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TRANSIENT THERMO-VISCOELASTIC RESPONSE OF A CRACK IN A LAYERED STRUCTURE

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This article examines the application of a numerical method of transient stress analysis for the study of a crack located in a viscoelatic layer bonded to an elastic substrate. The layered structure containing the defect is subjected to heat conduction and associated thermo-viscoelastic effects. The finite-element technique is used to examine the time-dependent variation in the stress intensity factor at the crack tip due to a sudden reduction in the temperature at the surface of the layered structure. Numerical results presented in the article illustrates the influence of the thermo-viscoelastic coupling on the crack behavior.

INTRODUCTION

The mechanics of layered structures is of interest to a variety of engineering applications including thermoplastic coatings, laminated composite media, and asphalt pavement structures. In particular, the study of the mechanical behavior of pavement structures that are subjected to low-temperature effects is regarded as an important aspect of improving pavement performance and design. These studies become more important in situations where defects such as cracks, delaminations, voids, etc., are induced in the asphalt pavements and overlays either during its construction or during its service life [1–3]. Recent studies [4, 5] have examined the behavior of a pavement structure with a surface crack that is subjected to the combined effects of heat conduction and associated thermoelastic effects. Such surface cracks could be created either during construction of the asphalt layer or during its service life. When the pavement is subjected to sudden cooling by ambient temperatures, thermal stresses can develop in the asphalt layer. Such thermal stresses have the adverse effect of enhancing the propagation of the crack. One limitation of previous studies [4, 5] is

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the lack of allowance for phenomena such as stress relaxation in the asphalt material. Asphaltic composite materials usually exhibit temperature-dependent stress relaxation phenomena, and the inclusion of such effects will be of benefit to the accurate modeling of the thermal response of the crack. The temperature dependence of asphaltic composite materials is well documented [6–9], and the theory of linearized thermo-viscoelasticity offers a useful first approximation for the study of thermomechanical fracture and deformation processes associated with temperature-sensitive materials such as asphalt. This article focuses on the analysis of the transient thermo-viscoelastic behavior of a viscoelastic asphalt layer with a surface crack that is bonded to an elastic substrate. The transient modeling accounts for heat conduction in the entire system and the presence of a crack located normal to the free surface of the layer. The transient response in the layered structure is initiated by the sudden cooling of its surface. The constitutive modeling of the thermo-viscoelastic response is achieved through a thermorheologically simple model for the viscoelastic behavior. The finite-element modeling of the problem is achieved through a Galerkin finite-element formulation, which also accounts for the singular behavior of the state of stress at the crack tip. The numerical scheme is used to evaluate the time-dependent variation of the flaw-opening-mode stress intensity factor at the crack tip. In the numerical computations, the thermorheological response of the surface layer is modeled by appeal to typical material characteristics applicable to asphalt-type materials. Specific numerical results given in the article illustrate the influence of the thermo-viscoelastic behavior of the asphalt material in attenuating the stress intensity factors in surface flaws that are subjected to sudden temperature reductions.

GOVERNING EQUATIONS

This section presents a brief account of the fundamental equations governing the transient linear thermo-viscoelastic behavior. Detailed accounts of these developments are also given by Williams [10], Lee [11], Leitman and Fisher [12], and Christensen [13]. The state of stress in the viscoelastic medium is defined by the stress tensor $\sigma_{ij}(x_i, t)$, and the displacement field is defined by $u_i(x_i, t)$, where t is the time variable and x_i are the coordinates. In the absence of dynamic effects, the equations of equilibrium are given by

$$\sigma_{ij,j} + b_i = 0 \qquad x_i \in D \tag{1}$$

where D is the region under consideration, the subscripts indicate directions x, y, z (or 1, 2, 3), and repeated indices are summed over the range of the subscripts. The equation governing the heat conduction in the viscoelastic body that includes the heat source (or sink) term due to mechanical effects can be written as (see, e.g., [14])

$$(kT_{,i})_{,i} - \rho c\dot{T} - T_0\beta\dot{\epsilon}_{\ell\ell} + Q = 0 \qquad x_i \in D \tag{2}$$

where k is the coefficient of thermal conductivity, c is the specific heat, ρ is the mass density, Q is the heat source term other than mechanical effect, T_0 is the initial temperature distribution when the medium is unstrained, and $(\cdot)$ denotes the time derivative. The linearized strain tensor is defined by

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{3}$$

The boundary conditions for the heat conduction process are

$$T = \overline{T} \qquad \text{on } x_i \in S_1 \tag{4}$$

$$k\frac{\partial T}{\partial n} = \overline{q} \qquad \text{on } x_i \in S_2 \tag{5}$$

where $\overline{T}$ is the value of the prescribed temperature on the surface S_1, $\overline{q}$ is the rate of heat loss at the boundary S_2, and the total surface is given by $S = S_1 \cup S_2$. The displacement and traction boundary conditions are given by

$$u_i = \overline{u}_i \qquad \text{on } x_i \in S_3 \tag{6}$$

$$\sigma_{ij}n_j = \overline{P}_i \qquad \text{on } x_i \in S_4 \tag{7}$$

where $\overline{u}_i$ are prescribed displacements and $\overline{P}_i$ are prescribed tractions. For the linear viscoelastic material, the stress or strain depends on the history of deformation. The appropriate constitutive relationship that takes into account such history effects can be expressed in the form of a hereditary integral of the form

$$\sigma_{ij}(x_i, t) = R_{ijkl}(0)\{\epsilon_{kl}(x_i, t) - \alpha(0)\theta(x_i, t)\delta_{kl}\} - \int_0^t \frac{\partial}{\partial\tau}[R_{ijkl}(t-\tau)]\{\epsilon_{kl}(x_i, \tau) - \alpha(T)\theta(x_i, t)\delta_{kl}\}\, d\tau \tag{8}$$

where $\theta(t)$ is the "pseudo-temperature," which will be discussed later, and $R_{ijkl}(t)$ is the relaxation modulus at time t, given by

$$R_{ijkl}(t) = K(t)\delta_{ij}\delta_{kl} + G(t)\left\{\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right\} \tag{9}$$

where $K(t)$ and $G(t)$ are the bulk modulus and shear modulus of the material, respectively. For example, for a linear elastic material, $K(t) = K(0) = K$ and $G(t) = G(0) = G$. The fourth-order tensor $R_{ijkl}(t)$ reduces to the generalized elasticity tensor C_{ijkl} at $t \to 0$. The parameter specification for $K(t)$ and $G(t)$ will be discussed in a later section.

The initial conditions governing the transient problem can take the forms

$$T = T_0 \qquad x_i \in D \tag{10}$$

and

$$u_i = u_i(0) \qquad x_i \in D \tag{11}$$

THERMO-VISCOELASTIC CONSTITUTIVE RELATIONS

Engineering materials such as concrete, geological media, polymeric materials, asphalts, and metals at elevated temperatures exhibit time-dependent behavior over certain ranges of loading and temperature. These time-dependent phenomena can be characterized either by creep processes or by viscoelastic processes. In general, the time-dependent processes governing materials such as asphalt, concrete, and other geological media are highly nonlinear in character. The nonlinearities can be attributed to the influence of large strains and rate dependence. The theory of linear viscoelasticity can be conveniently employed to examine the time-dependent processes in asphaltic materials [3, 15–17].

