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HEAT-INDUCED MOISTURE TRANSPORT IN THE VICINITY OF A SPHERICAL HEAT SOURCE

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SUMMARY

The present paper develops an analytical approach to the problem of heat-induced moisture movement in the vicinity of a spherical heat source embedded in an undeformable, moist porous solid of infinite extent. A transient-state distribution of temperature within the infinite medium is assumed to induce the moisture transport process. The numerical results, presented in the paper, illustrate the influence of the moisture transport characteristics on the time-dependent distribution of moisture within the porous medium.

Key words: heat and moisture flow; analytical solution; transient analysis; spherical heat source; coupled analysis; Green's function method

1. INTRODUCTION

The study of the coupled processes of heat transfer and moisture transport in porous materials is of interest to many branches of engineering. The early applications of the theories of heat and moisture movement were largely in the area of chemical engineering and agricultural science. In recent years, there has been renewed interest in the study of heat and moisture movement in porous geological materials, particularly in view of the potential application of such theories to geo-environmental problems. These theories have been used in the modelling of geothermal energy extraction problems, related to both natural and artificial sources. The primary motivation for the present study stems from an application relating to the disposal of heat emitting nuclear fuel waste in engineered geological media. In particular, several proposals for the disposal of such wastes calls for the provision of a porous geological buffer region immediately surrounding the heat emitting waste container.[1–3] The radiogenic heating, due to the waste containers, can induce moisture movement within the engineered barrier. The ability of the engineered geological barrier to maintain its capabilities for retarding the long-term radionuclide migration is greatly influenced by the heat-induced moisture transport processes within it. In particular, excessive moisture loss from a buffer region can result in the development of shrinkage cracks in the medium. If such cracks are permanent, the long-term radionuclide transport through the barrier will occur as a flow process through the cracks rather than by a diffusive process through an

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intact buffer region. For this reason, there are concerted efforts to examine, both theoretically and experimentally, the process of heat and moisture transport in engineered geological buffer materials.[4–9] Here, attention is primarily restricted to the study of coupled heat and moisture transport processes in porous geological media, which do not experience mechanical deformations. The theory proposed by Philip and De Vries[10] has found extensive application in the examination of heat and moisture flow in such unsaturated buffer regions, where the flow processes are largely governed by diffusive phenomena. In the ultimate application of these theories, it is foreseeable that computational methods have to be employed to model problems of practical interest to the waste disposal endeavors. There is, however, a great need for the development of analytical solutions, which can be used to test the accuracy of solutions that are based on finite element and boundary element schemes. The numerical schemes involve spatial and temporal discretization techniques, which will have an influence on the stability of the numerical schemes. The availability of analytical solutions will provide a benchmark for comparisons and provide a basis for the refinement of the procedures employed in the numerical schemes.

The objective of this paper is to develop an analytical solution to the problem related to the heat-induced moisture movement in the vicinity of a spherical heat source located in an unsaturated porous medium. The analytical solution utilizes the transient distribution of temperature within the porous medium and evaluates the moisture distribution within the medium in explicit form. The analytical model is in an explicit form, which can be attractive for researchers and scientists in the field since it offers general insight into the physical phenomena. The solution can also be used to provide an approximation to the physical parameters involved if experimental results for the temperature and moisture distributions are available. The numerical results presented in the paper indicate trends observed in experiments, which examine heat-induced moisture movement in unsaturated porous media. The more complete treatment of the heat-induced moisture movement problem must invariably consider the influence of the mechanical deformations of the porous medium. An example of such an application, involving consolidation around a spherical heat source, which is located in a Biot-type,[11,12] fully saturated, poroelastic medium, is given by Booker and Savvidou.[13]

