MECHANICS OF COHESIVE-FRICTIONAL MATERIALS
Mech. Cohes.-Frict. Mater. **3**, 291–303 (1998)

Enhanced consolidation in brittle geomaterials susceptible to damage

A. T. Mahyari and A. P. S. Selvadurai*

Department of Civil Engineering and Applied Mechanics, McGill University, 817 Sherbrooke Street West, Montreal, Q.C., Canada H3A 2K6

SUMMARY

This paper examines consolidation behaviour of saturated geomaterials with a matrix component which is susceptible to damage. Finite-element-based computational model accounts for the alteration in both the deformability and permeability characteristics of the porous material due to damage evolution. The isotropic damage criteria governing the evolution of elastic stiffness and hydraulic conductivity parameters are characterized by the dependency of the damage variable on the distortional strain invariant. The computational procedure is utilized to evaluate the extent to which the time-dependent axisymmetric indentation behaviour of a rigid circular punch on a poroelastic half-space can be influenced by the damage evolution in the porous skeleton. © 1998 John Wiley & Sons, Ltd.

KEY WORDS: poroelasticity; isotropic damage; brittle geomaterials; enhanced consolidation; saturated geomaterials; computational modelling; indentation of geomaterials

1. INTRODUCTION

The classical theory of poroelasticity developed by Biot[1,2] examines the coupled behaviour of fluid flow and elastic deformations on the consolidation process of porous materials saturated with either incompressible or compressible pore fluids. The theory of poroelasticity has been successfully applied to examine time-dependent transient phenomena in a variety of natural and synthetic materials, including geomaterials and biomaterials.[3] The assumption of linear elastic behaviour of the porous skeleton is a significant limitation in the application of the classical theory of poroelasticity to brittle geomaterials which could exhibit stiffness changes and, in particular, elastic stiffness degradation in the constitutive behaviour of the skeleton of the geomaterial. This non-linear behaviour can be due to development of microcracks and microvoids in the porous fabric of the geomaterial which essentially retains its elastic nature (i.e. absence of irreversible plasticity phenomena). Such damage or microvoid and microcrack evolution can result in the alteration of the permeability characteristics of the porous medium. The effect of such *damage* on either the degradation of elastic moduli, and in extreme situations, on the strength of the materials (i.e. strain softening) has been observed by Cook[4] and Bieniawski *et al.*[5] in experiments conducted on rocks and by Spooner and Dougill[6] in experiments conducted

* Correspondence to: A. P. S. Selvadurai, Dept. of Civil Engineering and Applied Mechanics, McGill University, 817 Sherbrooke Street West, Montreal, Q.C., Canada H3A 2K6.

CCC 1082-5010/98/03291–13$17.50

Received 25 May 1997
Revised 26 September 1997

on concrete. The theory of damage mechanics has been extensively applied to model the behaviour of such brittle geomaterials.[7–10] The effect of microcrack developments on permeability characteristics of saturated geomaterials has also been observed by Zoback and Byerlee,[11] Shiping *et al.*,[12] and Kiyama *et al.*[13] in rocks and by Samaha and Hover[14] in concrete.

When considering saturated poroelastic geomaterials, their consolidation response can be influenced by the evolution of damage in the porous skeleton. The notion of continuum damage is considered to be more relevant to geomaterials such as soft rocks and overconsolidated clays were progressive softening in an *elastic* sense can occur due to generation of microvoids or microcracks. The classical theory of continuum damage mechanics[15] can be extended to model such damage phenomena in porous saturated materials. This theory simulates the effect of microcrack developments on the behaviour of materials prior to the development of *macrocracks* (i.e. fractures). In such modelling, damage is interpreted as a reduction in the stiffness of the material due to the generation of microcracks and other microdefects. In this study attention is restricted primarily to *brittle elastic behaviour* of material where the associated damage processes before inception of strain softening (i.e. strictly pre-peak elastic behaviour) can be described by an isotropic damage model. Admittedly, the damage processes are expected to be highly anisotropic in nature and could invariably be restricted to localized zones. The effect of soil skeletal damage on the consolidation behaviour of saturated geomaterials can be examined by representing the stiffness properties and the permeability characteristics of porous medium as a function of the state of damage in the saturated geomaterial. Cheng and Dusseault[16] developed an anisotropic damage model to examine the poroelastic behaviour of saturated geomaterials. Their studies were, however restricted to the case where there was no corresponding evolution in the permeability characteristics of the geomaterials during the damage processes.

In this study a finite element technique is used to examine the influence of damage-induced alterations in both the elastic stiffness and the permeability characteristics of the porous geomaterial on the corresponding consolidation response of a saturated poroelastic medium. The isotropic damage evolution law used in the analysis is characterized by the dependency of damage parameters on the distortional strain invariant. Two different phenomenological damage criteria governing the evolution of permeability characteristics are postulated from experimental observations on saturated geomaterials. Finally, the numerical procedure is utilized to evaluate the extent to which the time-dependent indentation behaviour of a smooth rigid circular punch with a permeable base resting on a poroelastic half-space can be influenced by the damage evolution in the porous skeleton (Figure 1).