An extensive account of the theory of linear viscoelasticity is given by Christensen [9]. The simplest forms of representation of viscoelastic behavior utilize basic viscoelastic elements that consist of elastic and viscous responses. These basic responses can be combined to form viscoelastic models such as those described by Kelvin and Maxwell, and multiparameter models [18]. These discrete viscoelastic representations are, however, restrictive in their general applicability. An alternative procedure employs a combination of these basic models to form a generalized viscoelastic response. In general, such a viscoelastic stress-strain relationship can be written as

$$\sum_{r=0}^{N_\sigma} p_r \frac{\partial^r(\sigma)}{\partial t^r} = \sum_{r=0}^{N_\epsilon} q_r \frac{\partial^r(\epsilon)}{\partial t^r} \tag{12}$$

where N_σ and N_ϵ are the number of operators and p_r and q_r are constant coefficients. The time derivatives of Eq. (12) can be eliminated by the application of the Laplace transform, which is defined by

$$[\bar{\sigma}(s); \bar{\epsilon}(s)] = \int_0^\infty [\sigma(t); \epsilon(t)]e^{-st}\, dt \tag{13}$$

where s is the transform parameter. Therefore Eq. (12) can be rewritten in the form

$$\left[\sum_{r=0}^{N_\sigma} p_r s^r\right]\bar{\sigma}(s) = \left[\sum_{r=0}^{N_\epsilon} q_r s^r\right]\bar{\epsilon}(s) \tag{14}$$

which is an algebraic equivalent of Eq. (12), and the transform equivalent of the stress σ is given by

$$\bar{\sigma}(s) = \sum_{r=0}^{N_\sigma} \left(\frac{c_r}{a_r s + b_r} \right) \bar{\epsilon}(s) \tag{15}$$

where a_r, b_r, and c_r are constants derived from the representation of Eq. (14) in terms of partial fractions. Since the right-hand side of Eq. (15) is expressed by a series summation of two Laplace transforms, the inverse transforms of the left-hand side of Eq. (15) can be obtained by means of a convolution integral. Consequently, the stress-strain relationship for the linear viscoelastic material can be expressed in an integral form as

$$\sigma(t) = R(0)\epsilon(t) - \int_0^t \frac{\partial R(t-\tau)}{\partial \tau} \epsilon(\tau)\, d\tau \tag{16}$$

where $R(t)$ is known as the "relaxation modulus" and takes the value of either $K(t)$ or $G(t)$ depending on the "dilatational" or "distortional" processes, respectively; i.e.,

$$R(t) = \begin{cases} K(t) & \text{for dilatational effects} \\ G(t) & \text{for distortional effects} \end{cases} \tag{17}$$

Considering the right-hand side of Eq. (15), $R(t)$ is expected to have the form of a Dirichlet series representation,

$$R(t) = \sum_{r=0}^{N_\sigma} A_r \exp(-B_r t) \tag{18}$$

where $A_r = c_r/a_r$ and $B_r = b_r/a_r$. Such a representation has two advantages related to the following:

1. In relating experimental data to theoretical models, the constants A_r and B_r can be determined by a smooth curve-fitting procedure (see, e.g., [19]).
2. The formualtion of a numerical algorithm of a step-by-step time integration accounts for the progression of viscoelastic behavior.

The discussions related to viscoleastic constitutive behavior presented earlier do not take into account the effect of temperature on the viscoelastic stress-strain relationships. Hence, the relaxation modulus $R(t)$ given in Eq. (18) is essentially an isothermal viscoelastic response at $T = T_0$.

In general, for a thermo-viscoelastic material such as asphalt concrete,

$$R_{ijkl} = R_{ijkl}(t, T) \tag{19}$$

where the constitutive tensor has to be determined from experiments. The experimental evaluation of the thermo-viscoelasticity tensor is a complicated task. The number of experiments will be inordinately high, and the transient thermomechanical effect cannot be eliminated in the experimental procedure. Fortunately, both theoretical and experimental results indicate that for a certain class of material, the *effect* *of temperature and time on the relaxation modulus can be combined.* That is, the relaxation modulus at a certain temperature T can be obtained from the corresponding relaxation modulus at a given temperature T_0; i.e.,

$$R(T, t) = R[T_0, \zeta(t)] \tag{20}$$

where $\zeta(t)$ is the "reduced time," and the class of viscoelastic materials that satisfy a relationship of the type (20) are known as "thermorheologically simple materials" (see, e.g., [20]). The reduced time $\zeta(t)$ is given by [20]

$$\zeta(t) = \int_0^t \frac{d\tau}{\psi[T(\tau)]} \tag{21}$$

where $\psi(T)$ is the temperature shift factor. The most commonly employed shift factor in thermo-viscoelastic stress analysis is the Williams, Landel, and Ferry [21] result, or the WLF equations

$$\log_{10}[\psi(T)] = \frac{-g_1(T - T_0)}{g_2 + (T - T_0)} \tag{22}$$

where g_1 and g_2 are material constants. Further, if the temperature effect on the expansion properties of the thermo-viscoelastic material is considered, an average effect of the expansion/contraction processes can be derived. That is, the "pseudo-temperature" defined previously [see Eq. (8)] is given by

$$\theta(t) = \frac{1}{\alpha(0)} \int_{T_0}^{T} \alpha(\tau)\, d\tau \tag{23}$$

Therefore, when $\alpha(T) = \alpha(0)$, $\theta(t) = T(t)$. Using these developments, the integral law for the *dilatational effect* can be expressed by

$$\sigma_{ii}(t) = 3K(0)[\epsilon_{ii}(t) - 3\alpha(0)\theta(t)] - \int_0^t \frac{\partial}{\partial\tau} [3K(\zeta - \tau)]\{\epsilon_{ii}(\tau) - 3\alpha(T)\theta(\tau)\}\, d\tau \tag{24}$$

and for the *distortional effect,*

$$\sigma_{ij}^D(t) = 2G(0)\epsilon_{ij}^D(t) - \int_0^t \frac{\partial}{\partial\tau} [2G(\zeta - \tau)]\epsilon_{ij}^D(\tau)\, d\tau \tag{25}$$

The complete hereditary integral given in Eq. (8) can be obtained by the combined effect of dilatation (24) and distortion (25) responses.

FINITE-ELEMENT MODELING

The equations of transient thermo-viscoelasticity described in the previous sections can be solved in a variety of ways. These include analytical methods based on integral transform techniques and numerical methods based on finite-difference, finite-element, and boundary integral equation techniques. In this study, the finite-element technique is used to solve the equations governing thermo-viscoelasticity problem. The finite-element technique can, conveniently, take into account the constraints imposed by the pavement geometry, the crack tip behavior, and the time- and temperature-dependent constitutive properties. In particular, attention is focused on the development of a Galerkin finite-element technique (see, e.g., [22, 23]).

The Galerkin Formulation

The Galerkin method is applied to formulate the functional expressions necessary for a finite-element discretization. First, it is convenient to adopt the representation

$$\sigma_{ij} = R_{ijkl} * d\{\epsilon_{kl} - \alpha\theta\delta_{kl}\} \tag{26}$$

to replace the integral expression (8). We assume that δT and δu_i are the weighting functions. Therefore, for the temperature equations (2), (4), and (5) we have

$$\int_D \{(kT_{,i})_{,i} - \rho c\dot{T}\}\delta T\, dD = \int_{S_2} \left(k\frac{\partial T}{\partial \eta} - \bar{q}\right)\delta T\, dS \tag{27}$$

Integrating Eq. (27) by parts once, we obtain

$$\int_D \{k\delta T_{,i}T_{,i} + \rho c\delta T\dot{T}\}\, dD - \int_{S_2} \delta T\bar{q}\, dS = 0 \tag{28}$$

For the viscoelastic problem, Eq. (1) becomes

$$\int_D (\sigma_{ij,j} + b_i)\delta u_i\, dD = \int_{S_4} (P_i - \bar{P}_i)\delta u_i\, dS \tag{29}$$

Integrating Eq. (29) by parts once, we obtain the following:

$$\int_D \sigma_{ij}\delta\epsilon_{ij}\, dD = \int_D b_i\delta u_i\, dD + \int_{S_4} \delta u_i\bar{P}_i\, dS \tag{30}$$

Substituting the constitutive relationship (26), we obtain

$$\int_D \delta\epsilon_{ij}R_{ijkl} * d\{\epsilon_{kl} - \alpha\theta\delta_{kl}\}\, dD - \int_D \delta u_i b_i\, dD - \int_{S_4} \delta u_i\bar{P}_i\, dS = 0 \tag{31}$$

Rewriting Eq. (31) in matrix form, we have

$$\int_D \{\delta\epsilon\}^T[R] * d[\{\epsilon\} - \alpha\{\theta\}]\, dD - \int_D \{\delta u\}^T\{b\}\, dD - \int_{S_4} \{\delta u\}^T\{\bar{P}\}\, dS = 0 \tag{32}$$

Equations (28) and (32) are the expressions required for finite-element discretizations.