2. GOVERNING EQUATIONS

A theory for describing the process of heat and moisture transport in a non-deforming porous medium was developed by Philip and de Vries.[10] The theory takes into consideration heat conduction, which is governed by Fourier's law, and moisture transport, which is governed by Darcy's law. The system of partial differential equations governing the coupled processes of heat conduction and moisture transport in a porous medium is given by

$$\rho c \frac{\partial T^*}{\partial t^*} = \nabla \cdot (\lambda \nabla T^*) - \rho L \nabla \cdot (D_{\theta v} \nabla \theta^*) \tag{1}$$

$$\frac{\partial \theta^*}{\partial t^*} = \nabla \cdot (D_T \nabla T^*) + \nabla \cdot (D_\theta \nabla \theta^*) + \frac{\partial K_\theta}{\partial z^*} \tag{2}$$

where θ^* is the volumetric moisture content, T^* is the temperature, ∇ is the gradient operator, D_T is the thermal moisture diffusivity, D_θ is the isothermal moisture diffusivity, $D_{\theta v}$ is the isothermal vapour diffusivity, K_θ is the unsaturated hydraulic conductivity, ρc is the volumetric heat capacity of the porous medium, λ is the thermal conductivity, L is the latent heat of vaporization, and the superscript * stands for dimensional variables.

In the present work, the geometry of the problem is assumed to be of a spherical form, the domain of interest being the region outside the sphere. The effect of a sudden rise in the temperature at the surface of the sphere on the moisture and the heat transport in an infinite homogeneous and isotropic porous media is investigated. In such a case, the prime variables are assumed to be bounded at infinity and the temperature boundary condition at the sphere is of a Dirichlet type

$$T^* = T_0^* H(t) \quad r = a \tag{3}$$

where T_0^* is the specified temperature at the surface of the sphere $r = a$ and $H(t)$ is the Heaviside step function. As for the moisture transport problem, the auxiliary condition has the form of a mass conservation equation, which states that the total mass in the system is constant and equal to the initial mass in the system:

$$\int_a^\infty \theta^*(r, t)\, 4\pi r^2\, \mathrm{d}r = \int_a^\infty \theta^*(r, 0)\, 4\pi r^2 \mathrm{d}r \tag{4}$$

Equation (4) is used to determine the time variation of the moisture content at the surface of the sphere.

The initial condition consists of a uniform distribution of temperature and moisture content

$$T^*(r, 0) = T_i^* \tag{5}$$

and

$$\theta^*(r, 0) = \theta_i^* \tag{6}$$

A comprehensive treatment of heat and moisture movement in an unsaturated medium requires a considerable mathematical effort since the processes are non-linear and coupled. It is therefore desirable to examine a simplified model, which can be successfully applied to the study of heat and moisture flow. The simplified model examines the transient heat conduction without the vaporization effects and the transient moisture flow without the gravity effects. For cases involving localized high temperature, the omission of gravitational effects might be acceptable. Moreover, it has been shown that the simplified heat equation is sufficient to model the temperature distribution.[14] The additional effects of coupling due to moisture transport do not noticeably influence the temperature field to the extent that it affects the subsurface moisture fluxes.

The coupled non-isothermal system of equations then reduce to

$$\rho c \frac{\partial T^*}{\partial t^*} = \nabla \cdot (\lambda \nabla T^*) \tag{7}$$

$$\frac{\partial \theta^*}{\partial t^*} = \nabla \cdot (D_T \nabla T^*) + \nabla \cdot (D_\theta \nabla \theta^*) \tag{8}$$

The physical parameters contained in (7) and (8) are known to vary as a function of the prime variables.[15,16] As a first approximation, they are assumed to remain constant, which is generally not the case for the moisture diffusivity involving unsaturated soil conditions. However, a constant diffusivity term can be obtained by assuming that the moisture content variation and the hydraulic conductivity function follow a log-linear form.[17] Assuming that

$$K = K_0 \exp(\alpha\psi), \qquad \theta^* = \theta_r^* + (\theta_s^* - \theta_r^*)\exp(\alpha\psi) \tag{9}$$

where α, K_0, θ_s^* and θ_r^* are experimentally derived parameters and ψ is the suction (negative) head, the moisture diffusivity becomes constant

$$D_\theta = K\frac{\partial\psi}{\partial\theta} = \frac{K_0}{\alpha(\theta_s^* - \theta_r^*)} \tag{10}$$

Although the expressions (9) do not closely correlate with experimental data over the entire range of ψ observed, they are considered for purposes of the ensuing mathematical analysis. They are applicable to field situations when the moisture variations are relatively small.[18] The parameter α is a measure of the importance of gravitational effects relative to capillary effects. It is small in fine-textured soils where capillarity is dominant and large in coarse-textured soils where gravity is dominant.[19] Philip[20] stated that α^{-1} ranges between 0·2 and 5 m; and fitted values of K_0 and α for some 17 soils are also given by Bresler.[21] Although the constant diffusivity assumption has been used widely in the literature in infiltration,[22,18,20] it might severely limit the validity of the results for large variations in moisture.