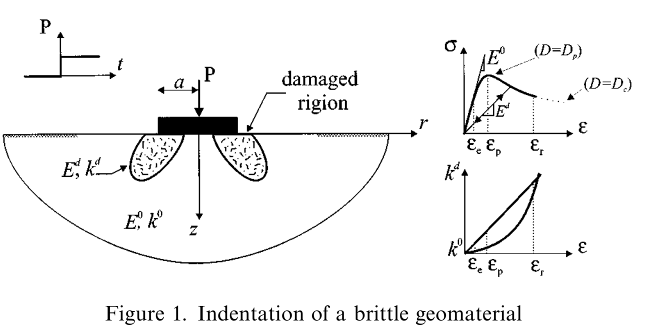

Figure 1. Indentation of a brittle geomaterial

2. GOVERNING EQUATIONS

The basic equations governing Biot's theory of poroelasticity are summarized for completeness. The constitutive equations governing the quasi-static response of a poroelastic medium, which consists of a porous isotropic elastic soil skeleton saturated with a *compressible pore fluid* take the forms

$$\sigma_{ij} = \frac{2\mu\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} - \frac{3(\nu_u - \nu)}{B(1-2\nu)(1+\nu_u)}p\delta_{ij} \tag{1a}$$

$$p = \frac{2\mu B^2(1-2\nu)(1+\nu_u)^2}{9(\nu_u-\nu)(1-2\nu_u)}\zeta_\upsilon - \frac{2\mu B(1+\nu_u)}{3(1-2\nu_u)}\varepsilon_{kk} \tag{1b}$$

where σ_{ij} is the total stress tensor, p is the pore fluid pressure, ζ_υ is the volumetric strain in the compressible pore fluid, ν is Poisson's ratio of the porous fabric, ν_u is the undrained Poisson's ratio, μ is the linear elastic shear modulus, and B is Skempton's pore pressure coefficient. Also ε_{ij} is the soil skeleton strain tensor, $2\varepsilon_{ij} = u_{i,j} + u_{j,i}$; where u_i are the displacement components, and a comma denotes a partial derivative with respect to a spatial variable. The effective stresses in the soil skeleton are given by

$$\sigma'_{ij} = \sigma_{ij} - \alpha p\delta_{ij} \tag{2}$$

where $\alpha = 3(\nu_u - \nu)/B(1-2\nu)(1+\nu_u)$. We consider body force free poroelastic media which are in quasi-static equilibrium (i.e. $\sigma_{ij,j} = 0$), and fluid flow in the pores is governed by the isotropic form of Darcy's law and the continuity equation

$$\upsilon_i = -\kappa p_{,i}, \quad \frac{\partial \zeta_\upsilon}{\partial t} + \upsilon_{i,i} = 0 \tag{3}$$

where υ_i is the specific discharge vector in the pore fluid and $\kappa = k/\gamma_w$, where k is the coefficient of hydraulic conductivity and γ_w is the unit weight of pore fluid.

Considering requirements for a positive-definite strain energy potential,[17] it can be shown that the material parameters should satisfy the following thermodynamic constraints: $\mu > 0$; $0 \leqslant B \leqslant 1$; $-1 < \nu < \nu_u \leqslant 0{\cdot}5$; $\kappa > 0$. The resulting equations of equilibrium for a poroelastic medium as introduced by Biot[1] and reformulated in more physically relevant variables by Rice and Cleary,[17] can be written in terms of the displacements and pore pressure as

$$\mu\nabla^2 u_i + \frac{\mu}{(1-2\nu)}\varepsilon_{kk,i} - \frac{3(\nu_u-\nu)}{B(1-2\nu)(1+\nu_u)}p_{,i} = 0 \tag{4a}$$

$$\frac{\partial p}{\partial t} - \frac{2\kappa\mu B^2(1-2\nu)(1+\nu_u)^2}{9(\nu_u-\nu)(1-2\nu_u)}\nabla^2 p = -\frac{2\mu B(1+\nu_u)}{3(1-2\nu_u)}\frac{\partial \varepsilon_{kk}}{\partial t} \tag{4b}$$

For a well-posed problem, boundary conditions and initial conditions on the variables u_i, σ_{ij} and p also need to be prescribed.

3. FINITE ELEMENT FORMULATIONS

Finite element methods have been widely applied for the study of problems in poroelasticity (see e.g. References 18–20). Reviews of both analytical and numerical approaches to the study of soil consolidation related to poroelastic media are given by Lewis and Schrefler[21] and Selvadurai.[3] The brief presentation here covers the general Galerkin finite element formulations of poroelasticity. The details of the Galerkin procedure are well documented by Sandhu and Wilson,[18] Lewis and Schrefler[21] and more recently by Selvadurai and Nguyen[22] in connection with the finite element modelling of isothermal consolidation of sparsely fractured porous media.

