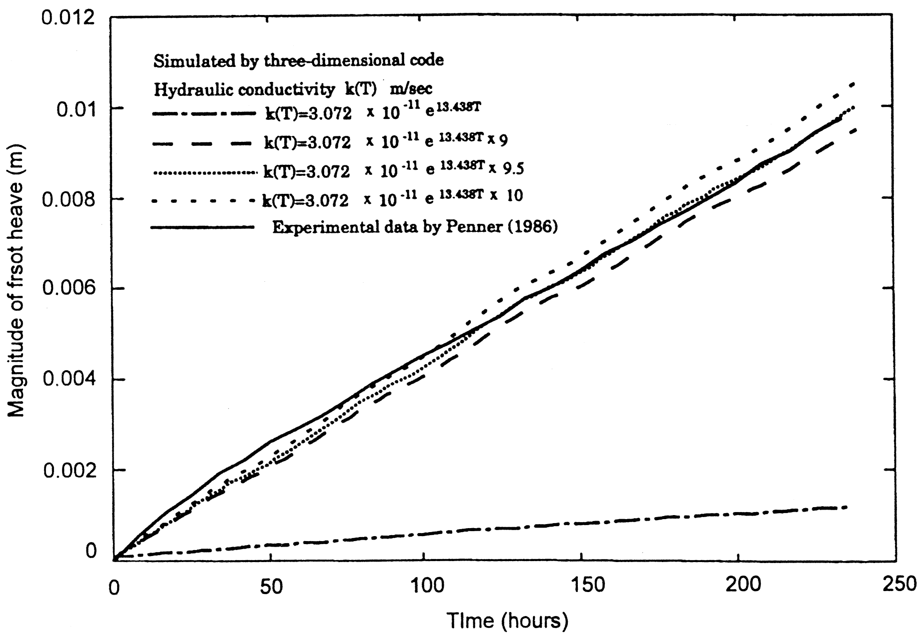

Fig. 3. Influence of hydraulic conductivity on the generation of frost heave.

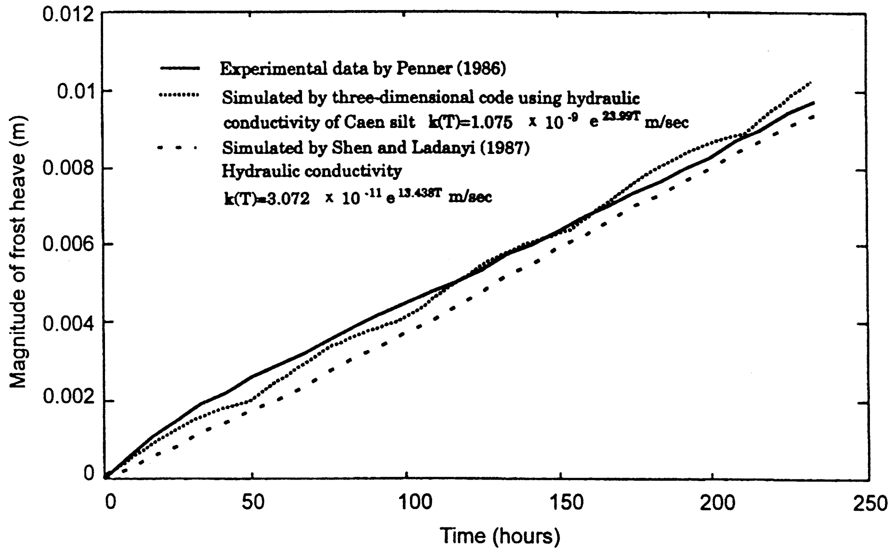

Fig. 4. Influence of hydraulic conductivity on the generation of frost heave.

applicable to the particular soil. The thermal conductivity and heat capacity parameters used in the computational modelling correspond to the generalized results given by Eqs. (24) and (25) and the associated constants defined by Eqs. (27) and (28). The hydraulic conductivity data are represented by the