3. ANALYTICAL SOLUTION

The following dimensionless variables are introduced:

$$R = \frac{r}{a}, \quad t = \frac{t^* D_\theta}{a^2}, \quad \mathrm{T} = \frac{T^*}{T_0^* - T_i^*}, \quad \theta = \frac{\theta^*}{\theta_i^*}, \quad k_1 = \frac{D}{D_\theta}, \quad k_2 = \frac{T_0^* - T_i^*}{\theta_i^*}\frac{D_T}{D_\theta} \tag{11}$$

where k_1 is a form of the *Lewis* number which gives the ratio of the heat diffusivity $D = \lambda/(\rho c)$ over the isothermal moisture diffusivity D_θ, and k_2 is a form of the *Posnov* number which is a ratio of the thermal moisture diffusivity D_T over the isothermal moisture diffusivity D_θ. The governing equations with their respective boundary conditions then become

$$\partial T/\partial t = k_1 \nabla^2 T \tag{12}$$

with

$$T(R, 0) = \frac{T_i^*}{T_0^* - T_i^*} \tag{13}$$

$$T(1, t) = \frac{T_0^*}{T_0^* - T_i^*} \tag{14}$$

and

$$\frac{\partial\theta}{\partial t} = \nabla^2\theta + k_2\nabla^2 T = \nabla^2\theta + \frac{k_2}{k_1}\frac{\partial T}{\partial t} \tag{15}$$

with

$$\theta(R, 0) = 1 \tag{16}$$

$$\theta(1, t) = \theta_0^*/\theta_i^* \tag{17}$$

where θ_0^* is the unknown moisture content at the surface of the sphere $R = 1$. The set of equations (12) and (15) can be considered as a useful first approximation for the study of heat and mass transport in unsaturated media. The solutions of equations (12) and (15) are derived sequentially in the order of presentation. The analytical result for the heat flow is substituted into the moisture flow equation through the forcing function, i.e. the last term on the right-hand side of (15), that models the thermally induced moisture movement, which is dominant at very low moisture contents. The solutions for both the heat conduction and the moisture transport equations are derived using a Green's function technique.

4. SOLUTION FOR HEAT CONDUCTION

Using a Green's functions technique, solution to (12) subject to the given initial (13) and boundary condition (14) involving radial spherical geometry of the domain is given by[23]

$$T = \int_1^\infty T(R', 0)\ G(R, t, R', 0)\, 4\pi R'^2\, \mathrm{d}R' + \int_0^t T(1, \tau)\ G'(R, t, 1, \tau)\, \mathrm{d}\tau \tag{18}$$

The first integral pertains to the initial conditions $T(R', 0)$ while the second integral is for the boundary conditions $T(1, \tau)$. G is the Green's function and $G' = \partial G/\partial R'$. For this particular geometry and boundary condition, G is expressed as[23]

$$G = \frac{1}{4\pi R R' \sqrt{(4\pi k_1 (t-\tau))}} \left\{ \exp\left[-\frac{(R-R')^2}{4k_1(t-\tau)} \right] - \exp\left[-\frac{(R+R'-2)^2}{4k_1(t-\tau)} \right] \right\} \tag{19}$$

Defining a new variable $T' = T - T_0^*/(T_0^* - T_i^*)$, initial condition (13) transforms to $T(R, 0) = -1$ and the boundary condition becomes zero, which reduces (18) to a single integral. After integration and back-transformation, we obtain

$$T = \frac{T_i^*}{T_0^* - T_i^*} + \frac{1}{R}\,\mathrm{erfc}\left(\frac{R-1}{\sqrt{(4k_1 t)}} \right) \tag{20}$$

The time derivative of (20), which appears on the right-hand side of (15), is

$$\frac{\partial T}{\partial t} = \frac{R-1}{Rt\sqrt{(4\pi k_1 t)}} \exp\left[-\frac{(R-1)^2}{4k_1 t} \right] \tag{21}$$

5. SOLUTION FOR MOISTURE TRANSPORT

The solution of the moisture flow equation is also obtained using the Green's function approach. The solution is similar to (18) but contains an additional integral term which handles the spherical heat source as a forcing function, i.e.