The governing equations (4) are discretized in the spatial domain using an eight-noded axisymmetric isoparametric element where the displacements within the element are interpolated as functions of all eight nodes, whereas the pore pressures are interpolated only as a function of the four corner nodes *i*, *k*, *m*, and *o* (Figure 2). The rationale for this procedure is now well documented (see e.g. References 21 and 23). The application of a Galerkin procedure to the governing equations of poroelasticity gives rise to the following distretized forms of the equations governing poroelastic media:

$$\begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \{-\gamma \Delta t \mathbf{H} + \mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t+\Delta t} \\ p_{t+\Delta t} \end{Bmatrix} = \begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \{(1-\gamma)\Delta t \mathbf{H} + \mathbf{E}\} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t} \\ p_{t} \end{Bmatrix} + \{\mathbf{F}\} \tag{5}$$

where **K** is the stiffness matrix of the soil skeleton; **C** the stiffness matrix due to interaction between soil and the pore fluid; **E** the compressibility matrix for the pore fluid; **H** the permeability matrix; **F** the force vectors due to external tractions, body forces and flows; $\mathbf{u}_t$ the nodal displacements at time t; p_t the pore pressure at time t; and Δt the time increment.

The time integration constant γ varies between 0 and 1. The criteria governing stability of the integration scheme given by Booker and Small[20] require that $\gamma \geqslant \frac{1}{2}$. According to Lewis and Schrefler[21] and Selvadurai and Nguyen,[22] the stability of solution can generally be achieved by selecting values of γ close to 1.

4. CONTINUUM DAMAGE MECHANICS

The concept of continuum damage mechanics is generally attributed to Kachanov[15] in recognition of the fundamental studies describing the tertiary creep of solids. Since these initial

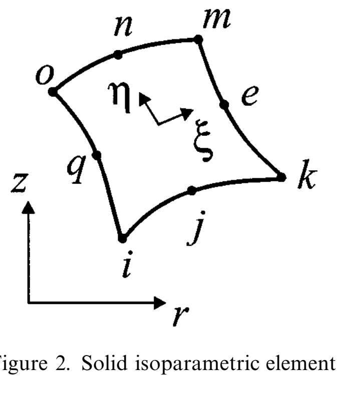

Figure 2. Solid isoparametric element

developments, the theory of continuum damage mechanics has been widely used to predict the non-linear response of variety of materials including metals, concrete, composites, ice, and geological materials (see e.g. References 8, 16, 24–27, etc.). The non-linear behaviour of most brittle materials is attributed to the initiation of new microdefects and growth of existing microdefects. This behaviour can be modelled by introducing local continuous damage variables in the analysis.[15] Damage variables reflect average material degradation at the macro-scale normally associated with the classical continuum description. In cases where the damage process results in microvoids with a spherical form or microcracks which have distribution and orientation without a preferred direction, the damage phenomena can be described by appeal to the scalar damage variable D given by

$$D = \frac{A_0 - \bar{A}}{A_0} \tag{6}$$

where A_0 is the initial area, $\bar{A}$ is the reduced net area. The damage variable D varies between 0 and D_c, where D_c is a critical value which corresponds to the fracture of material (the critical damage value D_c can also be regarded as a normalizing parameter against which the damage evolution can be measured). This definition facilitates the adaptation of the 'damage' concept within the theory of elasticity or in any other theory associated with classical continuum mechanics. The coupling of elasticity and damage models has been investigated by a number of researchers including Sidoroff,[28] Mazars,[7] Krajcinovic,[25] Chow and Wang,[29] and Lemaitre and Chaboche.[30]

The introduction of a damage variable D leads directly to the concept of a net stress which is the stress defined in relation to the net area. For isotropic damage, the net stress tensor σ^n_{ij} is related to the stress tensor σ_{ij} in the undamaged state by the following:

$$\sigma''_{ij} = \frac{\sigma_{ij}}{1 - D} \tag{7}$$

Furthermore, the hypothesis of 'strain equivalence' proposed by Lemaitre[31] ensures that the constitutive laws applicable to the virgin material are also applicable to the damaged material, with the stress in damaged state being replaced by the net stress. The constitutive equation for the damaged skeletal material which exhibits elastic isotropy and isotropic damage can then be written in following form:

$$\sigma_{ij} = 2\bar{\mu}\varepsilon_{ij} + \frac{2\bar{\mu}v}{1-2v}\varepsilon_{kk}\delta_{ij} = 2(1-D)\mu\varepsilon_{ij} + \frac{2(1-D)\mu v}{1-2v}\varepsilon_{kk}\delta_{ij} \tag{8}$$

where $\bar{\mu} = (1 - D)\mu$ is the shear modulus applicable to damaged material. This implies that Poisson's ratio always remains constant (i.e. $\bar{v} = v$) which is an added constraint when considering three-dimensional states of stress.[32]