Finite-Element Formulations

The domain D is divided into N_{el} finite elements such that

$$D = \sum_{e=1}^{N_{el}} F_e \tag{33}$$

where F_e is an element with interpolation functions defined by

$$\{u\} = [N]\{u\}^n \tag{34}$$

Also, the other variables, such as strain or temperature, can be defined in a similar manner. Substituting these representations into Eq. (28), we have

$$\sum_e \{\delta T\}^T[[K_{TT}^{(e)}]\{T\}^{(e)} + [M_T^{(e)}]\{\dot{T}\}^{(e)} - \{P_T^{(e)}\}] = 0 \tag{35}$$

where the element matrices are defined as

$$[K_{TT}^{(e)}] = \int_{F_e} k[B_T]^T[B_T]\, dD \tag{36}$$

$$[M_T^{(e)}] = \int_{F_e} \rho c[N_T]^T[N_T]\, dD \tag{37}$$

and

$$\{P_T^{(e)}\} = \int_{F_e \cap S_2} [N_T]^T\bar{q}\, dS \tag{38}$$

respectively. Summation over all elements N_{el} gives the global form of Eq. (35) as follows:

$$[K_{TT}]\{T(t)\} + [M_T]\{\dot{T}(t)\} = \{P_T(t)\} \tag{39}$$

From the discretization of the expression for the viscoelastic response (32), we have

$$\sum_e \{\delta u(t)\}^T\Bigg[[K_{uu}^{(e)}(0)]\{u(t)^{(e)}\} - \int_0^t [K_{uu}^{(e)}(\zeta - \tau)]\{u(\tau)^{(e)}\}\, d\tau$$
$$+ [K_{uT}^{(e)}(0)]\{\theta(t)^{(e)}\} - \int_0^t [K_{uT}^{(e)}(\zeta - \tau)]\{\theta(\tau)^{(e)}\}\, d\tau$$
$$- \{P_b^{(e)}\} - \{P_u\}^{(e)}\Bigg] = 0 \tag{40}$$

where the element matrices are defined as follows:

$$[K_{uu}^{(e)}(0)] = \int_{F_e} [B_u]^T[R(0)][B_u]\, dD \tag{41}$$

$$[K_{uT}^{(e)}(0)] = -\int_{F_e} [B_u]^T 3K(0)\alpha(0)[N_T]\, dD \tag{42}$$

$$[K_{uu}^{(e)}(\zeta - \tau)] = \int_{F_e} [B_u]^T \frac{\partial}{\partial \tau} [R(\zeta - \tau)][B_u]\, dD \tag{43}$$

$$[K_{uT}^{(e)}(\zeta - \tau)] = -\int_{F_e} [B_u]^T \frac{\partial}{\partial \tau} [3K(\zeta - \tau)]\alpha(\tau)[N_T]\, dD \tag{44}$$

$$\{P_b^{(e)}\} = \int_{F_e} [N_u]^T\{b\}\, dD \tag{45}$$

and

$$\{P_u^{(e)}\} = \int_{F_e \cap S_4} [N_u]^T\{P\}\, dD \tag{46}$$

respectively.

Summation over all elements gives the following:

$$[K_{uu}(0)]\{u(t)\} - \int_0^t [K_{uu}(\zeta - \tau)]\{u(\tau)\}\, d\tau + [K_{uT}(0)]\{\theta(t)\} - \int_0^t [K_{uT}(\zeta - \tau)]\{\theta(\tau)\}\, d\tau = \{P_u(t)\} \tag{47}$$

where

$$\{P_u(t)\} = \sum_e [\{P_b^{(e)}\} + \{P_u^{(e)}\}] \tag{48}$$

Therefore, Eqs. (39) and (47) provide the solution for a thermo-viscoelastic problem. The temperature equation (39) can be solved using the usual time-integration scheme. The displacement equation (47) is a system of integral equations of Volterra type. It follows that the current displacement in the viscoelastic medium has to be determined from the consideration of the complete history of the deformation and the temperature.

Solution Procedure

Following previous studies [4, 5], for the temperature unknowns, the time integration is given by

$$[[M_T] + f\,\Delta t[K_{TT}]]\{T(t + \Delta t)\} = \{P_T\} - [[M_T] + (1 - f)\,\Delta t[K_{TT}]]\{T(t)\} \tag{49}$$

where Δt is the time step and f is the scale factor ($0.5 < f < 1.0$). For the displacement field, the governing matrix equation is a system of simultaneous Volterra integral equations of the second kind. A standard numerical method for solving this class of equations is the step-forward integration procedure, where all past solutions are required in the integration with respect to time. However, the implementation of such a procedure in the finite-element scheme is virtually impossible owing to the limitations of computer storage and CPU time. In this context, the Dirchlet series representation of the relaxation moduli allows us to formulate a step-by-step time integration procedure such as the one proposed by Taylor et al. [24] and by Srinatha and Lewis [25].

For simplicity, the system of integral equations is replaced by

$$K(0)X(t) + \int_0^t \frac{\partial K(t-\tau)}{\partial \tau} X(\tau)\, d\tau = F(t) \tag{50}$$

where $X(t)$ represents the unknowns, $K(t)$ is the stiffness coefficient, and $F(t)$ is the loading term. If the relaxation modulus discussed earlier can be represented by a Dirichlet series, the function $K(t - \tau)$ can be written as

$$K(t-\tau) = \sum_j A_j \exp(-B_j t) \exp(B_j \tau) \tag{51}$$

Substituting Eq. (51) into Eq. (50), we have, at $(t + \Delta t)$,

$$K(0)X(t+\Delta t) + \sum_j A_j \exp[-B_j(t+\Delta t)] \int_0^{t+\Delta t} \frac{\partial \exp(B_j\tau)}{\partial \tau} X(\tau)\, d\tau$$

$$= F(t+\Delta t) \tag{52}$$

Considering the integral in Eq. (52), we can write

$$\int_t^{t+\Delta t} \frac{\partial \exp(B_j\tau)}{\partial \tau} X(\tau)\, d\tau \simeq \frac{1}{2}[X(t+\Delta t) + X(t)]\{\exp[B_j(t+\Delta t)] - \exp(B_j t)\} \tag{53}$$

Substituting Eq. (53) into Eq. (52), we obtain

$$\left[K(0) + \frac{1}{2}\sum_j A_j \exp[-B_j(t+\Delta t)]\{\exp[B_j(t+\Delta t)] - \exp(B_j t)\}\right] X(t+\Delta t)$$

$$= F(t+\Delta t) + F^{me}(t)$$

$$+ \frac{1}{2}\sum_j A_j \exp[-B_j(t+\Delta t)]\{\exp[B_j(t+\Delta t)] - \exp(B_j t)\}X(t) \tag{54}$$

where $F^{me}(t)$ is the memory loading up to time t, which is given by

$$F^{me}(t) = -\sum_j A_j \exp\,[-B_j(t + \Delta t)] \int_0^t \frac{\partial \exp\,(B_j\tau)}{\partial\tau} X(\tau)\, d\tau \tag{55}$$

By the series expression in Eq. (51), Eq. (54) can be simplified to the form:

$$[K(0) + K^*(\Delta t)]X(t + \Delta t) = F(t + \Delta t) + F^{me}(t) + K^*(\Delta t)X(t) \tag{56}$$

which is a time integration expression with a memory load. Following the similar development from Eq. (51) to Eq. (56), we can rewrite the system of integral equations (47) into a recurrence relationship of the form:

$$\begin{aligned}
&[K_{uu}(0) + K^*_{uu}(\Delta\zeta)]\{u(t + \Delta t)\} \\
&\quad = \{P_u(t + \Delta t)\} + \{P_u^{me}(t)\} + \{P_T^{me}(t)\} \\
&\qquad + [K^*_{uu}(\Delta\zeta)]\{u(t)\} + [K^*_{uT}(\Delta\zeta)]\{\theta(t)\} \\
&\qquad - [K_{uT}(0) + K^*_{uT}(\Delta\zeta)]\{\theta(t + \Delta t)\}
\end{aligned} \tag{57}$$

where $\{P_u^{me}(t)\}$ and $\{P_T^{me}(t)\}$ are the memory loads due to, respectively, the deformation and the temperature up to time t, and they are obtained from the expression given in (55).