$$\theta = \int_1^\infty \theta(R', 0)\ G(R, t, R', 0)\ 4\pi R'^2\, \mathrm{d}R' + \int_0^t \theta(1, \tau)\ G'(R, t, 1, \tau)\, \mathrm{d}\tau$$
$$+ \int_0^t \int_1^\infty \frac{k_2}{k_1} \frac{\partial T}{\partial t}\ G(R, t, R', \tau) 4\pi R'^2\, \mathrm{d}R' \mathrm{d}\tau \tag{22}$$

where $G(R, t, R', \tau)$ is the Green's function which is similar in form to (19), except for the presence of the parameter k_1, since the geometry and the boundary conditions are the same

$$G = \frac{1}{4\pi RR'\sqrt{(4\pi(t-\tau))}}\left\{\exp\left[-\frac{(R-R')^2}{4(t-\tau)}\right] - \exp\left[-\frac{(R+R'-2)^2}{4(t-\tau)}\right]\right\} \tag{23}$$

The integrand $\partial T/\partial t$ in the last integral is given by (21) but with the arguments R and t replaced by R' and τ, respectively.

Defining a new variable $\theta' = \theta - (\theta_0^*/\theta_i^*)$, boundary condition (17) becomes zero and the corresponding integral in (22) is also zero. The solution in integral form is then

$$\theta = \frac{\theta_0^*}{\theta_i^*} + \int_1^\infty \left(1 - \frac{\theta_0^*}{\theta_i^*}\right) G(R, t, R', 0) 4\pi R'^2 \, \mathrm{d}R' + \int_0^t \int_1^\infty \frac{k_2}{k_1}\frac{\partial T}{\partial t} G(R, t, R', \tau) 4\pi R'^2 \, \mathrm{d}R' \, \mathrm{d}\tau \tag{24}$$

Substituting equation (21) into (24) and integrating on R', we obtain

$$\begin{aligned}\theta = {} & \frac{\theta_0^*}{\theta_i^*} + \left(1 - \frac{\theta_0^*}{\theta_i^*}\right)\left[1 - \frac{1}{R}\operatorname{erfc}\left(\frac{R-1}{\sqrt{(4t)}}\right)\right] \\ & + \int_0^t \frac{k_2}{\sqrt{(4\pi)}}\frac{R-1}{R}\frac{1}{(t-\tau+k_1\tau)^{3/2}}\exp\left[-\frac{(R-1)^2}{4(t-\tau+k_1\tau)}\right]\mathrm{d}\tau\end{aligned} \tag{25}$$

Completing the integration on τ, we obtain

$$\theta = 1 - \left(1 - \frac{\theta_0^*}{\theta_i^*}\right)\frac{1}{R}\operatorname{erfc}\left(\frac{R-1}{\sqrt{(4t)}}\right) + \frac{k_2}{R(k_1-1)}\left[\operatorname{erf}\left(\frac{R-1}{\sqrt{(4t)}}\right) - \operatorname{erf}\left(\frac{R-1}{\sqrt{(4k_1t)}}\right)\right] \quad k_1 \neq 1 \tag{26}$$

For the particular case of $k_1 = 1$, (25) yields

$$\theta = 1 - \left(1 - \frac{\theta_0^*}{\theta_i^*}\right)\frac{1}{R}\operatorname{erfc}\left(\frac{R-1}{\sqrt{(4t)}}\right) + \frac{k_2}{\sqrt{(4\pi t)}}\frac{R-1}{R}\exp\left[-\frac{(R-1)^2}{4t}\right] \tag{27}$$

The boundary condition of θ_0^* at the surface of the sphere is still an unknown. To determine θ_0^*, we use the mass conservation equation (4) written here in dimensionless form

$$\int_1^\infty (\theta - 1) 4\pi R^2 \, \mathrm{d}R = 0 \tag{28}$$

Substituting (26) in (28), we obtain

$$\int_1^\infty \left(1 - \frac{\theta_0^*}{\theta_i^*}\right)\frac{1}{R}\operatorname{erfc}\left(\frac{R-1}{\sqrt{(4t)}}\right) 4\pi R^2 \, \mathrm{d}R = \int_1^\infty \frac{k_2}{R(k_1-1)}\left[\operatorname{erf}\left(\frac{R-1}{\sqrt{(4t)}}\right) - \operatorname{erf}\left(\frac{R-1}{\sqrt{(4k_1t)}}\right)\right] 4\pi R^2 \, \mathrm{d}R \tag{29}$$