The gradual degradation in the constitutive properties of the material is as a result of continuing growth of either already existing microdefects or the progressive nucleation of new microdefects. For a given state of stress the extent of damage is an intrinsic property of the material which is characterized by a damage evolution law. The damage evolution criteria can either be postulated by appeal to micromechanical considerations or determined by experiment. Following observations of experiments conducted on rocks, Cheng and Dusseault[16] assumed that damage evolution is a function of the shear strain energy and proposed the following damage evolution equation for rocks:

$$\frac{\partial D}{\partial \xi_{\mathrm{d}}} = \eta \frac{\gamma \xi_{\mathrm{d}}}{1 + \gamma \xi_{\mathrm{d}}} \left(1 - \frac{D}{D_{\mathrm{c}}} \right) \tag{9}$$

where the equivalent shear strain ξ_{d} is defined as

$$\xi_{\mathrm{d}} = (e_{ij} e_{ij})^{1/2}, \quad e_{ij} = \varepsilon_{ij} - \tfrac{1}{3} \varepsilon_{kk} \delta_{ij} \tag{10}$$

and η, γ are material constants which are positive. In this formulation, the normalizing damage measure is the critical damage D_{c} which is associated with the damage corresponding to a residual value of the strength of the geomaterial under uniaxial compression. We note that D_{c} need not be the only normalizing variable; the formulation can be presented in terms of D_{p} the damage at peak loads which can limit the development of localization effects that can result when $D \to D_{\mathrm{c}}$.

For saturated geomaterials susceptible to damage, the elastic properties and permeability characteristics can alter due to development of microcracks in the porous fabric.

4.1. *Deformability characteristics*

The constitutive parameters applicable to an isotropic poroelastic material which experiences micromechanical damage in the porous fabric can be represented as a function of intact elastic properties by invoking the hypothesis of strain equivalence.[31] The generalized constitutive tensor applicable to damaged materials which exhibit isotropic damage takes the form

$$C^{\mathrm{d}}_{ijkl} = (1 - D) C_{ijkl} \quad (i = 1, \ldots, 4,\ j = 1, \ldots, 4) \tag{11}$$

where C_{ijkl} is the elasticity tensor applicable to virgin elastic materials (see e.g. Reference 33). The damage evolution law can specify the variation of the damage variable (D) with the state of strain in material. The damage evolution law proposed by Cheng and Dusseault[16] is employed in this study to model the elastic stiffness degradation of materials. The evolution of damage variable can be obtained by the integration of (9) (between the limits D_0 to D) as follows:

$$D = D_{\mathrm{c}} - (D_{\mathrm{c}} - D_0)(1 + \gamma \xi_{\mathrm{d}})^{\eta/\gamma D_{\mathrm{c}}} \exp(-\eta \xi_{\mathrm{d}} / D_{\mathrm{c}}) \tag{12}$$

where D_0 is the initial value of damage variable corresponding to intact state of material (e.g. $D_0 = 0$ for virgin state of materials).

4.2. *Hydraulic conductivity characteristics*

Development of damage criteria which can account for alterations in the hydraulic conductivity during evolution of damage in saturated geomaterials is necessary for computational modelling of such phenomena in poroelastic media. Literature on the coupling between microcrack developments and permeability evolution in saturated geomaterials is primarily restricted to experimental observations. The effect of microcrack development on the permeability characteristics of fluid saturated geomaterial was first investigated by Zoback and

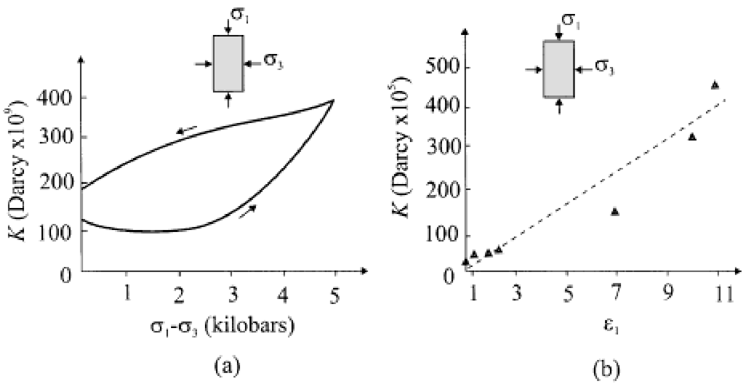

Figure 3. Permeability evolution in saturated geomaterials (a) after Zoback and Byerlee;[11] and (b) after Shiping *et al.*[12]

Byerlee,[11] who conducted triaxial tests on granite. Figure 3 illustrates their experimental observations which indicate that the permeability coefficient is first reduced slightly due to closure of pre-existing microcracks and void channels as a result of the elastic deformations of the intact material, and begins to increase as a result of the growth of existing microcracks and the nucleation of new microcracks. Shiping *et al.*[12] examined the permeability evolution of sandstone for a series of complete triaxial stress–strain paths (Figure 3). They observed that permeability characteristics of material can increase nearly by one order of magnitude, prior to the peak values of the stress and can increase up to two orders of magnitude in the strain-softening regime where microcracks tend to localize in shear faults. Kiyama *et al.*[13] also observed similar results for permeability evolution of granites in triaxial experiments. This would suggest that localization phenomena can result in significant changes in the permeability in the localization zones. It must be emphasized that in this study the process of localization is excluded from the analysis and all changes in permeability are assumed to materialize in a distributed fashion at stress states well below those which can initiate localization.