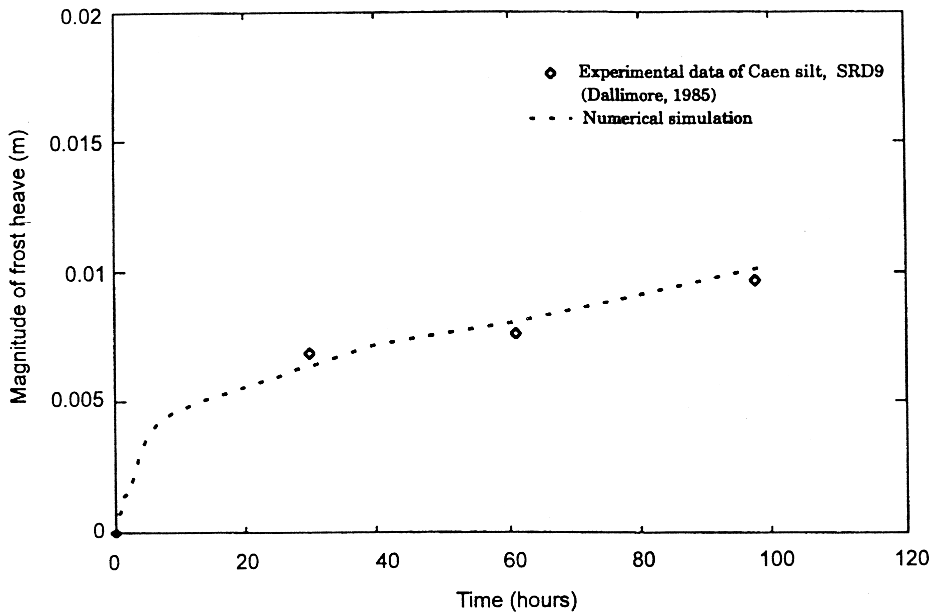

Fig. 5. One-dimensional frost heave generation in a sample of Caen silt.

result (Eq. (26)). Since the hydraulic conductivity of the frost susceptible soil is an important parameter in the problem, its value is varied according to

$$k=\begin{cases}3.072C\times10^{-11}e^{13.438T}; & -0.3^{\circ}\mathrm{C}<T<T_{\mathrm{f}};\\ 5.453C\times10^{-13}; & T\leq-0.3^{\circ}\mathrm{C}\end{cases}\ \mathrm{m/s} \tag{30}$$

where $C\in(1,10)$.

Fig. 2 shows the comparison of the results of the finite element techniques proposed in this study with the experimental results obtained by Penner (1986). These calibration results have been obtained by varying the value of the hydraulic conductivity and the specific value used in the computations corresponding to the case where $C=9.5$. Fig. 3 illustrates the influence of k on the magnitude of frost heave. As is evident, the reduction in the hydraulic conductivity results in an attendant reduction in the magnitude of the frost heave.

Computations were also performed by assigning the hydraulic conductivity values that are applicable to Caen silt. This silty clay material was used in the experiments conducted to determine the influence of discontinuous frost heave developments on the behaviour of a buried pipeline (Dallimore and Williams, 1984). Estimates for the hydraulic conductivity applicable to Caen silt given by Shen and Ladanyi (1991) are as follows;

$$k=\begin{cases}1.075\times10^{-9}e^{23.99T}; & -0.3^{\circ}\mathrm{C}<T<T_{\mathrm{f}};\\ 8.0499\times10^{-13}; & T\leq-0.3^{\circ}\mathrm{C}\end{cases}\ \mathrm{m/s} \tag{31}$$

Fig. 4 illustrates the time-dependent evolution of one-dimensional frost heave in a cuboidal element with edge length of 10 cm. The results obtained by Shen and Ladanyi (1987) for the specific case when the permeability is defined by Eq. (31) and the experimental results obtained by Penner (1986) are also shown for purposes of comparison. The results given in Fig. 4 indicate that the development of frost heave under one-dimensional conditions obtained by the computational modelling can be matched reasonably well with observations of one-dimensional experiments. In particular, the *trends of the computational results* agree very closely with experimental results.