Now, Eqs. (49) and (57) are the time-stepping equations for both the temperature and the displacement. The solution procedure can be started by making use of the initial conditions (10) and (11).

Crack-Tip Elements

In the available literature on viscoelastic fracture mechanics analysis (see, e.g., [26–28]), the stress distribution near the tip of a stable crack is assumed to be of $(1/\sqrt{r})$ type. Therefore, the quarter-point element of Henshell and Shaw [29] and Barsoum [30] is applicable in the present modeling. In the previous study [5], the stress intensity factor can be evaluated by the nodal displacements along rays emanating from the crack tip (Fig. 1). For completeness, the expressions are given below as

$$K_I \begin{Bmatrix} (2\gamma - 1)\cos\dfrac{\bar\theta}{2} - \cos\dfrac{3\bar\theta}{2} \\ (2\gamma + 1)\sin\dfrac{\bar\theta}{2} - \sin\dfrac{3\bar\theta}{2} \end{Bmatrix} = \frac{4G(t)}{\sqrt{\ell_i/2\pi}} \begin{Bmatrix} 4u_{x2} - u_{x3} - 3u_{x1} \\ 4u_{y2} - u_{y3} - 3u_{y1} \end{Bmatrix} \tag{58}$$

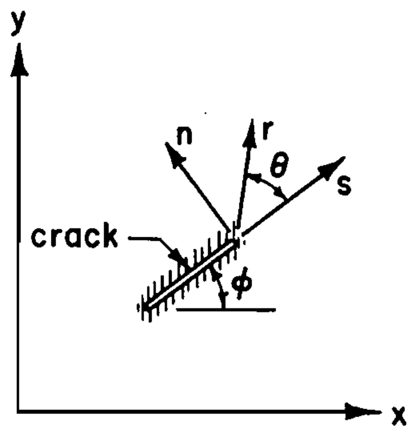

Fig. 1 Geometry and coordinate system at the crack tip.

and

$$K_{II}\left\{\begin{matrix}(-2\gamma+3)\sin\frac{\tilde{\theta}}{2}-\sin\frac{3\tilde{\theta}}{2}\\(2\gamma-3)\cos\frac{\tilde{\theta}}{2}+\cos\frac{3\tilde{\theta}}{2}\end{matrix}\right\}=\frac{4G(t)}{\sqrt{\ell_i/2\pi}}\left\{\begin{matrix}4u_{x2}-u_{x3}-3u_{x1}\\4u_{y2}-u_{y3}-3u_{y1}\end{matrix}\right\} \tag{59}$$

with $\ell_i = \ell_1$ or ℓ_2 defined in Fig. 2. Also, $\tilde{\theta}$ is considered to be either 0° or 180°. In Eqs. (58) and (59), γ is given by

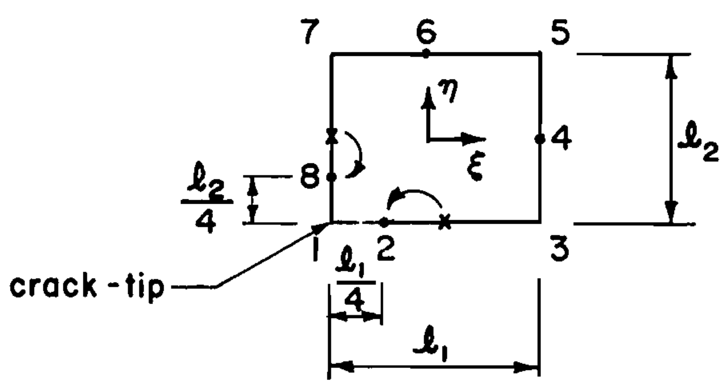

Fig. 2 Quarter-point element.

$$\gamma = \begin{cases} \dfrac{3K(t) + 7G(t)}{3K(t) + G(t)} & \text{(plane strain)} \\ \dfrac{15K(t) + 8G(t)}{9K(t)} & \text{(plane stress)} \end{cases} \tag{60}$$

where $K(t)$ and $G(t)$ are the relaxation moduli given by Eq. (17).

SURFACE CRACK IN A VISCOELASTIC MEDIUM

The numerical procedure developed in the preceding section is first applied to examine the thermo-viscoelastic behavior of a homogeneous viscoelastic layer with a surface crack (Fig. 3). The attention is restricted to the examination of two-dimensional plane-strain problem in which the surface of the viscoelastic layer is subjected to a sudden decrease in the temperature, in the form of a step function of time. The results of a previous investigation [5] have shown that the rapid reduction in the surface temperature of an elastic asphalt layer results in the development of the maximum stress intensity factor at the crack tip. The geometry of the region in relation to the surface crack is shown in Fig. 3, and the elastic layer is restrained from

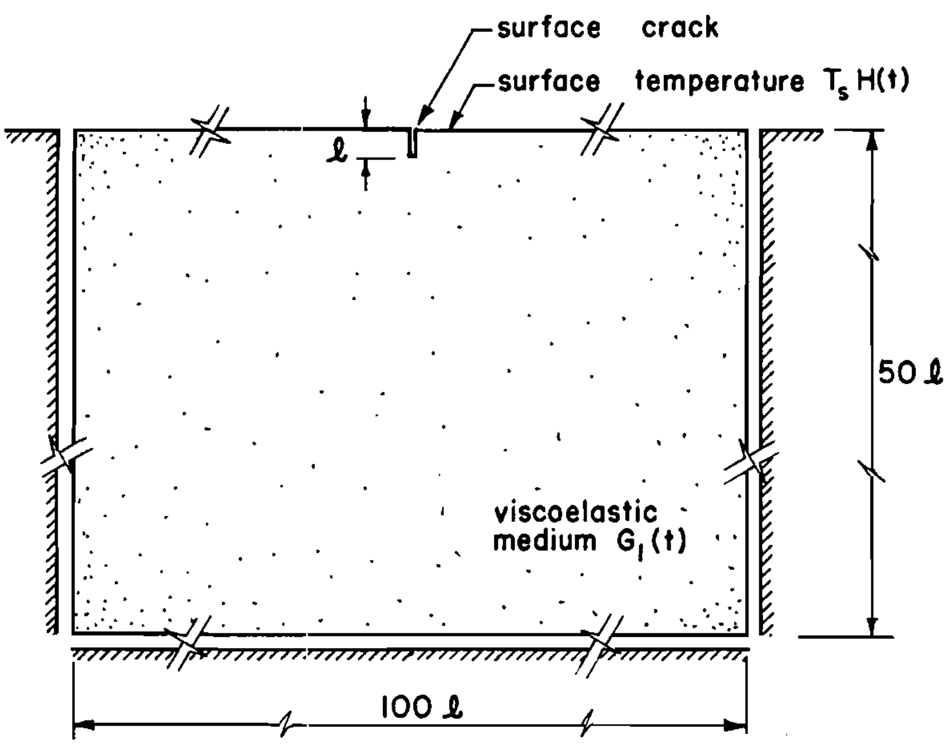

Fig. 3 Surface crack in a viscoelastic medium.

horizontal movement at the extreme boundaries. These are frictionless restraints. The surface temperatures are prescribed by the following conditions:

$$T(x, y) = T_0 \qquad \text{for } t = 0$$

$$T(x, 0) = T_s H(t) \qquad \text{for } t > 0 \tag{61}$$

The finite-element discretization adopted for the numerical analysis is shown in Fig. 4. The result of primary interest concerns the time-dependent stress intensity factor at the crack. In order to perform the numerical analysis, it is necessary to define the thermo-viscoelastic parameters associated with the constitutive modeling. The basic groups of materials parameters relate to:

1. Heat conduction within the layered structure [governed by the temperature-dependent values of the coefficient of volumetric expansion $\alpha(T)$, the thermal conductivity $k(T)$, and the heat capacity $\rho c(T)$]
2. Viscoelastic behavior [governed by the time-dependent relaxation moduli associated with dilatational behavior, $K_1(t)$, and distortional behavior, $G_1(t)$, of the viscoelastic medium]

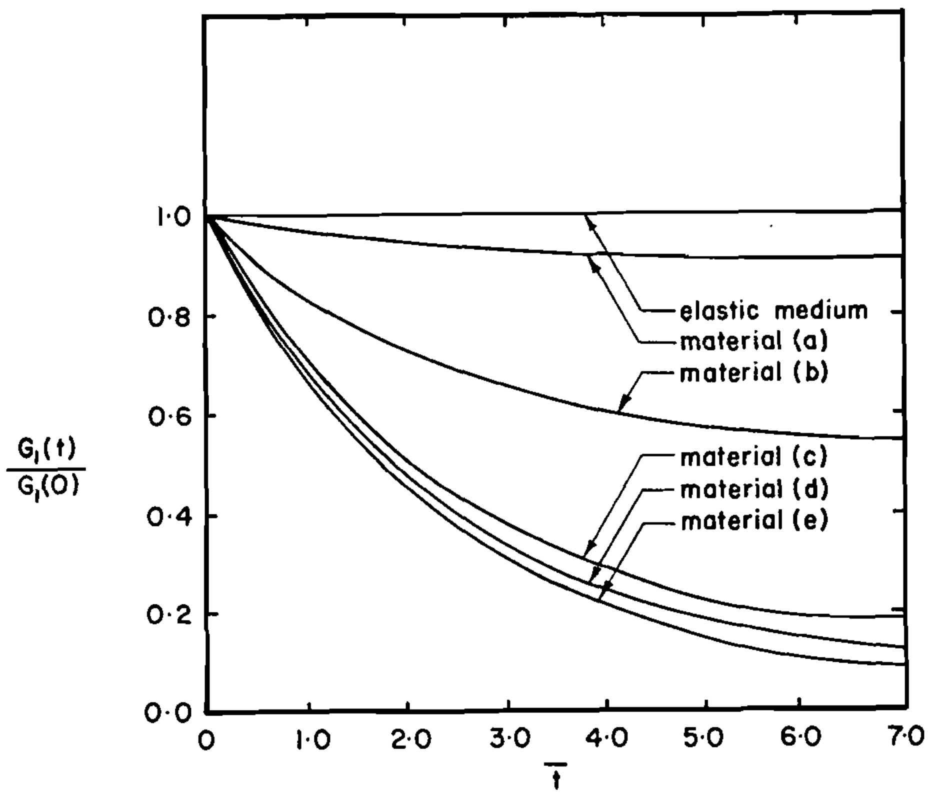

Fig. 4 Relaxation functions for the viscoelastic shear modulus $G_1(t)$. Material types (a)–(e) are given in Table 1.

Table 1 Viscoelastic material type (a) to (e)

Material type	$G(t)/G(0)$	$K(t)/K(0)$
(a)	$0.9 + 0.1 \exp(-x)$	1.0
(b)	$0.5 + 0.5 \exp(-x)$	1.0
(c)	$0.1 + 0.9 \exp(-x)$	1.0
(d)	$0.05 + 0.95 \exp(-x)$	1.0
(e)	$0.01 + 0.99 \exp(-x)$	1.0

Here $x = 8t/\ell^2(\rho c/k)$; $G(0) = 6 \times 10^5$ kN/m^2, $K(0) = 4 \times 10^5$ kN/m^2.

For the purpose of the numerical computations we shall assume that the thermal properties $\alpha(T)$, $k(T)$, and $\rho c(T)$ are relatively uninfluenced by the temperature and are assumed to be constant. We shall further assume that the dominant mode of viscoelastic behavior relates to the shear behavior of the material [defined by $G_1(t)$] and that the dilatation or volumetric behavior [defined by $K_1(t)$] of the material is

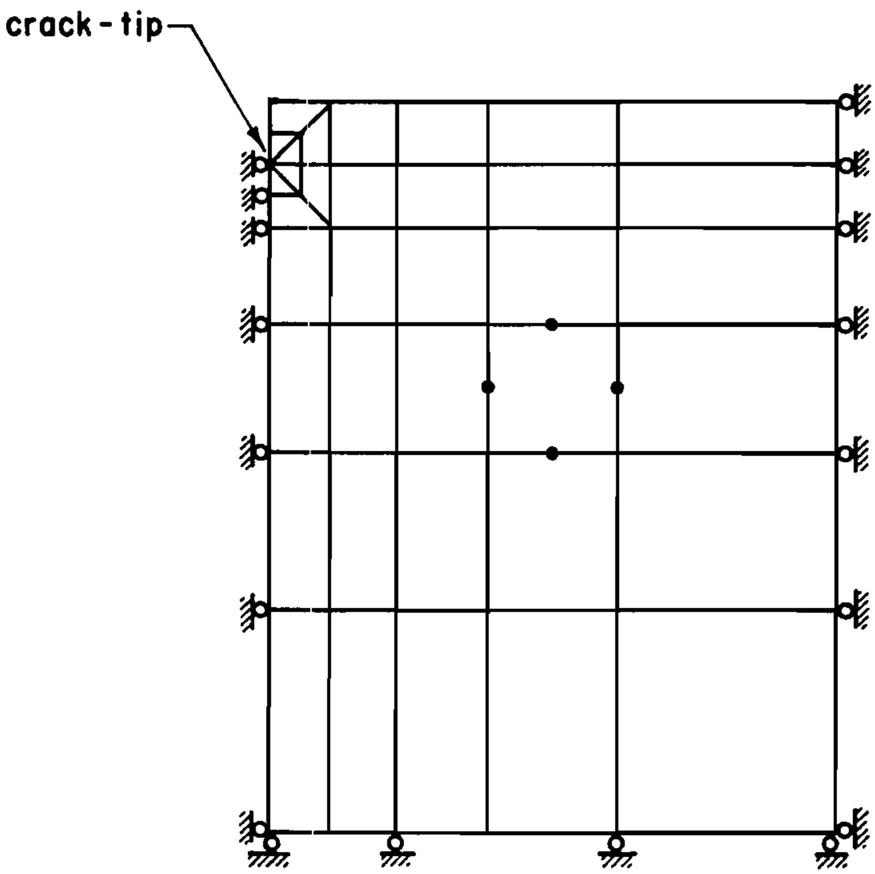

Fig. 5 The finite element discretization of the vicoelastic medium with surface crack.

nonrelaxing and constant in time. For the linear viscoelastic shear behavior we consider five types of typical materials where the shear moduli $\epsilon_1(t)$ have decaying variations with time. The specific expressions for $G_1(t)$ are shown in Table 1, and Fig. 4 shows the variations of $G_1(t)/G_1(0)$ with time. The surface of the layer is subjected to the rapid reduction in the surface temperature T_s such that the thermal jump is given by $T = |T_s - T_0|$. The effect of temperature on $G_1(t)$ is accounted for by the shift function $\psi(t)$ of the $A \exp[-(T_s - T)/T_s]$ type. The values of $\alpha(T_0)$, $k(T_0)$, and $\rho c(T_0)$ are those used in the previous investigation [5]. For completeness, we note that

$$\alpha(T_0) = \alpha_0 = 5 \times 10^{-6}\,{}^{\circ}\mathrm{C}^{-1}$$

$$k(T_0) = k_0 = 2.0\ \mathrm{J/m\ s\ {}^{\circ}C}$$

$$\rho c(T_0) = (\rho c)_0 = 4 \times 10^6\ \mathrm{J/m^3\ {}^{\circ}C} \tag{62}$$