There has been only limited work in the context of constitutive modelling of permeability characteristics in damaged porous materials. In this study two phenomenological constitutive models based on the experimental observations by Zoback and Byerlee[11] and Shiping *et al.*[12] are proposed for the permeability evolution criteria in poroelastic media. The slight reduction in the hydraulic conductivity of saturated geomaterials in the elastic range prior to the onset of microcrack developments is neglected. The hydraulic conductivity (k) is assumed to have either linear or quadratic variations with respect to equivalent shear strain ξ_{d} (equation (10)) as follows:

$$k^{\mathrm{d}} = (c_1 + c_2 \xi_{\mathrm{d}}) k^0 \tag{13a}$$

$$k^{\mathrm{d}} = (c_3 + c_4 \xi_{\mathrm{d}}^2) k^0 \tag{13b}$$

where k^{d} is the hydraulic conductivity applicable to damaged materials and c_1, c_2, c_3 and c_4 are material constants.

5. COMPUTATIONAL PROCEDURES AND NUMERICAL RESULTS

The effect of soil skeletal damage on the time-dependent poroelastic behaviour of saturated geomaterials can be examined by incorporating continuum damage mechanics within the classical theory of poroelasticity. This is achieved by the representation of elastic parameters and permeability characteristics of the porous medium as a function of the state of damage in material. In the computational procedures, damage evolution criteria governing the alteration of elastic stiffness and permeability parameters of the porous material are described in relation to the damage variable and the equivalent shear strain.

An incremental finite element procedure is developed using the damage evolution criteria in the numerical treatment of damaged poroelastic materials. The scalar damage variables are first obtained at nine Gauss points within finite elements. The constitutive matrix C^{d}_{ijkl} and the hydraulic conductivity k^{d} are updated at these locations to incorporate the evolution of damage. The discretized governing equations are then solved to obtain the state of strains at each integration point using the updated matrix C^{d}_{ijkl} and parameter k^{d} in the incremental analysis. The coupling between the state of strain and state of damage in each time step is solved by an iterative process using the standard Newton–Raphson technique. The convergence criterion adopted is based on the norm of the evolution of the damage variable[8] in relation to a specified tolerance ω (see Box 1). The initial state of damage (i.e. D_0) can also be prescribed for a given element. The computational procedures are performed at each time step until the convergence criterion is satisfied in the time-dependent analysis. The details of the incorporating damage evolution algorithm are given in Box 1.

(i) Compute D at Gauss integration points

$$D = D_c - (D_c - D_0)(1 + \gamma\xi_d)^{\eta/\gamma D_c} \exp(-\eta\xi_d/D_c)$$

$$\xi_d = (e_{ij}e_{ij})^{1/2}, \quad e_{ij} = \varepsilon_{ij} - \tfrac{1}{3}\varepsilon_{kk}\delta_{ij}$$

(ii) Update the poroelastic parameters

$$C^{d}_{ijkl} = (1 - D)C_{ijkl}$$

$$k^{d} = (c_1 + c_2\xi_d)k^0$$

or

$$k^{d} = (c_3 + c_4\xi_d^2)k^0$$

(iii) Solve the governing equations for u_t, p_t and calculate the strain tensor

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{i,j})$$

(iv) Check convergence criteria

$$\mathrm{SQRT}\{[(D_i^{itr} - D_i^{itr-1})/D_i^{itr-1}]^2\} \leqslant \omega$$

No: Damage growth. Return to (i).
Yes: No further damage evolution. Exit.

Box 1. Damage evolution algorithm.

As an example of the application of the modelling concepts and computational developments, we consider the axisymmetric indentation of a poroelastic half-space susceptible to damage by a rigid porous smooth circular indentor. The poroelastic medium is subjected to a total load P with a time variation in form of a Heaviside step function (Figure 1). The soil skeleton has a Poisson's ratio ν and the pore fluid within the geomaterial is assumed to be nearly incompressible ($\nu_u = 0{\cdot}499$). Again we emphasize the fact that damage evolution precedes strain softening and the formulations are applicable to elastic responses prior to strain softening. The material parameters employed in numerical simulations for an isotropic damage model are those adopted by Cheng and Dusseault[16] for a material behaviour shown in Figure 4 and are as follows:

$$E = 8300\ \text{MPa}, \quad \nu = 0{\cdot}195, \quad \gamma = \mu = 130, \quad D_c = 0{\cdot}75$$

The hydraulic conductivity is assumed to increase one order of magnitude from the initial virgin state (k^0) to the peak stress ($\sigma_{max} = 30$ MPa) consistent with experimental observations.[12,13] In the development of the computational results, both linear and quadratic variations of the hydraulic conductivity are considered. Figure 4 illustrates the uniaxial stress–strain behaviour and the evolution of hydraulic conductivity of material.