which, upon integration, yields

$$\left(1 - \frac{\theta_0^*}{\theta_i^*}\right) 4\sqrt{(\pi t)}\,(2 + \sqrt{(\pi t)}) = k_2\left[4\pi t + 8\sqrt{(\pi t)}\left(\frac{\sqrt{k_1} - 1}{k_1 - 1}\right)\right] \tag{30}$$

and the moisture content at the surface of the sphere θ_0^* can then be obtained from the result

$$\left(1-\frac{\theta_0^*}{\theta_i^*}\right)=k_2\left[\frac{2+\sqrt{(\pi t)}\ (\sqrt{k_1}+1)}{(2+\sqrt{(\pi t)})\ (\sqrt{k_1}+1)}\right] \tag{31}$$

Substituting (31) into (26), the final solution becomes

$$\theta=1-\frac{k_2}{R}\left[\frac{2+\sqrt{(\pi t)}\,(\sqrt{k_1}+1)}{(2+\sqrt{(\pi t)})\ (\sqrt{k_1}+1)}\right]\mathrm{erfc}\left(\frac{R-1}{\sqrt{(4t)}}\right)$$

$$+\frac{k_2}{R(k_1-1)}\left[\mathrm{erf}\left(\frac{R-1}{\sqrt{(4t)}}\right)-\mathrm{erf}\left(\frac{R-1}{\sqrt{(4k_1 t)}}\right)\right] \tag{32}$$

Equation (32) is the solution for the temporal and spatial distribution of moisture under the influence of a spherical heat source which is maintained at a constant temperature T_0 in a medium with an initial uniform temperature of T_i. The moisture movement exhibits spherical symmetry since the gravity term is neglected, and the distillation effect is omitted in the derivation of the solution for the heat conduction problem. The solution is more appropriate for early times than for large times since the gravitational effects are expected to be less dominant in the initial stages. Note also that the moisture transport characteristics such as the diffusivity coefficient has been assumed to be constant.

6. PARAMETER ESTIMATION

The above results can provide a means by which the physical parameters can be estimated from experimental measurements of the moisture distribution with time. Equation (31) can be expressed as

$$F=\frac{1-\theta_0}{k_2}=\left[\frac{2+\sqrt{(\pi t)}+\sqrt{(\pi t k_1)}}{(2+\sqrt{(\pi t)})\,(1+\sqrt{k_1})}\right] \tag{33}$$

where F is a dimensionless parameter expressing the dimensionless moisture content at the surface of the sphere θ_0 as a function of time and the parameter k_1. We notice from (33) that for $t=0$, the parameter F is equal to $1/(\sqrt{k_1}+1)$ and that as time increases, the actual moisture content θ_0^* decreases in value from $\theta_i(1-k_2/(\sqrt{k_1}+1))$. Figure 1 gives the variation of the dimensionless parameter F at the surface of the sphere as a function of time for various values of the parameter k_1. Decreasing values of k_1 implies higher moisture diffusivities resulting in lower values of the moisture content (F increasing) at the surface of the sphere.

The time derivative of (33) gives the rate of moisture movement at $R=1$

$$\frac{1}{k_2}\frac{\mathrm{d}\theta_0}{\mathrm{d}t}=-\frac{\sqrt{(\pi k_1)}}{\sqrt{t}(1+\sqrt{k_1}((2+\sqrt{(\pi t)})^2} \tag{34}$$

Dividing (34) by (33) yields an expression more suitable for parameter estimation, namely

$$A=\frac{1}{1-\theta_0}\frac{\mathrm{d}\theta_0}{\mathrm{d}t}=\frac{\sqrt{(\pi k_1)}}{\sqrt{t}(2+\sqrt{(\pi t)})\,(2+\sqrt{(\pi t)}+\sqrt{(\pi t k_1)})} \tag{35}$$