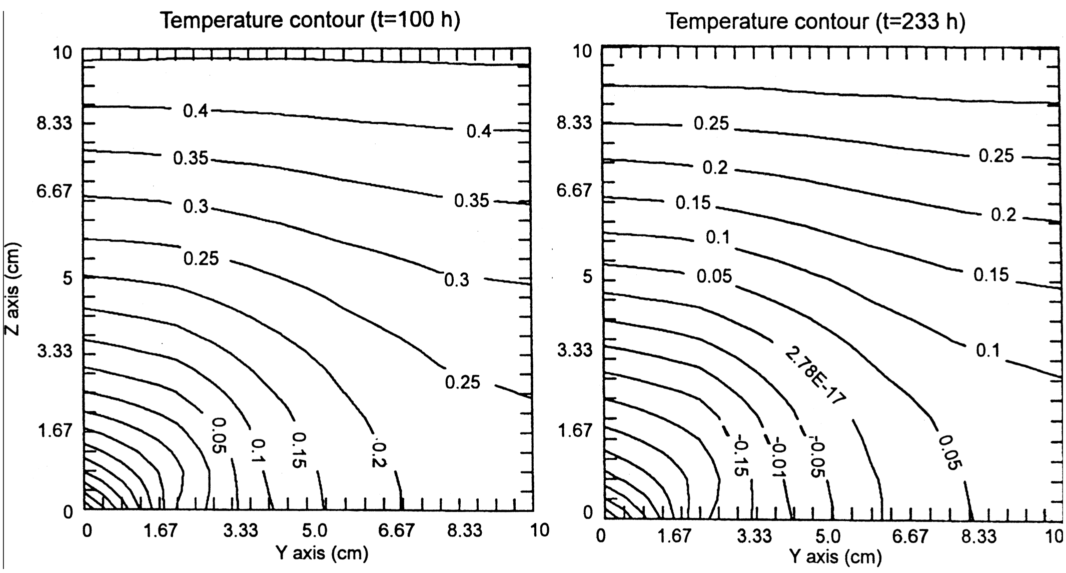

Fig. 6. Temperature contours within a cuboidal of Caen silt.

Extensive experiments were conducted by Dallimore (1985) to determine the frost heave characteristics of Caen silt. The results given by Dallimore (1985) are for short-term durations and the conditions of the testing which lead to wide variations in the measured heave are not discussed in detail. The selection of an extensive set of results for the purposes of comparison with the computational predictions cannot, therefore, be achieved with confidence. In this study, some typical experimental results have been selected to cover a 100 h period. Also, the hydraulic conductivity has been modified to the form

$$k = \begin{cases} 16.255 \times 10^{-9} e^{23.99T}; & -0.3°\mathrm{C} < T < T_f; \\ 12.07 \times 10^{-12}; & T \leq -0.3°\mathrm{C} \end{cases} \mathrm{m/s} \qquad (32)$$

Fig. 5 shows the set of experimentally derived frost heave data from the tests on Caen silt obtained by Dallimore (1985). It also illustrates the results of the corresponding numerical simulation. It is evident that the trends in a particular experiment can be