The finite-element discretization employed in the numerical analysis of the viscoelastic region with a surface crack is shown in Fig. 5. The formulation utilizes eight-noded isoparametric elements to model the domain, and the quarter-point element is used to model the crack-tip singular behavior. The numerical results for the time-dependent flaw opening or mode I stress intensity factor at the crack tip are shown in Figs. 6–9. In these figures, the notations (a), (b), (c), . . ., (e) refer to the viscoelastic material types referred to in Table 1. Also, the results are presented in

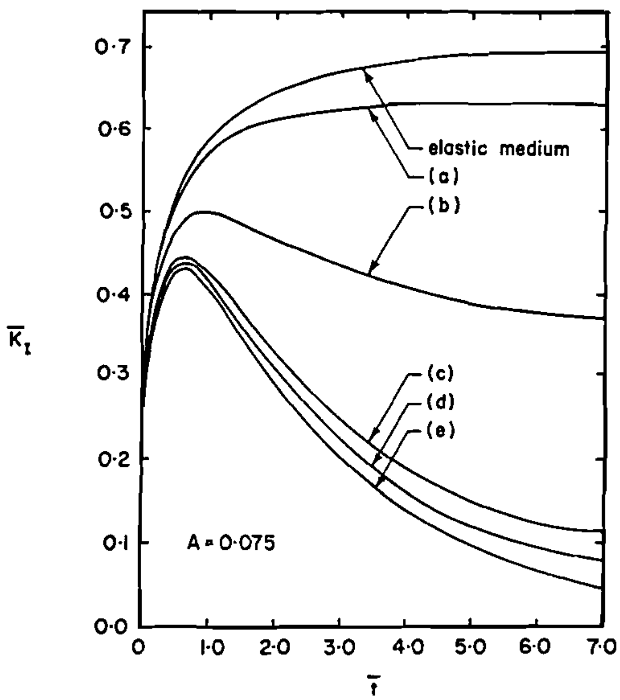

Fig. 6 Influence of thermo-viscoelastic properties on the stress intensity factor at a surface crack. $\bar{K}_I = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

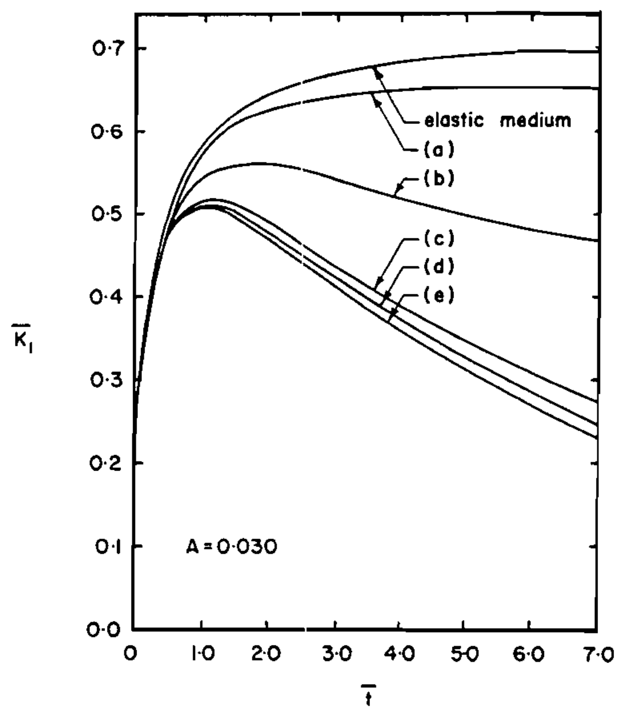

Fig. 7 Influence of thermo-viscoelastic properties on the stress intensity factor at a surface crack. $\bar{K}_I = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

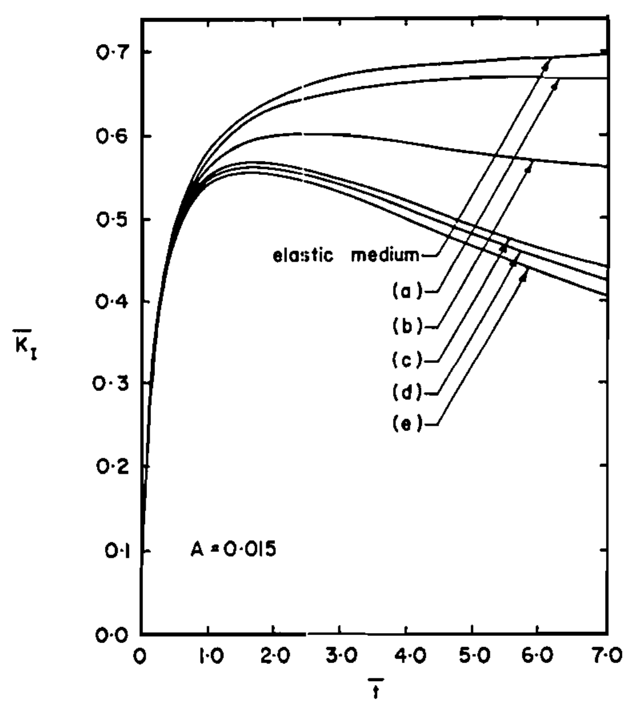

Fig. 8 Influence of thermo-viscoelastic properties on the stress intensity factor at a surface crack. $\bar{K}_I = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

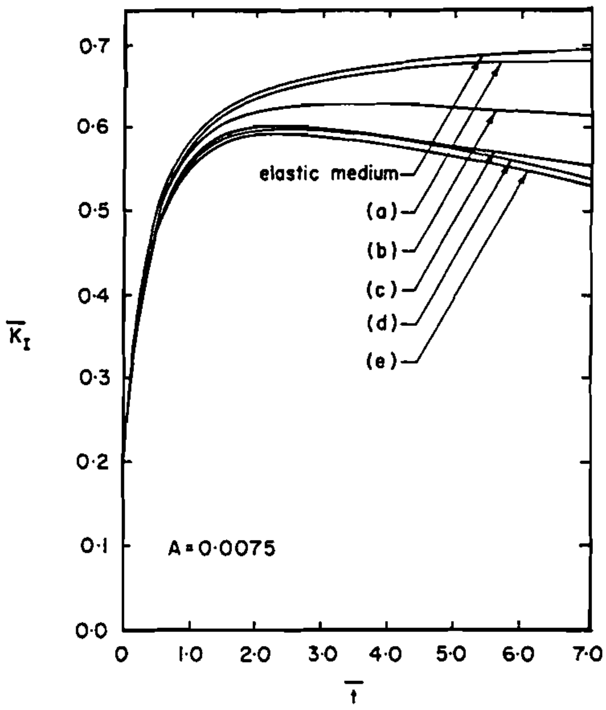

Fig. 9 Influence of thermo-viscoelastic properties on the stress intensity factor at a surface crack. $\bar{K}_I = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

the form of nondimensional quantities pertaining to the stress intensity factor $\bar{K}_I$ and time $\bar{t}$, where

$$\bar{K}_I = \frac{K_I\sqrt{2\ell}}{G_1(0)\alpha_0 T_s} \tag{63}$$

$$\bar{t} = \frac{tk_0}{\ell^2(\rho c)_0} \tag{64}$$

The results given in Figs. 6–9 also account for the variations in the $G_1(t)$ due to temperature effects, as indicated by the variations in the shift function

$$\psi(T) = A\exp\left[-\frac{(T_s - T)}{T_s}\right] \qquad A \in 10^{-3}(7.5, 75.0) \tag{65}$$

As the nondimensional parameter A becomes small, the temperature dependence of $G_1(t)$ becomes less significant. The results of the computations indicate that both the relaxation phenomena and temperature dependence on $G_1(t)$ results in the reduction of the stress intensity factor at the crack tip.

