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# Mechanics and fluid transport in a degradable discontinuity

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## Abstract

This paper briefly examines the mechanical and hydraulic response of discontinuity in a geologic medium that can experience degradation of its contacting regions due to shear and normal stresses acting on it. The paper discusses the methodologies that can be adopted to describe the mechanics and fluid transport behaviour of such a joint. The calibration capabilities of the model are illustrated by appeal to a comparison of experimental results with computational predictions. © 1999 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

The study of joints and discontinuities in geologic media has a long history in terms of experimental studies, field investigations, theoretical developments and computational modelling (e.g. see Stephansson, 1985; Barton and Stephansson, 1990; Selvadurai and Boulon, 1995). The studies by Patton (1966), Ladanyi and Archambault (1970), and Barton and Choubey (1977) investigated the mechanical response of joints subjected to a range of stress states that can induce dilatancy or degradation at the asperities of both conforming and non-conforming surfaces. The incremental relationships (e.g. see Goodman and Dubois, 1972) involve piecewise linear relationships between the

increments of stress and the increment of strain. These relationships are derived from direct shear tests under constant normal stress. The second category of constitutive relationships are the elasto–plastic models derived by appeal to a theory of plasticity. These models are characterized by recoverable elastic behaviour of the joint prior to sliding and irrecoverable plastic behaviour during sliding. The plastic deformations are derived from a yield criterion and a flow rule applicable to an interface. The literature in geomechanics contains an extensive record of the applications of plasticity-based models for the study of interface phenomena (Selvadurai and Boulon, 1995). Among these developments, the model proposed by Plesha (1987) is particularly attractive owing to its simplicity and its ability to capture certain fundamental aspects of the mechanical behaviour of actual joints, including dilatancy under shear and softening due to asperity degradation. For developing the hydraulic behaviour of rock joints, the parallel plate model, developed by considerations of flow

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of an incompressible Newtonian viscous fluid between two parallel plates, is employed. This procedure is widely used to calculate the effective permeability  $k$  of a fracture (e.g. see Gale, 1982; Benjelloun, 1991). The permeability of the joint is thus expressed as a function of its effective opening to fluid flow, termed the ‘hydraulic aperture’. Since a natural fracture is unlike the parallel plate idealization, the hydraulic aperture is different from the ‘mechanical aperture’.

In this paper, we adopt the methodology proposed by Plesha (1987) to formulate the elasto–plastic stress–strain relationship for a rock joint. We illustrate a procedure whereby most of the parameters required to postulate the elasto–plastic constitutive relationships for the joint could be estimated from two widely used empirical coefficients proposed by Barton and Choubey (1977) and Bandis et al. (1981) (i.e. the JRC and JCS). In order to extend these developments to consideration of the influence of gouge production on hydraulic conductivity, we note that gouge production in the joint can be related to plastic work of the shear forces. The extended version of the model proposed by Plesha (1987) is implemented in the finite element code FRACON (fracture consolidation). The analysis is restricted to joints without infilling.

## 2. The elasto–plastic behaviour of the rock joints

In this study a dilatant rock joint is assumed to contain surface asperities that are irregular in shape and height. A perfectly matching joint will have surfaces that can be idealized by the two-dimensional saw tooth pattern proposed by Patton (1966). The asperities are assumed to have regular angles of inclination  $\alpha^*$  with respect to the horizontal direction (Fig. 1). Along a typical asperity, inclined at an angle  $\alpha^*$ , the relationship between the ‘macro’ values of the stresses  $\tau$  and  $\sigma$  the ‘local’ or ‘micro’ values of the stresses  $\tau_\alpha$  and  $\sigma_\alpha$  can be obtained by appeal to local equilibrium at the inclined sliding plane. Avoiding details of calculation, it can be shown that the yield criterion for