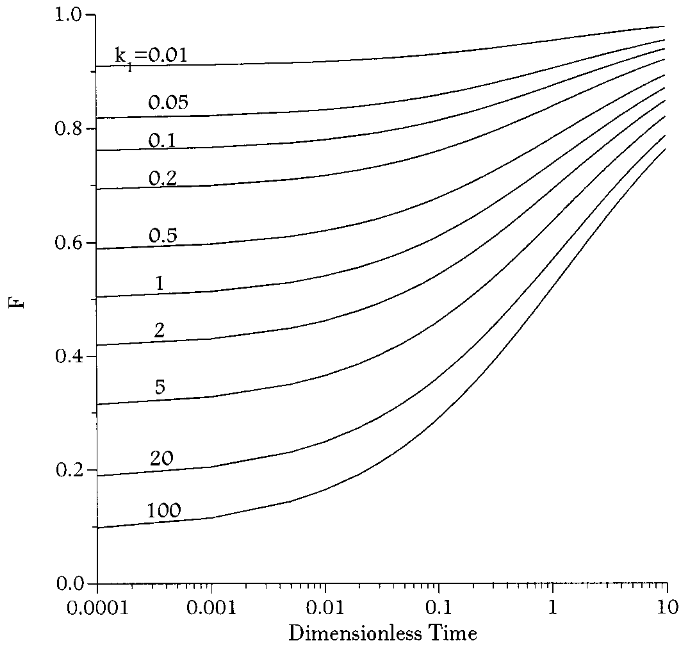

Figure 1. The variation of the dimensionless parameter F at the surface of the sphere for various values of k_1. Note that $F = (1 - \theta_0)/k_2$ and is given by equation (33)

where A is a dimensionless parameter expressing the dimensionless moisture content and its rate $\mathrm{d}\theta_0/\mathrm{d}t$ at the surface of the sphere as a function of the parameter k_1. For a particular moisture-time distribution, the parameter k_1 can be evaluated by solving (35) for k_1 which yields

$$k_1 = \frac{A^2 t}{2\pi}(2 + \sqrt{(\pi t)})^2 \left(\pi t + \frac{2}{A}\sqrt{\frac{\pi}{t}} + \sqrt{(\pi t)} \sqrt{\left(\pi t + \frac{4}{A}\sqrt{\frac{\pi}{t}} \right)} \right) \tag{36}$$

The second dimensionless parameter k_2 can then be easily obtained from (31). These two equations provide a suitable means of estimating the physical parameters involved in the coupled heat and moisture flow.

Equation (35) has been used to generate the plot shown in Figure 2. The family of curves shown in Figures 1 and 2 could also be considered as 'type-curves' which can be used to identify the physical parameters by curve matching techniques if experimental measurements of the moisture variation at the surface of the sphere are available. Figure 2 can be used to determine the dimensionless parameter k_1 whereas Figure 1 can be used to determine the second dimensionless parameter k_2.

7. RESULTS AND DISCUSSION

Equation (32) expresses the variation of the moisture content as a function of radial coordinate and time. Figure 3 gives the variation of the moisture content for various times. The physical

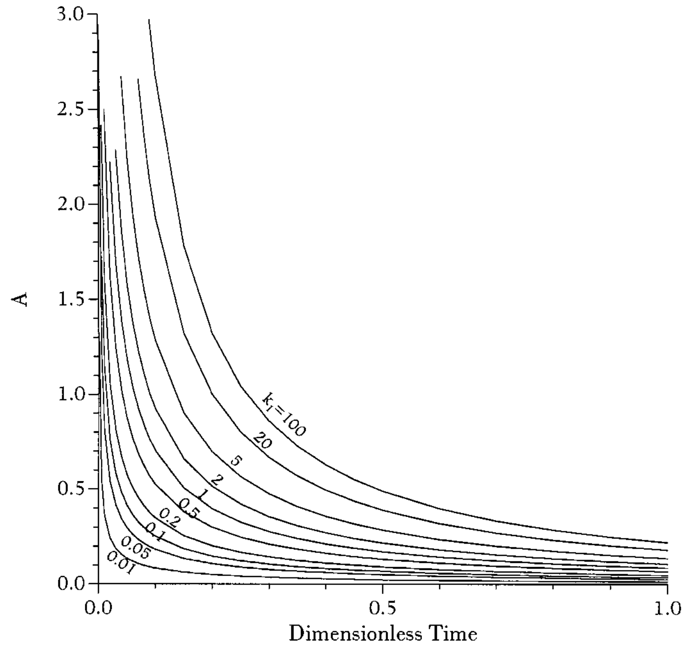

Figure 2. The variation of the dimensionless parameter A at the surface of the sphere for various values of k_1. Note that $A = 1/(1-\theta_0)\, \mathrm{d}\theta_0/\mathrm{d}t$ and is given by equation (35)