CRACK IN A VISCOELASTIC PAVEMENT LAYER

We now apply the numerical technique to examine the thermo-viscoelastic behavior of a crack located in an asphalt layer that rests on an elastic subgrade. The geometry of the problem under consideration is shown in Fig. 10. The pavement structure is laterally constrained at a finite distance from the surface crack 50ℓ, where ℓ is the depth of the surface crack. The interface between the elastic subgrade and the viscoelastic layer is assumed to be perfectly bonded. For purposes of the example, the thermal properties of both the asphalt layer and the elastic subgrade are assumed to be time and temperature independent. It is also assumed that the thermo-viscoelastic behavior of the asphalt layer is characterized by a dilatation behavior that is nonrelaxing. The shear relaxation behavior is identical to that assumed previously. In addition to these, two other nondimensional parameters are important to the discussion of the thermo-viscoelastic behavior of the composite pavement; these relate to the pavement-subgrade relative shear modular ratio, defined by $G_1(t)/G_2$, where G_2 is the shear modulus of the elastic subgrade; and the layer thickness-crack length aspect ratio, h/l. Again, for purposes of illustration we assign the following specific values for these parameters: $h/l = 2$ and $G_1(0)/G_2 = 0.1$, 0.01. The finite-element discretization of the asphalt layer–subgrade system follows an arrangement similar to that adopted in Fig. 5. Figures 11–16 illustrate the manner in which the normalized stress intensity factor $\bar{K}_I^*$ at the crack tip located in the viscoleastic layer is influenced by the relaxation phenomena in the viscoelastic layer, the temperature shift $\psi(T)$, and the modulus mismatch between the elastic subgrade and the layer.

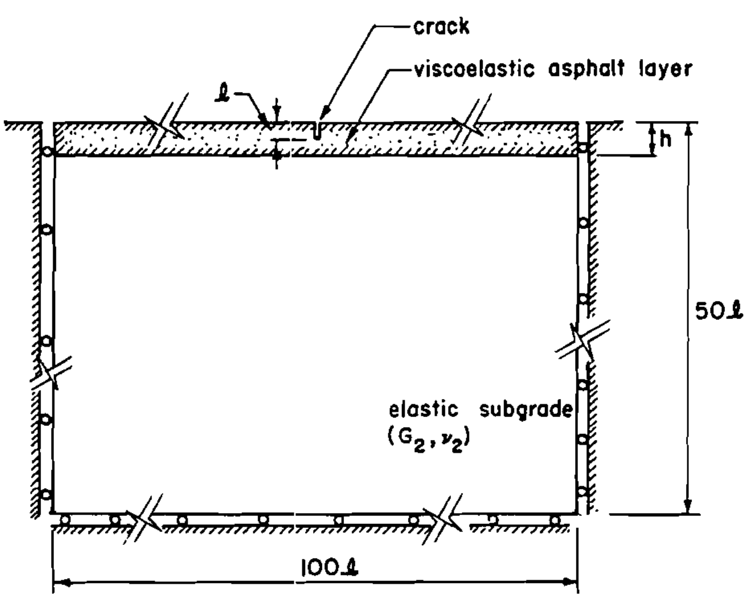

Fig. 10 Surface crack in a viscoelastic layer resting on an elastic medium.

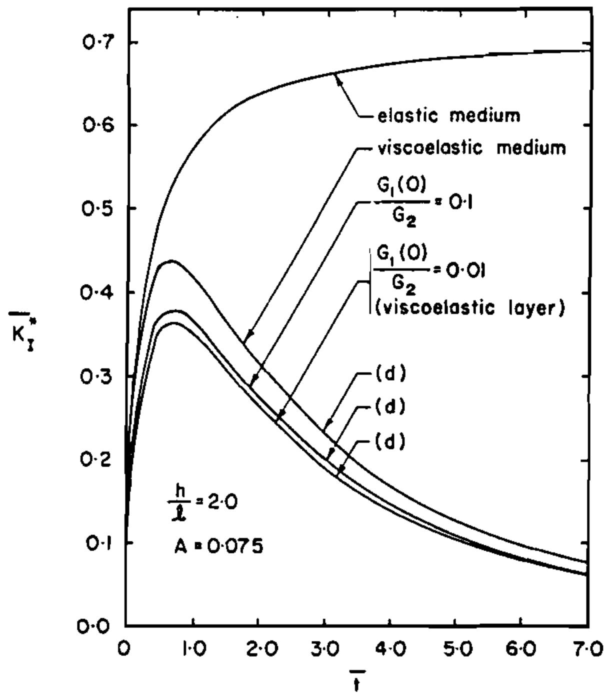

Fig. 11 Influence of thermo-viscoelastic properties and modulus mismatch on the stress intensity factor at a surface crack in a viscoelastic layer. $\overline{K}_I^* = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

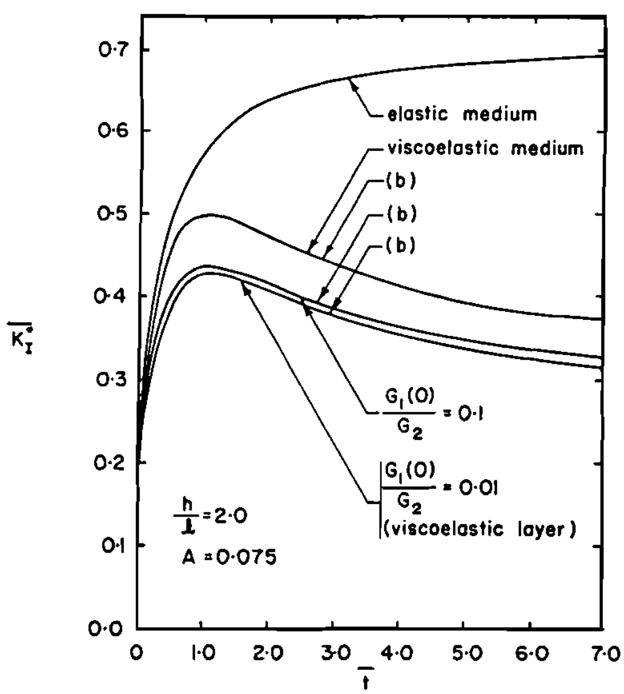

Fig. 12 Influence of thermo-viscoelastic properties and modulus mismatch on the stress intensity factor at a surface crack in a viscoelastic layer. $\overline{K}_I^* = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

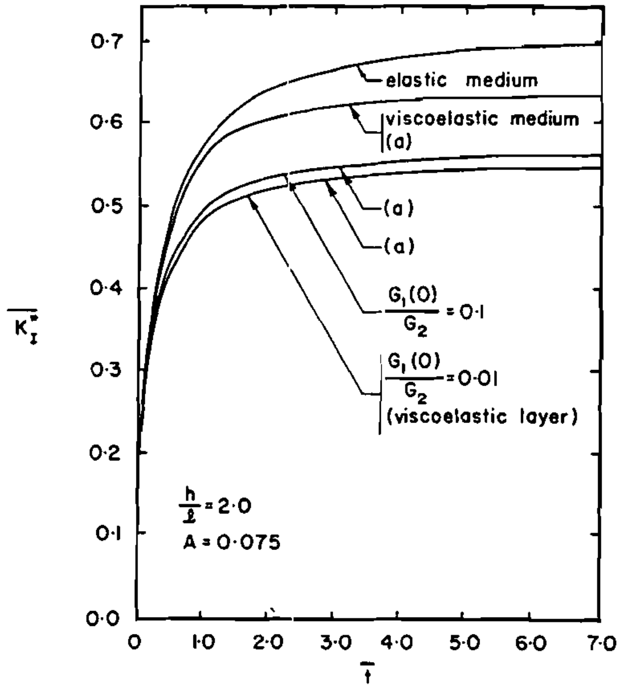

Fig. 13 Influence of thermo-viscoelastic properties and modulus mismatch on the stress intensity factor at a surface crack in a viscoelastic layer. $\overline{K}_I^* = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