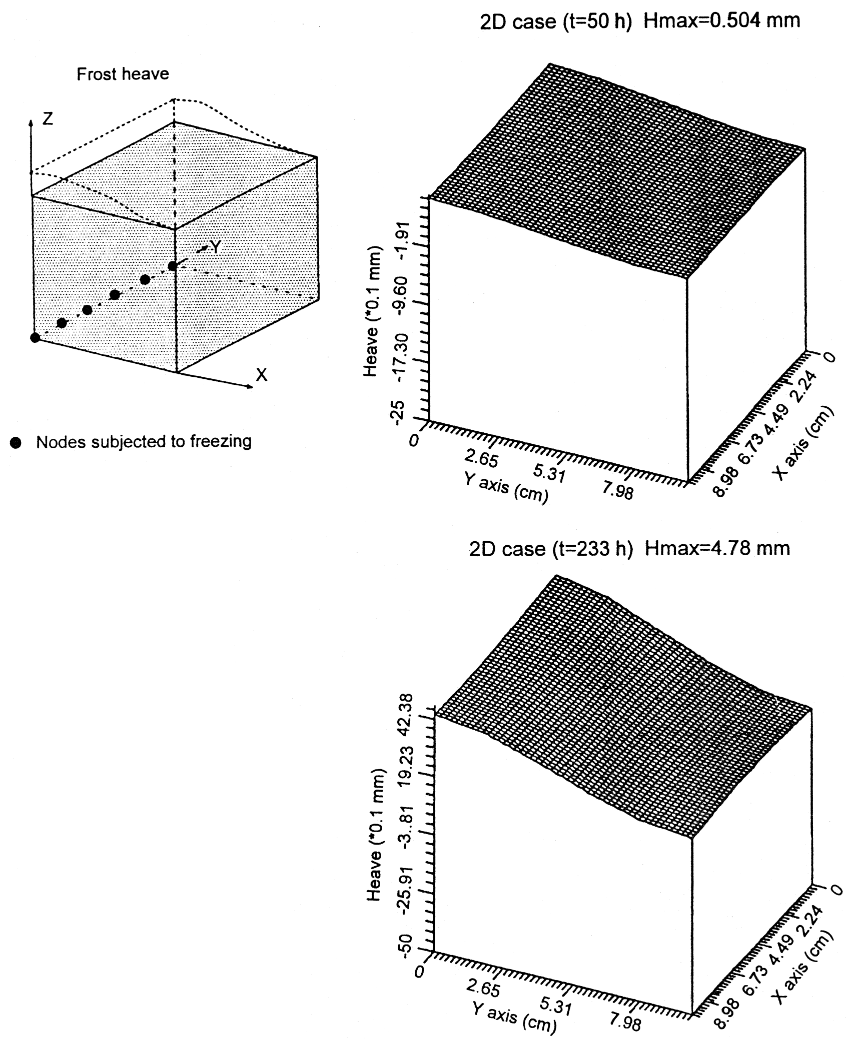

Fig. 7. Surface heave of a cuboidal element subjected to base freezing along a line of edge nodes.

modelled reasonably well by the computational model. The absence of continuous monitoring of the frost heave generation in the experiment makes it difficult to establish complete confidence in the correlation between the experimental result and the computational estimates particularly in the initial stages of the experiment.

4.2. A two-dimensional problem of frost heave development

The computational modelling procedure was used to examine the two dimensional problem of frost heave development in a cuboidal element with an edge length of 10 cm. The two-dimensional process of the frost heave development is achieved by allowing the freezing action to develop along a line of elements located at the edge of the cuboidal element. For the purposes of the computational modelling, the thermal and mechanical properties of the soil are identical to those used in the one-dimensional modelling and the hydraulic conductivity values are defined by those applicable to Caen silt (Eq. (31)). The surface temperature of the cuboidal element is maintained at 0.55°C. Fig. 6 illustrates the temperature contours within one plane ($x = 0$) of the cuboidal element for lapsed times of $t = 100$ and 233 h. It is evident that the pattern of heat conduction is consistent with the imposed thermal boundary conditions.