The finite element discretization of the problem and the associated boundary conditions are shown in Figure 5. Figure 6 illustrates the numerical results related to the degree of consolidation settlement of indentor and the evolution of a zone of damage (i.e. where $D/D_c > 0{\cdot}05$) with time. For the non-linear damage model where no permeability modifications are implemented, the rate of consolidation decreases due to softening of material as a result of microcrack development. The generation of damage in the soil skeleton results in a higher displacement and pore pressure. When the damage-induced alteration of hydraulic conductivity is taken into consideration, the rate of consolidation increases and the excess pore pressures dissipate at a faster rate (Figure 6). The effect of an increase in hydraulic conductivity overrides the effect of reduction in elastic stiffness properties on the consolidation behaviour of material. Therefore, the damaged material with no modification of hydraulic characteristics requires a longer time to achieve the same degree of consolidation. Also, this difference is more significant when a linear form of the evolution for the hydraulic conductivity is considered where the increase in hydraulic

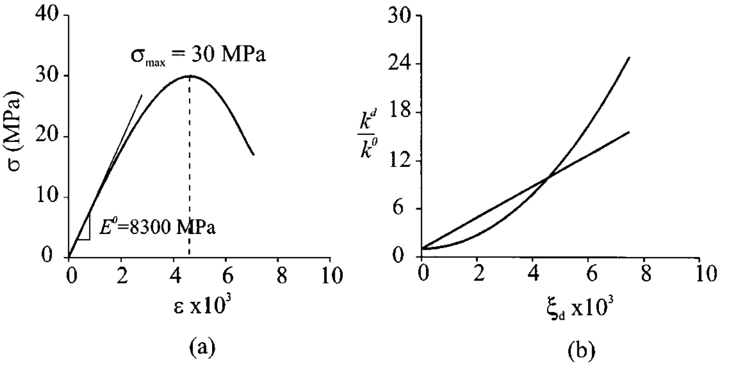

Figure 4. The stress–strain behaviour and permeability evolution of saturated geomaterial

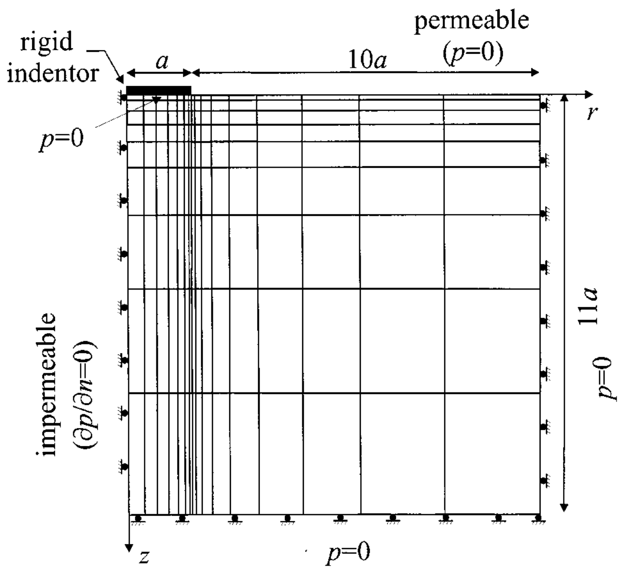

Figure 5. Finite element discretizations

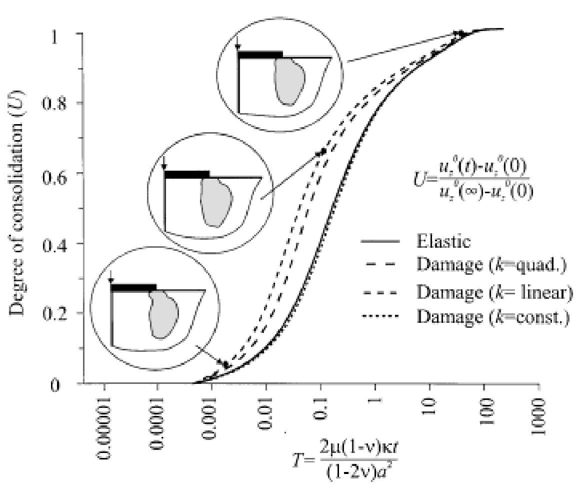

Figure 6. Degree of consolidation and evolution of damaged zone (where $D/D_c > 0{\cdot}05$) with time

conductivity is maximum. Figure 6 also indicates that much of the damage generation takes place instantaneously and further pore pressure diffusion which results in the change of effective stresses, does not appreciably alter the extent of damage. Admittedly no generalizations can be made of this observation since the rate of load transfer will depend on the prescribed damage evolution laws.