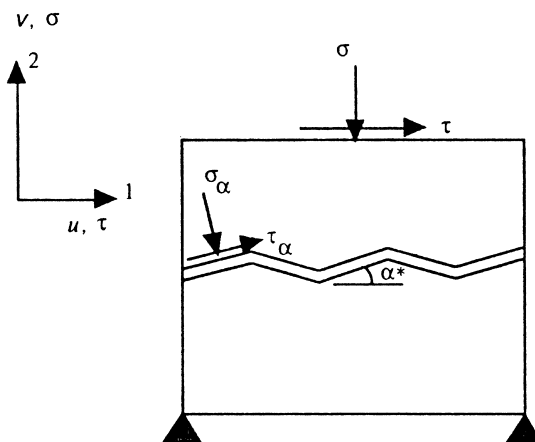


Fig. 1. The joint model.

the saw tooth joint is

$$F = |\sigma \sin \alpha^* + \tau \cos \alpha^*| + \tan \phi (\tau \cos \alpha^* - \sigma \sin \alpha^*). \quad (1)$$

Similarly, the plastic potential is defined as

$$Q = |\sigma \sin \alpha^* + \tau \cos \alpha^*|. \quad (2)$$

When plastic deformations occur, the asperities of the joint are damaged and such degradation processes can result in a decrease in the asperity angle. Plesha (1987) assumed that, during plastic deformations, the asperity angle varies according to the relationship

$$\alpha^* = \alpha_0^* \exp\left(-\int_0^{W^p} c \, dW^p\right), \quad (3)$$

where  $\alpha_0^*$  is the original asperity angle,  $c$  is a degradation coefficient and  $W^p$  is the plastic work of the shear stresses such that

$$dW^p = \tau \, du, \quad (4)$$

where  $u$  is the relative shear displacement at the joint.

When asperity damage is considered, strain softening phenomena can occur and the yield function  $F$  and the plastic potential  $Q$  will depend on  $\tau$ ,  $\sigma$  and  $W^p$  such that

$$F = F(\tau, \sigma, W^p) \quad Q = Q(\tau, \sigma, W^p). \quad (5)$$

Following conventional developments, it can be

shown that the increments of elastic displacement are given by

$$d\sigma_i = D_{ij}^e du_j^e, \quad (6)$$

where  $d\sigma_i$  are the incremental stresses ( $d\sigma_1 = d\tau$  and  $d\sigma_2 = d\sigma$ ); and the increments of plastic displacement are given by

$$du_i^p = d\lambda \frac{\partial Q}{\partial \sigma_i}, \quad (7)$$

where

$$d\lambda = \frac{1}{\psi - H} \frac{\partial F}{\partial \sigma_i} D_{ij}^e du_j \quad (8)$$

$$\psi = \frac{\partial F}{\partial \sigma_l} D_{lm}^e \frac{\partial Q}{\partial \sigma_m}; \quad H = \frac{\partial F}{\partial W^p} \tau \frac{\partial Q}{\partial \tau}. \quad (9)$$

Consequently, we can write Eq. (9) in the form

$$d\sigma_i = D_{ij}^{ep} du_j = \left( D_{ij}^e - \frac{1}{\psi - H} \frac{\partial Q}{\partial \sigma_l} D_{il}^e D_{mj}^e \frac{\partial F}{\partial \sigma_m} \right). \quad (10)$$

### 3. Mechanical parameters for the joint model

The parameters governing the model proposed by Plesha (1987) are the elastic stiffnesses, the degradation factor  $c$ , the initial asperity angle  $\alpha^*$  and the friction angle  $\phi$ . Assuming symmetry considerations of induced elastic behaviour during imposed shear, it can be shown that

$$D_{ij}^e = k_s; \quad D_{22}^e = k_n; \quad D_{12}^e = D_{21}^e = 0, \quad (11)$$

where  $k_s$  and  $k_n$  are elastic shear and normal stiffnesses. Plesha (1987) estimated these model parameters by calibrating the results of experimental data derived from shear tests under constant normal stress. It can be shown, if no asperity degradation occurs before peak shear stress is mobilized, that

$$\alpha_0^* = \text{JRC} \log_{10} \left( \frac{\text{JCS}}{\sigma} \right). \quad (12)$$

Using the empirical relationship for  $u_{\text{peak}}$  (the shear displacement corresponding to the peak

shear stress  $\tau_{\text{peak}}$  under constant normal stress conditions) proposed by Barton et al. (1985), it can be shown that

$$k_s = \frac{|\tau_{\text{peak}}|}{u_{\text{peak}}} = \frac{\sigma \tan \left[ \text{JRC} \log_{10} \left( \frac{\text{JCS}}{\sigma} \right) + \phi \right]}{\frac{L}{500} \left( \frac{\text{JRC}}{L} \right)^{1/3}}, \quad (13)$$

where  $L$  is the size of the sample.