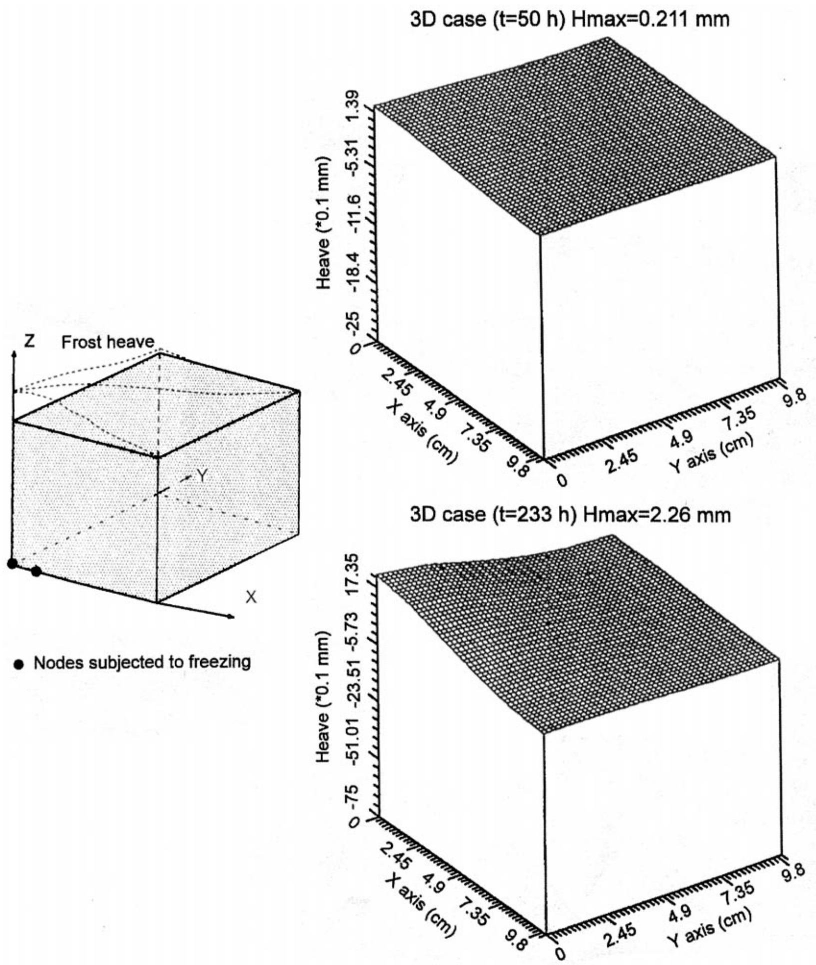

Fig. 8. Surface heave of a cuboidal element subjected to base freezing at two edge nodes.

We now focus on the development of frost heave at the surface of the element due to the internal cooling along a line of nodes located along the base of the element. Fig. 7 illustrates the development of frost heave at the surface of the cuboidal element for elapsed times of 50 and 233 h. Again, the frost heave profiles are consistent with the boundary conditions associated with the cuboidal element. The surface heave profile above the line of nodes subjected to freezing has a zero slope. This is a requirement of symmetry.

4.3. A three-dimensional problem of frost heave development

We now consider the three-dimensional problem where the cuboidal element is subjected to cooling at the base of the element at two node locations. The hydrothermal parameters and the mechanical parameters used in the computations are identical to those used in the one- and two-dimensional problems described previously. Fig. 8 illustrates the development of frost heave at the surface of the cuboidal element for elapsed times of 50 and 233 h. Again, it is evident that the frost heave patterns are consistent with the cooling of isolated nodes located at the base of the cuboidal element.

5. Concluding remarks

An objective of this study is to develop a plausible frost heave model which can be used to examine the problem of frost heave induced soil–pipeline interaction at a discontinuous frost heave zone. The requirements of such a model are that it should take into account the basic coupled processes of heat conduction and moisture transport within an evolving frost heave zone and be capable of demonstrating trends in frost heave generation consistent with experimental data. Also, the thermo-physical parameters required to conduct computational modelling should be obtainable from either laboratory tests and/or in situ tests. The heat conduction and hydrodynamic coupled model of frost action in soils presented by Shen and Ladanyi (1987) is proposed as a model which could be utilized for frost heave modelling. A generalized computational procedure which incorporates this model is used to develop numerical solutions to one-, two- and three-dimensional problems of frost heave generation. Comparisons with results of one-dimensional experiments of frost heave development indicates that the computational procedure can adequately duplicate the *trends* observed in the experiments. The results of two- and three-dimensional problems indicate heat conduction and surface heave patterns which could be expected in experimental situations. The parametric studies conducted using this coupled model indicates that the *hydraulic conductivity* of the soil is a key parameter which governs the rate of fluid influx and consequently the magnitude of the frost heave. The accurate determination of the hydraulic conductivity of the frost susceptible soil is regarded as an essential prerequisite for the accurate estimation of frost heave.

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