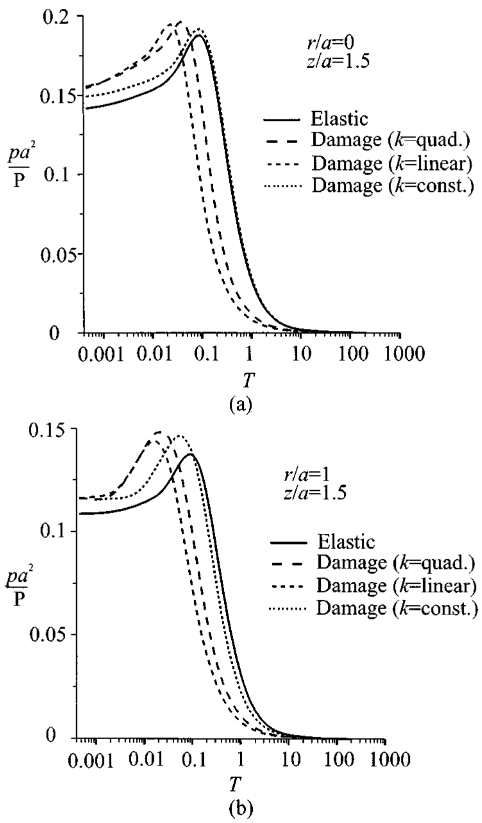

Figure 7. Pore pressure evolution at different depths

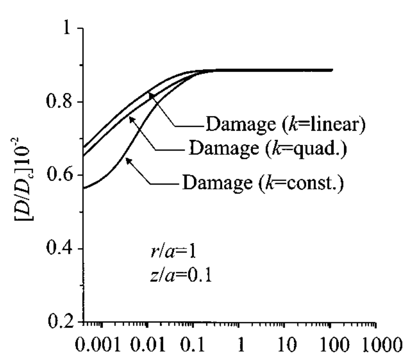

Figure 8. Evolution of damage variable at the edge of indentor

Figure 7 illustrates the evolution of pore pressure at two locations of the indentor; the centre of indentor ($r/a = 0$) and the edge of indentor ($r/a = 1$) corresponding to a depth of $z/a = 1{\cdot}5$ within the poroelastic half-space. The damage models predict higher excess pore pressures in the porous medium which is consistent with observations by Cheng and Dusseault.[16] The evolution of the damage variable with time at the edge of indentor ($r/a = 1$) at a depth of $z/a = 0{\cdot}1$ is also shown in Figure 8.

6. CONCLUDING REMARKS

The classical theory of poroelasticity for a fluid saturated brittle geomaterial has been extended through computational modelling to include the influence of both damage evolution in the geomaterial fabric and alterations in the fluid transport behaviour due to damage evolution. This latter modification to the modelling is considered to be a novel development in the application of damage mechanics concepts to the study of poroelastic phenomena. The studies to date are based on plausible damage evolution laws which are derived from a limited database of experimental results. The damage evolution laws based on micromechanical considerations, on the other hand, will require considerably more analytical efforts and the incorporation of the possible influences of scale at which micromechanical processes generate microcrack evolution which manifests in the form of damage. The procedure applied in this paper is intended to capture phenomenological processes which can be modelled by appeal to experimentation. The computational modelling of damage evolution in the geomaterial fabric and the alteration in the hydraulic transport characteristics can be easily accomodated with a conventional formulation of computational modelling of transient processes in poroelastic media. The indentation problem modelled in this paper illustrates an example where damage evolution and permeability alterations can occur in zones of high local contact stresses. The modelling strictly excludes the possible development of strain localization phenomena. It is appreciated that such strain localization phenomena can contribute to both non-homogeneity and anisotropy in the permeability characteristics which merits further consideration. Also, the incorporation of such localization effects will require a consistent formulation of the computational scheme to account for scale effects, numerical stability and mesh dependency (see e.g. Reference 34). The numerical results presented in the paper illustrate the various influences of geomaterial skeletal softening and alterations in the hydraulic transport characteristics on the consolidation rate for the indentor. For the damage laws considered in this paper, the influence of hydraulic property alterations due to damage appears to have a greater influence on the time-dependent consolidation rate of the indentor.

ACKNOWLEDGMENTS

The work described in this paper was supported through a Natural Sciences and Engineering Research Council of Canada Research Grant A3866 awarded to A.P.S. Selvadurai. The authors would like to thank the referees for their critical comments which led to substantial improvement in the presentation of the paper.

REFERENCES

1. M. A. Biot, 'General theory of three-dimensional consolidation', *J. Appl. Phys*., **12**, 155–164 (1941).
2. M. A. Biot, 'Theory of elasticity and consolidation for a porous anisotropic solid', *J. Appl. Phys*., **26**, 182–185 (1955).
3. A. P. S. Selvadurai (ed), *Mechanics of Poroelastic Media*, Kluwer Academic Publishers, Dordrecht, Netherlands, 1996.
4. N. G. W. Cook, 'Failure of rock', *Int. J. Rock Mech. Min. Sci. Geomech. Abs*., **2**, 289–403 (1965).
5. Z. T. Bieniawski, H. G. Denkhaus and U. W. Vogler, 'Failure of fractured rock', *Int. J. Rock. Mech. Min. Sci. Geomech. Abs*., **6**, 323–346 (1967).
6. D. C. Spooner and J. W. Dougill, 'A quantitative assessment of damage sustained in concrete during compressive loading', *Mag. Conc. Res*., **27**, 151–160 (1975).