The normal stiffness  $k_n$  can be determined by performing uniaxial compression tests on both intact and jointed specimens (Bandis et al., 1981). It can be shown that the normal stiffness at any level of normal stress is

$$k_n = \frac{d\sigma}{dv} = k_{ni} \left( 1 - \frac{\sigma}{v_m k_{ni} + \sigma} \right)^{-2}. \quad (14)$$

The parameters  $k_{ni}$  and  $v_m$  can be determined from compression tests on jointed rock samples (e.g. see Benjelloun, 1991; Boulon et al., 1993). There is limited experimental data on the determination of the degradation constant  $c$ . Experiments by Hutson and Dowding (1990) and Qiu et al. (1993) indicate that  $c$  increases with the normal stress, whereas Benjelloun (1991) observed a reverse trend. It is likely that  $c$  will be related to JRC, JCS and  $\sigma$ . However, further experimental studies are necessary to establish clearly the relationship between  $c$  and  $\sigma$ .

### 4. Hydraulic behaviour of the rock joint

The conventional parallel plate model developed by consideration of laminar flow of an incompressible Newtonian viscous fluid between two plates with parallel flat surfaces gives the relationship

$$k = \frac{e_h^2}{12}, \quad (15)$$

where  $e_h$  is the hydraulic aperture.

The empirical relationship proposed by Barton (1982) relates the hydraulic aperture  $e_h$  to the

mechanical aperture  $e_m$  according to

$$e_h = \frac{e_m^2}{\text{JRC}^{5/2}}, \quad (16)$$

where  $e_h$  and  $e_m$  are expressed in micrometres. [Note that Eq. (16) is an empirical relationship; as such, the apparent dimensional inconsistency should be disregarded.]

Witherspoon et al. (1979) proposed a linear relationship between the hydraulic and mechanical apertures of the form

$$e_h = e_{h0} + f\Delta e_m, \quad (17)$$

where  $e_{h0}$  is the initial hydraulic aperture,  $\Delta e_m$  is the variation in the mechanical aperture due to the combined effects of compression and shear and  $f$  is a constant. Benjelloun (1991) has confirmed the validity of Eq. (17) and found that  $f$  varies between 0.5 and 1.0. The factor takes into consideration the surface regularity of the parallel plates. The factor  $f=1$  applies to the case of parallel plates with no surface irregularities (i.e. relatively open joint with apertures of the order of millimetres). For most other cases,  $f < 1$  and depends on the geometry of the flow path. For rectilinear flow  $f$  generally approaches 0.8 and for radial flow  $f$  approaches 0.5.

In this paper, we propose a relationship that is intended to account for (i) variation in joint permeability due to the dilation of the joint, which should increase with joint shear, and (ii) decrease in the joint permeability, which results in asperity degradation and gouge production. This latter aspect is considered to be a novel development and intended to model observations of the hydraulic behaviour of joints made by Bandis et al. (1985). The basic theoretical justification rests on the assumption that gouge production is related to the plastic work of shear tractions on the joint. We assume that  $f$  in Eq. (17) can be expressed in the form

$$f = f_0 \exp\left(-\int_0^{W^p} c_f dW^p\right), \quad (18)$$

where  $c_f$  is a gouge production factor. There is a high likelihood that the additional parameters  $f_0$  and  $c_f$  may also be related to JRC, JCS and  $\sigma$ . This assertion can only be verified through an extensive program of experimental research.

## 5. Computational modelling

The basic concepts advocated for the mechanical and hydraulic behaviour of the joint were implemented in the finite element code FRACON (Selvadurai and Nguyen, 1995; Nguyen, 1995). The code is a finite element scheme for the computational modelling of coupled thermal, hydraulic and mechanical (THM) behaviour of both intact and jointed rock masses. The THM processes in the intact geomaterial are represented by an extension to the generalized theory of poroelasticity proposed by Biot (1941) (see also Selvadurai, 1996) and the flow in fractures is by consideration of a parallel plate simulation. There is full thermo-hydro-mechanical coupling between the fractures and the intact poroelastic geomaterial. In addition, the fractures exhibit non-linear constraints through the interface plasticity simulations described previously. Such variable non-linear interface constraints also account for the variations in the fluid transport characteristics at the joints.