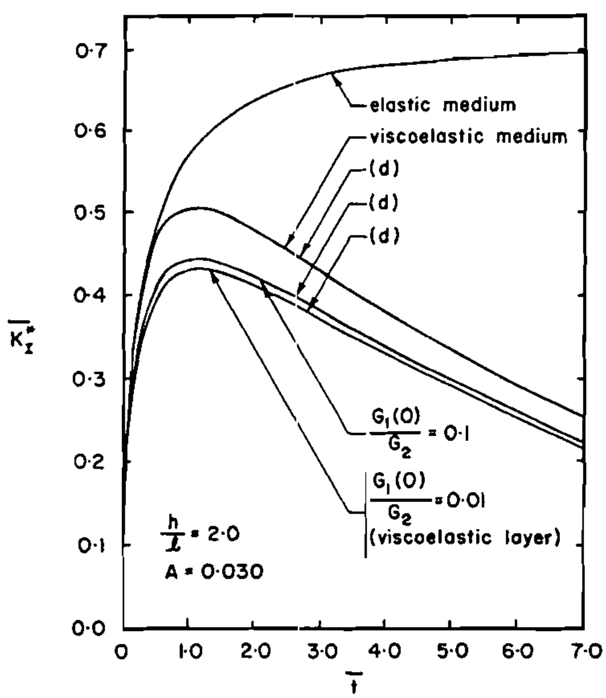

Fig. 14 Influence of thermo-viscoelastic properties and modulus mismatch on the stress intensity factor at a surface crack in a viscoelastic layer. $\overline{K}_I^* = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

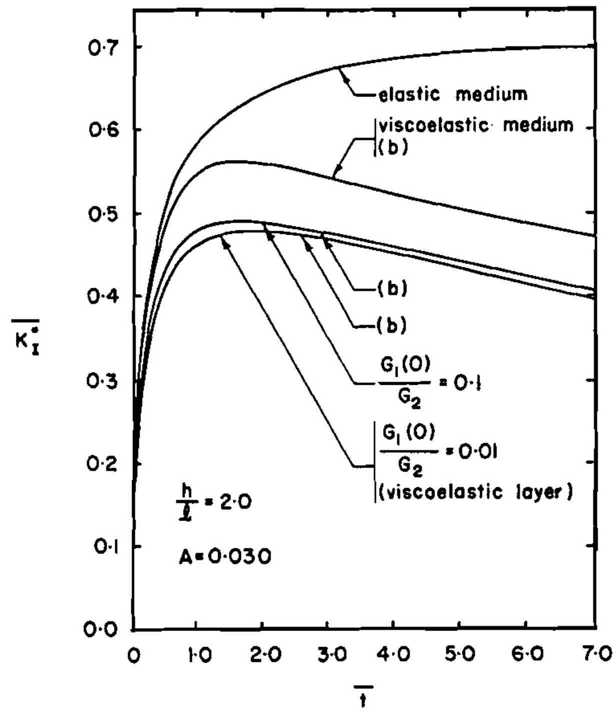

Fig. 15 Influence of thermo-viscoelastic properties and modulus mismatch on the stress intensity factor at a surface crack in a viscoelastic layer. $\overline{K_I^*} = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

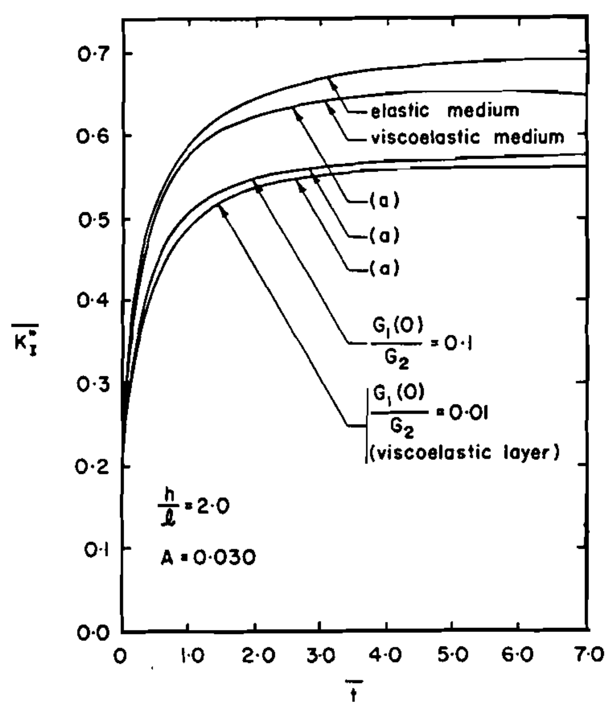

Fig. 16 Influence of thermo-viscoelastic properties and modulus mismatch on the stress intensity factor at a surface crack in a viscoelastic layer. $\overline{K_I^*} = K_I\sqrt{2\ell}/G_1(0)\alpha_0 T_S$; $\bar{t} = tk_0/\ell^2(\rho c)_0$.

The normalized stress intensity factor $\bar{K}_I^*$ is related to the stress intensity factor K_I in the viscoelastic layer according to the following:

$$\bar{K}_I^* = \frac{K_I\sqrt{2\ell}}{G_1(0)\alpha_0 T_s}$$

The equivalent results for the stress intensity factors derived for the surface crack located in viscoelastic medium and the surface crack in an elastic layer underlain by an elastic subgrade are also provided for purposes of comparison.

CONCLUSIONS

This article presents a theoretical model and the associated numerical developments for the stress analysis of layered structures that exhibit thermo-viscoelastic behavior. Particular attention is focused on the examination of the influence of thermo-viscoelastic behavior of the asphalt on the behavior of surface cracks located in a pavement structure. The basic conclusions can be summarized as follows.

1. Asphalt concrete materials possess complex time- and temperature-dependent stress-strain phenomena. The theory of linear viscoelasticity serves as a useful first approximation in the study of time-dependent and temperature-dependent mechanical behavior of asphalt materials. The assumption of linear viscoelastic behavior enables the description of the viscoelastic constitutive relations in terms of relaxation functions applicable to dilatational and distortional behavior of the material.
2. Many viscoelastic materials that exhibit temperature dependence can be conveniently described as thermorheologically simple materials. The basic postulate of thermorheologically simple behavior is that the viscoelastic relaxation functions at a series of different temperatures when plotted against the logarithm of time can be superimposed to form a single curve simply by shifting the various curves at different temperatures along the time axis. By invoking this assumption, the temperature dependence can be described by a "shift factor" that accounts for the reduction in the asphalt stiffness with increases in temperatures or vice versa.
3. Numerical methods of stress analysis can be employed to examine transient phenomena associated with thermo-viscoelastic behavior of pavement structures containing defects. In this study the finite-element technique is used to examine the time- and temperature-dependent behavior of the stress intensity factor at the tip of a surface crack located either in a viscoelastic medium or a viscoelastic layer underlain by an elastic subgrade. The magnitude of the stress intensity factor is an indicator of the tendency for the crack to further propagate due to low-temperature-induced stresses. The particular thermal loading corresponds to a sudden reduction in the surface temperature of the medium or the layer containing the crack.

4. The numerical results obtained for the time-dependent variation in the stress intensity factor at the surface crack located in a viscoelastic medium indicate that, in general, the viscoelastic behavior has the tendency of reducing the magnitude of the stress intensity factor at the crack tip. This reduction is greatly influenced by the form of the relaxation function governing, in this case, the shear behavior of the viscoelastic medium and the shift factor. Similar conclusions apply to the results for the time-dependent variation in the stress intensity factor at the crack tip located in a viscoelastic layer underlain by a subgrade. Additional variables in this example include the crack length-to-layer thickness ratio and the modulus mismatch between the viscoelastic layer and the underlying elastic medium. The results presented in the report indicate only a small degree of sensitivity of the stress intensity factor to the modulus mismatch between the viscoelastic layer and the underlying elastic medium.
5. The computational methodologies developed in connection with this study can be used to study the processes associated with both the initiation and propagation of surface cracks in viscoelastic asphalt layers that are subjected to low-temperature phenomena. In such situations the low-temperature-induced stresses are assumed to initiate and propagate cracks in a quasi-static manner. In order to apply these computational methodologies for the prediction of the occurrence of low-temperature-induced surface cracking, it is necessary to determine additional strength and fracture toughness criteria. The strength criteria can be deduced from uniaxial tension tests conducted at a range of low temperatures. The fracture toughness of the asphalt material can similarly be determined from mode I fracture tests conducted at a range of low temperature.

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