7. J. Mazars, 'Mechanical damage and fracture of concrete structures', *Advances in Fracture Research*, Vol. 4, Pergamon Press, Oxford, pp. 1499–1506, 1982.
8. J. C. Simo and J. W. Ju, 'Strain- and stress-based continuum damage models-II. Computational aspects', *Int. J. Solids Struct*., **23**, 841–869 (1987).
9. S. Yazdani and H. L. Schreyer, 'Anisotropic damage model with dilation for concrete', *Mech. Mater*., **7**, 231–244 (1988).
10. R. de Borst, J. Piman, R. H. J. Peerlings and L. J. Sluys, 'On gradient-enhanced damage and plasticity models for failure in quasi-brittle and frictional materials', *Comput. Mech*., **17**, 130–141 (1995).
11. M. D. Zoback and J. D. Byerlee, 'The effect of microcrack dilatancy on the permeability of Westerly Granite', *J. Geophys. Res*., **80**, 752–755 (1975).
12. L. Shiping, L. Yushou, L. Yi, W. Zhenye and Z. Gang, 'Permeability-strain equations corresponding to the complete stress–strain path of Yinzhuang Sandstone', *Int. J. Rock Mech. Min. Sci. Geomech. Abs*., **31**, 383–391 (1994).
13. T. Kiyama, H. Kita, Y. Ishijima, T. Yanagidani, K. Aoki and T. Sato, 'Permeability in anisotropic granite under hydrostatic compression and triaxial compression including post-failure region', *Proc. 2nd North American Rock Mech. Sym*., pp. 1643–1650, 1996.
14. H. R. Samaha and K. C. Hover, 'Influence of microcracking on the mass transport properties of concrete', *ACI Mat. J*., **89**, 416–424 (1992).
15. L. M. Kachanov, 'Time of rupture process under creep conditions', *Isv. Akad. Nauk SSR Otd. Teck*., **8**, 26–31 (1958).
16. H. Cheng and M. B. Dusseault, 'Deformation and diffusion behaviour in a solid experiencing damage: a continuous damage model and its numerical implementation', *Int. J. Rock Mech. Min. Sci. Geomech. Abs*., **30**, 1323–1331 (1993).
17. J. R. Rice and M. P. Cleary, 'Some basic stress diffusion solution for fluid-saturated elastic porous media with compressible constituents', *Rev. Geophy. Space Phys*., **14**, 227–241 (1976).
18. R. S. Sandhu and E. L. Wilson, 'Finite element analysis of flow in saturated porous media', *J. Engng. Mech. ASCE*, **95**, 641–652 (1969).
19. J. Ghaboussi and E. L. Wilson, 'Flow of compressible fluid in porous elastic media', *Int. J. Num. Meth. Engng*., **5**, 419–442 (1973).
20. J. R. Booker and J. C. Small, 'An investigation of the stability of numerical solution of Biot's equations of consolidation', *Int. J. Solids. Struct*., **11**, 907–917 (1975).
21. R. W. Lewis and B. A. Schrefler, *The Finite Element Method in the Deformation and Consolidation of Porous Media*, Wiley, New York, 1987.
22. A. P. S. Selvadurai and T. S. Nguyen, 'Computational modelling of isothermal consolidation of fractured porous media', *Comput. Geotech*., **17**, 39–73 (1995).
23. I.M. Smith and D. V. Griffiths, *Programming the Finite Element Method*, Wiley, New York, 1988.
24. D. Krajcinovic and G. U. Fonseka, 'The continuous damage theory of brittle materials', *J. Appl. Mech*., **48**, 809–815 (1981).
25. D. Krajcinovic, 'Continuous damage mechanics', *Appl. Mech. Rev*., **37**, 1–6 (1984).
26. A. P. S. Selvadurai, 'Transverse loading of unidirectionally reinforced composites: the role of matrix fracture', in *Damage Mechanics of Composites*, ASME, New York; D.H. Allen and J.W. Ju eds, Vol. **185**, pp. 117–127, 1994.
27. A. P. S. Selvadurai and J. Hu, 'The axial loading of foundations embedded in frozen soils', *Int. J. Offshore and Polar Engng*., **6**, 96–103 (1995).
28. F. Sidoroff, 'Description of anisotropic damage application to elasticity', in *Int. The. Appl. Mech. Symp. Senlis, France. Phys. Nonlin. Struc. Analy*., J. Hult and J. Lemaitre eds, Springer, Berlin, Heidelberg, New York, pp. 237–244, 1980.
29. C. H. Chow and J. Wang, 'An anisotropic theory of elasticity for continuum damage mechanics', *Int. J. Fracture*, **33**, 3–16 (1987).
30. J. Lemaitre and J. L. Chaboche, *Mechanics of Solid Materials*, Cambridge University Press, Cambridge, England, 1990.
31. J. Lemaitre, 'How to use damage mechanics', *Nucl. Engng. Design*, **80**, 233–245 (1984).
32. J. W. Ju, 'Isotropic and anisotropic damage variables in continuum damage mechanics', *J. Engng. Mech. ASCE*, **116**, 2764–2769 (1990).
33. R. O. Davis and A. P. S. Selvadurai, *Elasticity and Geomechanics*, Cambridge University Press, Cambridge, England, 1996.
34. B. A. Schrefler, L. Sanavia and C. E. Majorana, 'A multiphase medium model for localisation and post-localisation simulation in geomaterials', *Mech. Cohesive-Frictional Mat*., **1**, 95–114 (1996).