The influence of joint shearing on the permeability characteristics of rock joints composed of gneiss was investigated by Bandis et al. (1985), using a uniaxial cell. A schematic configuration of the cell

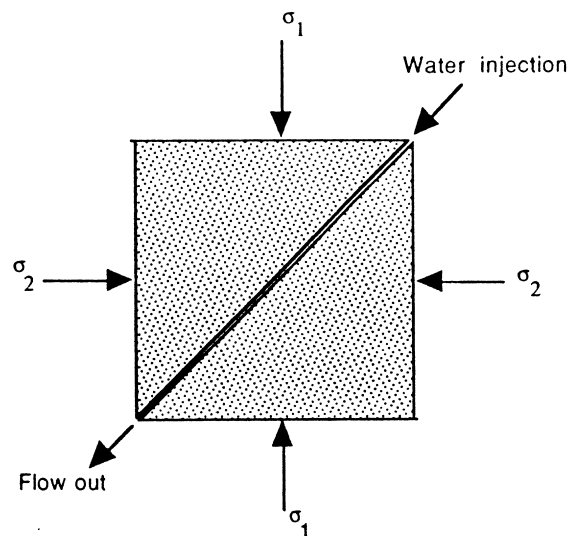


Fig. 2. Schematic configuration of the experimental facility used to perform hydromechanical experiments on gneiss (Bandis et al., 1985; Makurat et al., 1990).

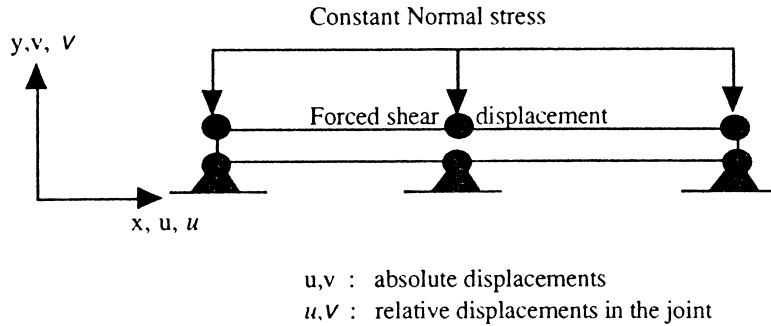


Fig. 3. Finite element model for joint shear under constant normal stress.

is shown in Fig. 2. The joint sample was first compressed by equal biaxial normal stresses. The joint was then sheared by maintaining one load constant and increasing the other. At specific values of the shear displacement, the permeability of the joint was determined by injecting water through the joint and measuring the flow rate. Bandis et al. (1985) recorded the evolution of the joint permeability with increasing shear displacement. Although both the normal and shear stresses varied during the experiment, Bandis et al. (1985) assumed constant normal stress conditions to simulate the evolution of joint permeability during one typical experiment. The assumed constant normal stress was the average value of the actual

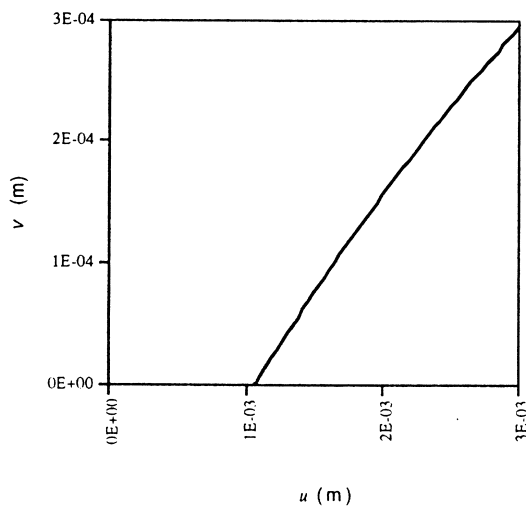


Fig. 4. Shear dilation calculated via the FRACON code for the experiment conducted by Bandis et al. (1985).

