Medium-Term Urban Water Demand Forecasting with Limited Data Using an Ensemble Wavelet–Bootstrap Machine-Learning Approach

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Abstract: Accurate and reliable weekly and monthly water demand forecasting is important for effective and sustainable planning and use of urban water supply infrastructure. This study explored a hybrid wavelet–bootstrap–artificial neural network (WBANN) modeling approach for weekly (one-week) and monthly (one- and two-month) urban water demand forecasting in situations with limited data availability. The performance of WBANN models was also compared with that of standard artificial neural networks (ANN), bootstrap-based ANN (BANN), and wavelet-based ANN (WANN) models. The proposed WBANN method is aimed at improving the accuracy and reliability of water demand forecasting by incorporating the capability of wavelet transformation and bootstrap analysis using artificial neural networks. Daily and monthly maximum temperature, total precipitation, and water demand data for almost three years obtained from the city of Calgary, Alberta, Canada were used in this study. For weekly and monthly lead-time forecasting, the hybrid WBANN and WANN models were determined to be more accurate compared with the ANN and BANN methods. The WANN and WBANN models simulated peak water demand very effectively. The better performance of the WANN and WBANN models for weekly and monthly water demand forecasts indicated that wavelet analysis significantly improved the model's performance, whereas the bootstrap technique improved the reliability of water demand forecasts by producing ensemble forecasts. WBANN models were also found to be effective in assessing the uncertainty associated with water demand forecasts in terms of confidence bands, which is helpful in operational water demand forecasting. This study was conducted with a very short length of available data, indicating the effectiveness of WANN and WBANN modeling approaches in situations with limited data availability. **DOI: 10.1061/(ASCE)WR.1943-5452.0000454.**

Author keywords: Artificial neural networks; Resampling; Ensemble; Water demand; Forecasting; Wavelets; Canada.

Introduction

Rapid population and industrial growth have increased water consumption in many cities around the world. The efficient operation and management of an urban water supply system requires accurate water demand forecasts, and the estimation of future urban water demand is critical to the sustainable planning of regional water supply systems (Zhou et al. 2002). The resulting issues of water stress and scarcity are complex, and the tightly coupled interactions of human and natural systems at multiple scales pose many urban water demand management challenges (House-Peters and Chang 2011). Long-range urban water demand (i.e., greater than a monthly time step) forecasting helps in the planning and design of water supply systems, whereas short-term water demand forecasting helps in the operation and management of water supply systems. Short-term urban water demand forecasts (i.e., less than a week time step) allow for optimal pump, well, reservoir, and mains operations, balanced allocation among urgent water needs, and the development of short-term demand management strategies (Kame'enui 2003; Herrera et al. 2010).

Medium-term (weekly and monthly) urban water demand forecasts help water managers make more informed water management decisions when balancing the needs of water supply, residential/industrial demands, and stream flows for fish and other habitats. Accurate forecasts also aid with decision making, such as when to implement regulatory water use restrictions in times of water stress or drought (Jain and Ormsbee 2002; Kame'enui 2003; Ghiassi et al. 2008; Herrera et al. 2010), or when to start drawing from auxiliary supplies (Jain and Ormsbee 2002). Urban water is generally supplied on the basis of the experience of operators, although accurate and reliable forecasts of water demand helps operators provide water more sustainably (Zhou et al. 2002). Temperature, precipitation, and past water demand are usually the most significant input variables that affect urban water demand forecasting (Jain et al. 2001; Bougadis et al. 2005; Adamowski et al. 2012).