parameters used in the calculations were obtained from Thomas[24] and are representative of values applicable to a sandy fine soil:

$$\frac{\lambda}{\rho c} = 0{\cdot}008\,\mathrm{cm^2/s} \quad D_T = 10^{-5}\,\mathrm{cm^2/s^\circ C} \quad D_\theta = 10^{-2}\,\mathrm{cm^2/s} \tag{37}$$

$$T_i^* = 10^\circ\mathrm{C} \quad T_0^* = 50^\circ\mathrm{C} \quad \theta_i^* = 0{\cdot}4$$

For the above set of parameters, $k_1 = 0{\cdot}8$ and $k_2 = 0{\cdot}1$. Figures 3–5 give the reduction in the dimensionless moisture content at the surface of the sphere for various dimensionless times, k_1 and k_2, respectively. From Figure 3, one can see the typical trend of the moisture propagation in the porous media under the influence of a heat source. The moisture content decreases at the surface of the sphere, due to the action of the spherical heat source. This sharp drop of the moisture content at the heat source is followed by a rise in the moisture content further away from the source; this peak value propagates radially outward and attenuates with time. The '*hump*' in the moisture content distribution is to be expected, as the total moisture distribution in the porous media must be constant in order that the global mass balance is satisfied (see also experimental data given by Selvadurai[16]). Therefore, a drop in the moisture content below the initial value in one region must certainly be followed by a rise above the initial value in another region. However, this rise is localized and the localization is indicative of the thermal and moisture diffusivity characteristics of the medium.

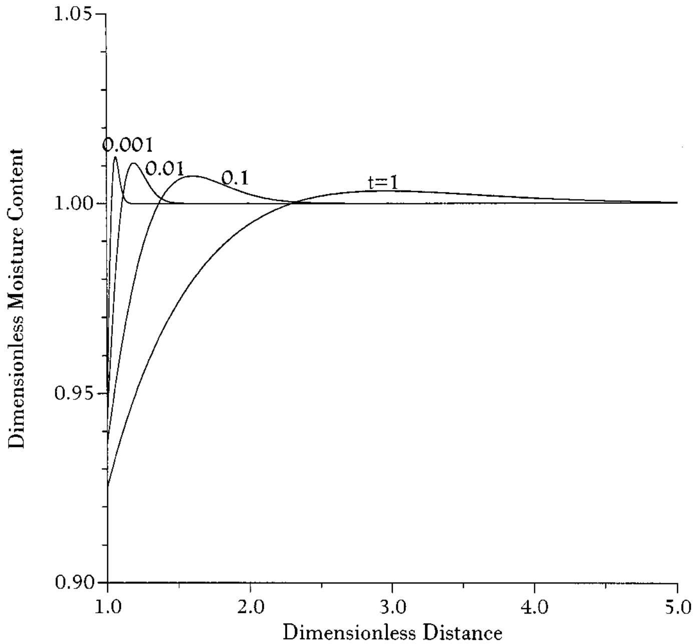

Figure 3. The variation of the dimensionless moisture content at various times for $k_1 = 0{\cdot}8$ and $k_2 = 0{\cdot}1$