stress. In this study, we computationally simulate the experiment by assuming constant normal stress conditions. The finite element model used in the FRACON simulation is shown in Fig. 3. The joint properties estimated by Bandis et al. (1985) are as follows:  $JRC = 7$ ;  $JCS = 110$  MPa, joint length  $L = 0.15$  m, average normal stress 1.5 MPa. In addition to these parameters, an asperity degradation factor  $c = 1.5 \times 10^{-4}$  m/N is assumed. The joint exhibits dilatancy during shear. The dilation calculated by FRACON is shown in Fig. 4. This dilation is accompanied by a corresponding increase in the permeability of the joint (Fig. 5). However, this permeability decreases with continued shear deformation, as gouge development takes place by asperity breakage. In their studies, Bandis et al. (1985) could not simulate this permeability decrease, purely by appeal to the model proposed by Barton (1982). The FRACON code simulation, with no gouge production ( $c_f = 0$ ) produces results similar to those presented by Bandis et al. (1985). By assuming  $f_0 = 1$  and  $c_f = 0.001$  m/N [see Eq. (18)], the FRACON code is able to provide predictions, where the trends agree well with the experimental data. Most importantly, the tendency for the gouge-induced reduction in the permeability of the joint with increasing relative shear  $u$  is correctly predicted. Further details can be found in the article by Nguyen and Selvadurai (1998).

## 6. Concluding remarks

The concept of a degradable discontinuity attempts to examine the influence of asperity

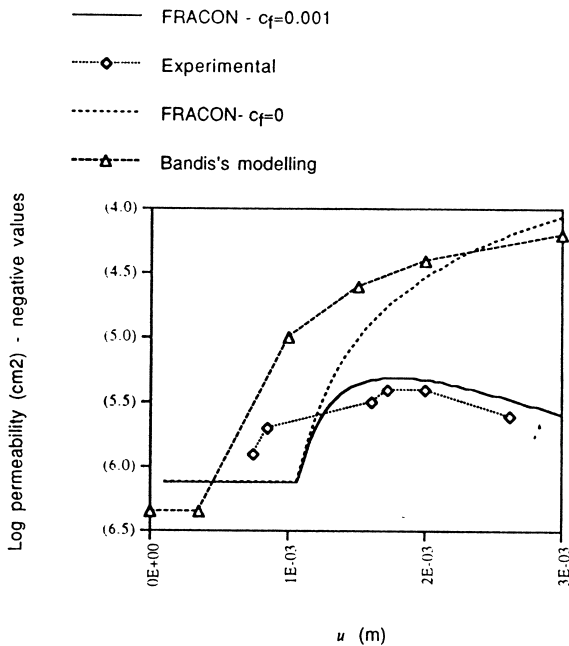


Fig. 5. Effect of shear on the joint permeability: FRACON code simulation of the experiment by Bandis et al. (1985).

degradation on the hydraulic behaviour of a joint. The paper reviews the formulation of the mechanics of a joint by recourse to an incremental theory of plasticity applicable to a dilatant joint where asperity degradation is coupled to the plastic work of shear forces. The basic methodology is also extended to cover the alterations in the hydraulic conductivity of the joint by initial dilation of the joint due to shear and the subsequent production of gouge material during asperity degradation. The paper proposes a postulate whereby the hydraulic aperture varies with the plastic work of shear forces. Two additional material parameters are required to implement the characteristics related to gouge-production-induced alteration in the permeability of the joint. The parameters governing the mechanical response of the joint are defined in relation to the joint friction angle  $\phi$ , the empirical measures JRC and JCS and their variations. The constitutive model for the degradable discontinuity is implemented in the two-dimensional finite element code FRACON. The performance of the model is calibrated against a set of experimental results reported in the literature. These correlations

are not intended to be validations of the model; instead, the trends observed in the computational simulations address several aspects of the alteration of the permeability characteristics of joint-induced asperity degradation and gouge production. The computational simulations adequately address the behaviour of the joint under constant normal stress and constant normal stiffness. Such implementations are also able to address the influence of shear on joint permeability and scale effects. The methodology provides a useful extension to the current state of characterization of shear-induced irreversible alterations in the permeability of a joint.

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