This study focused on medium-term urban water demand forecasts for one week and one-to two-month lead times. Traditional or conventional urban water demand forecasting at these lead times used linear regression, trend-extrapolation, and time-series techniques. Maidment and Parzen (1984) investigated the combination of regression and time series analysis for forecasting monthly water demand in six cities in Texas. Maidment et al. (1985) used multivariate time series analysis for daily urban water demand forecasting, and Franklin and Maidment (1986) used the cascade modeling approach to forecast weekly water demand data. Jain et al. (2001) used artificial neural networks (ANNs) for short-term water demand forecasts at IIT Kanpur using two meteorological variables (rainfall and maximum air temperature), in addition to past water

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Note. This manuscript was submitted on September 11, 2013; approved on March 10, 2014; published online on July 10, 2014. Discussion period open until December 10, 2014; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Water Resources Planning and Management***, © ASCE, ISSN 0733-9496/04014053(12)/$25.00.

demand. Jain and Kumar (2007) and Bougadis et al. (2005) applied ANN for monthly and weekly lead-time water demand forecasting, respectively, and found that the ANN models performed better compared with simpler/more traditional models. Simpler models such as regression models have been used extensively in urban water demand forecasting. Several studies compared the performance of linear regression models with newly developed models such as ANN to benchmark performance (e.g., Adamowski 2008; Caiado 2010; Adamowski et al. 2012; Tiwari and Adamowski 2013); generally, these studies found that ANN models outperform the simpler/more traditional models.

More recently, Adamowski and Karapataki (2010) found that ANN models performed better than multiple linear regression models for peak weekly water demand forecasts. Nasseri et al. (2011) developed a hybrid model combining the extended Kalman filter with genetic programming for monthly water demand forecasting in Tehran. Campisi-Pinto et al. (2012) carried out municipal water demand forecasting for one- to six-month lead times using ANNs coupled with wavelet denoising, and demonstrated the significance of data preprocessing using the selection of an appropriate wavelet transform and decomposition levels. Mohammed and Ibrahim (2012) developed wavelet-based ANN (WANN) hybrid models for daily and monthly municipal water demand forecasting in the city of Tampa in the United States, and found accurate results for both daily and monthly lead-time forecasts. Adamowski et al. (2013) investigated the use of the continuous wavelet transform to determine changes in the temporal pattern of urban water demand and its potential meteorological drivers, and found that an inverse relationship exists between urban water demand and precipitation during the summer months in areas with low precipitation (e.g., Calgary, Canada). However, one major drawback of ANNs is their inability to address data nonstationarity. Wavelet analysis was found to be a potentially useful method for detecting nonstationarity in urban water demand (Adamowski et al. 2013). To address this issue, WANN models have been explored during the past few years. To date, most studies found WANNs to be more accurate than multiple linear regression, time-series, and regular ANN models, such as in regional drought forecasting (Kim and Valdes 2003), rainfall–runoff forecasting and streamflow forecasting (Kamruzzaman et al. 2013; Nayak et al. 2013; Tiwari et al. 2013), monthly groundwater level forecasting (Adamowski and Chan 2011), and short-term and long-term urban water demand forecasting (Adamowski et al. 2012; Odan and Reis 2012; Huang et al. 2013). However, further development of the WANN method in urban water demand forecasting applications is still needed.

Although ANNs, including WANNs, were found to be more accurate than traditional models, such as multiple linear regression (MLR) and autoregressive integrated moving average (ARIMA) for water demand forecasting, ensemble forecasting is an emerging approach that has not been explored in any detail in the urban water demand forecasting literature. Ensemble forecasts help with decision making by providing a level of uncertainty associated with a particular urban water demand forecast (Donkor et al. 2014). A hybrid approach that was very recently proposed in the hydrological forecasting literature to provide ensemble probabilistic forecasts is the bootstrap-based ANN (BANN) method (Tiwari and Chatterjee 2010a). The bootstrapping method allows for a reduction in uncertainty by mimicry of randomness, thus reducing the uncertainty in the variance (Efron 1979). Generally, the BANN method has provided better results than regular ANN approaches when used in hydrological forecasting applications, such as in long term runoff (Sharma and Tiwari 2009), flow (Tiwari and Chatterjee 2010a), and flood forecasts (Han et al. 2007; Tiwari and Chatterjee 2010a).