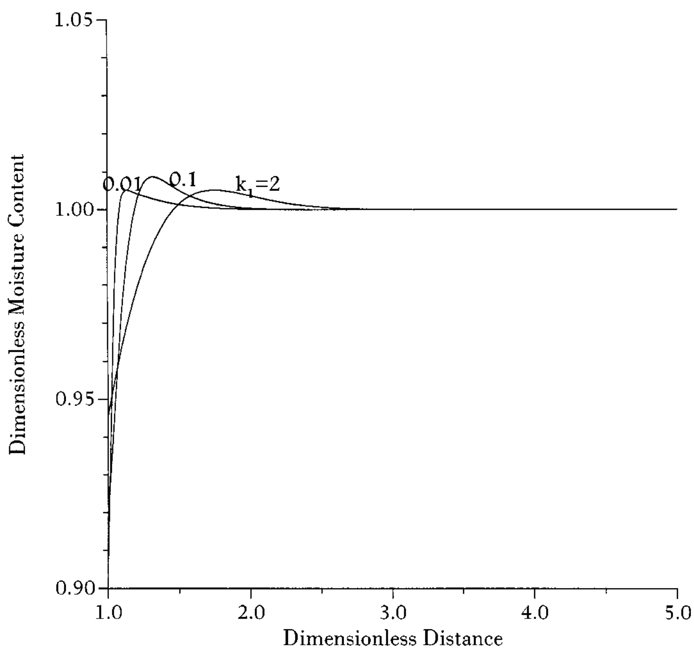

Figure 4. The variation of the dimensionless moisture content for various values of k_1 for $k_2 = 0{\cdot}1$ and $t = 0{\cdot}1$

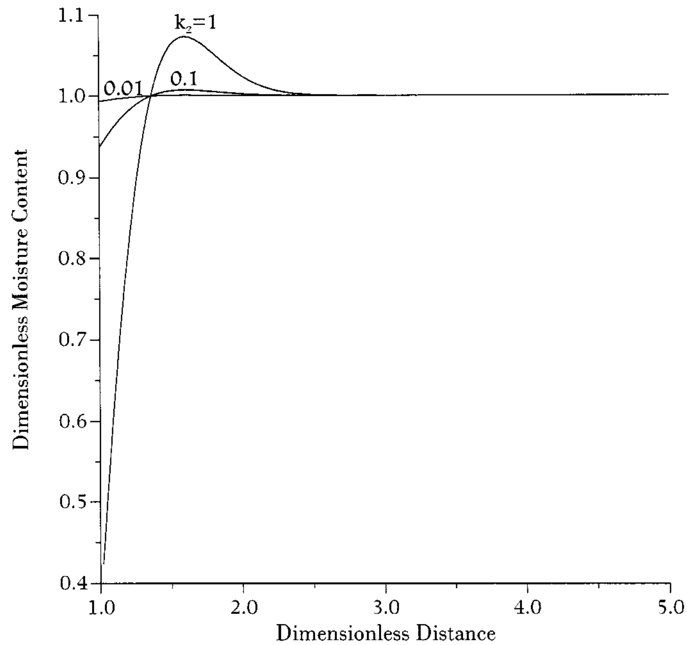

Figure 5. The variation of the dimensionless moisture content for various values of k_2 for $k_1 = 0{\cdot}8$ and $t = 0{\cdot}1$

Figures 4 and 5 present respectively the influence of the parameters k_1 and k_2 on the moisture spatial distribution. Both figures clearly show that as the parameters k_1 and k_2 increase, the moisture movement propagates outward faster. As k_1 increases, the *hump* in the moisture distribution increases while the reduction in the moisture content at the surface decreases. However, as can be observed in Figure 4, there reaches a point whereby an increase in k_1 leads to a decrease in the peak value of the moisture. A higher value of k_1 implies a higher diffusion of heat into the domain but with a varying rate, first increasing and then decreasing, as can be deduced from the forcing function (21). Similarly, as k_2 increases, the moisture distribution develops a pronounced crest shape feature, whose maximum value (see Figure 5) is further away from the spherical heat source. This is expected since an increase in k_2 implies an increase in the temperature rise $T_0^* - T_i^*$ at the surface of the sphere.

8. CONCLUSIONS

The analysis of coupled heat transfer and moisture transport in porous geomaterials presents a complex problem in mathematical analysis. Such solutions are usually obtained by considering numerical treatment of the coupled equations. The accuracy of the numerical schemes, as they relate to spatial and temporal discretizations involved in the numerical treatments, can be assessed only by appeal to analytical solutions. In this paper, a convenient analytical result is developed by invoking a non-steady-state distribution of the temperatures associated with the heat conduction process. The gradient of the temperatures together with the concentration gradients induce moisture movement within the porous medium. The paper develops the analytical expressions for the moisture profile in a problem where a spherical heat source is embedded in a medium of infinite extent. The occurrence of a pulse of moisture movement within the porous medium is predicted via the analytical method and it is observed that the peaks associated with the pulse attenuate both spatially and in time. The result can be used to examine the overall distribution of residual moisture under spherically symmetric non-steady heating conditions. Alternatively, an experimental configuration can be developed whereby the moisture distribution profiles in conjunction with the analytical results can be used to determine the diffusivity parameters associated with the moisture transport process.

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