Both of these described approaches (i.e., wavelet transforms and bootstrapping) may be combined to form a wavelet–bootstrap-ANN (WBANN) model that has the potential to increase accuracy and reliability in urban water demand forecasting. Although the WBANN method has not yet been applied in urban water demand forecasting, preliminary studies by one of the authors of this paper found the method to be highly accurate and reliable for daily river discharge (Tiwari and Chatterjee 2011) and hourly flood forecasts (Tiwari and Chatterjee 2010b). Therefore, the purpose of this study was to explore the use of the new WBANN method for medium-term urban water demand forecasting. First, a robust WBANN model was developed for one week and one- to two-month lead times, and the performance of the WBANN model was then compared with the bootstrap based ANN (BANN) and WANN methods to assess their effectiveness.

Methodology

A brief theoretical background is provided for ANNs, wavelet analysis, and the bootstrap method.

Artificial Neural Networks

ANNs are computational methods designed to mimic the functioning of the human brain when performing a particular task of interest. They observe complex relationships between input and output data by sorting patterns and trial and error methods using a set of interconnected simple processing nodes or neurons. These computational nodes are organized into a layer and with links between the nodes designated as weighted synaptic connections. These nodes and synaptic weights use multiple simple functions that are combined to construct the relationship between the input and output dataset. The multilayer feed-forward neural network that is also called a multilayer perceptron (MLP) consists of an input layer, one or more hidden layers of computation nodes, and an output layer of computation nodes. The ANN approach has been found to be very rapid and robust even in noisy environments, and has been applied to solve a wide variety of real world problems such as, for example, time series predictions (Abrahart et al. 2012; Nayak et al. 2013). An output node of a MLP-ANN is calculated as (Jeong and Kim 2005)

$$O_k = g_2\left[\sum_j V_j w_{ij} g_1\left(\sum_i w_{ji} I_i + w_{j0}\right) + w_{k0}\right] \tag{1}$$

where I_i = input variable to the ith node of the input layer; w_{ji} = adjustable weight connecting the ith input node and the jth hidden node of the respective input and hidden layers; w_{j0} = bias weight for the jth hidden node; and g_1 = activation function of the hidden layer. This process is iterated and the output of the g_1 function becomes the new input that is weighted using w_{ij}, w_{k0}, and, V_j, the hidden value of jth node of the hidden layer. O_k is the output at the ith node through the activation function g_2. The activation functions are usually continuous, bounded, and nonlinear transfer functions, such as the sigmoid and hyperbolic tangent functions (Ozbek and Fidan 2009). Bishop (1995) and Haykin (1999) provided further explanations of the general properties of ANNs, whereas Maier and Dandy (2010) discussed various applications of ANNs in water resources.

Wavelet Analysis

Wavelet analysis utilizes a wavelet function known as a mother wavelet, $\psi(t)$, and is defined as $\int_{-\infty}^{+\infty}\psi(t)dt = 0$, and successive wavelets $\psi_{a,b}(t)$ are derived as

$$\psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right) b \in R, \qquad a \in R, \qquad a \neq 0 \quad (2)$$

where a = scale or frequency factor, b = time factor, and R = domain of real numbers.

The timescale wavelet transform of a continuous time signal with a finite energy signal $f(t) \in L^2(R)$, is defined as (Kisi 2010)

$$W_f(a,b) = |a|^{-1/2}\int_R f(t)\psi^*\left(\frac{t-b}{a}\right)dt \quad (3)$$

where $W_f(a,b)$ = wavelet coefficient and ψ^* corresponds to the complex conjugate function.

The wavelet transform decomposes the signal $f(t)$ into different components by searching for correlations between the signal and the wavelet function at different scales of a and locally around the time of b, and forming a contour map known as a scalogram. The effort to generate several wavelet coefficients at every possible scale and time may be reduced by constraining the wavelet dilation (a) and translation (b) parameters, and defining the discrete wavelet transformation as (DWT) (Mallat 1989)

$$\psi_{m,n}\left(\frac{t-b}{a}\right) = a_o^{-m/2}\psi^*\left(\frac{t-nb_o a_o^m}{a_o^m}\right) \quad (4)$$

where m and n are integers that determine the magnitude of wavelet dilation and translation, respectively, a_0 is a specified dilation step greater than 1 (most commonly $a_0 = 2$), and b_0 is the location parameter that must be greater than 0 (most commonly $b_0 = 1$).

For a discrete time series $f(t)$ occurring at a different time t, where t = integer time step and assuming $a_0 = 2$ and $b_0 = 1$, the DWT simplifies to (Kisi 2010)

$$W_f(m,n) = 2^{-m/2}\sum_{t=0}^{N-1} f(t)\psi^*(2^{-m}i - n) \quad (5)$$

where $W_f(m,n)$ = wavelet coefficient for the DWT of scale $a = 2^m$ and location $b = 2^m n$. $f(t)$ is a finite time series ($t = 0, 1, 2, \ldots, N-1$), where the maximum $t = N$, defined as an M integer power of 2 ($N = 2^M$), n = the time translation parameter in the range $0 < n < 2^{M-m}$–1, and m = magnitude dilation parameter with the range $1 < m < M$.

The DWT operates two functions (one a high-pass and the second a low-pass filter) and separates the original time series at these two different scales. The wavelet components that identify the high frequencies and fast events, and that capture small features of interpretational value in the data are recognized as Details (d). The wavelet component or the residual term, representing the background low-frequency information (approximation) of data, and that captures longer trend cycles (Kucuk and Agiralloglu 2006) is known as the Approximation (A).

Bootstrap Technique

Bootstrap resampling is a computational, data-driven simulation method that generates multiple realizations from one dataset of a distribution or process (Efron 1979). Bootstrap resampling is very similar to the jack-knife approach and both are widely used for data resampling (Shao and Tu 1995). However, in this study, only the bootstrap resampling method was used considering its wide application in water resources modeling. The bootstrap method is a widely accepted and simple method for the construction of confidence intervals (CIs) for ANN point forecasts and was found to be reliable compared with other techniques (Tiwari and Adamowski 2013; Wang et al. 2013). The bootstrap samples are created through intensive resampling using a replacement method, and this expansion in the number of realizations provides a better understanding of the average and variability of the original, unknown distribution or process, reducing uncertainty (Efron 1979). Assuming a population of an unknown probability distribution F, where $t_i = (x_i, y_i)$ is a realization drawn independently and identically distributed (i.i.d.) from F, x_i is a predictor vector with y_i, the corresponding output variable, and n is a random dataset sample drawn from F. The bootstrap resample denoted as $T_n = [(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)]$ is obtained from an empirical distribution function, $\hat{F}$, selection n number of t_i by placing a probability of $1/n$ for each $t_1, t_2, \ldots, t_i$. Similarly, a set of bootstrap samples such as $T^1, T^2, \ldots, T^s, \ldots, T^S$ may be produced, where S = total number of bootstrap samples, usually ranging from 50–200 samples (Efron 1979).

In this study, several bootstrap resamples were generated and used to train several different ANN models, and an ensemble forecast was obtained (Tiwari and Chatterjee 2010a, b; Tiwari and Chatterjee 2011). For each T^s, an ANN model is developed and trained using all n observations and the ANN output, $f_{NN}(x_i, w_s/T^s)$, is then evaluated using a set A^s of observation pairs $t_i = (x_i, y_i)$ that were not included to generate bootstrap resamples. Subsequently, the performance of the ANNs in these validation tests is averaged/ensemble, which also represents the generalization error for the ANN models relative to T_n. This generalization error is denoted as E_0, which is estimated as (Twomey and Smith 1998)

$$\hat{E}_0 = \frac{\sum_{s=1}^{S}\sum_{i\in A_s}[y_i - f_{NN}(x_i, w_s/T^s)]^2}{\sum_{s=1}^{B}(A_s)} \quad (6)$$

where, again, $f_{\mathrm{ANN}}(x_i, w_s/T^s)$ = output of the ANN developed from the bootstrap sample T^s in which x_i is a particular input vector and w_s is the weight vector. Finally, the BANN estimate $\hat{y}(x)$ of all developed ANNs is given by the average of the S bootstrapped estimates (Jia and Culver 2006)

$$\hat{y}(x) = \frac{1}{S}\sum_{s=1}^{S} f_{NN}(x, w_s) \quad (7)$$

and the variance is given by

$$\sigma^2(x) = \frac{\sum_{s=1}^{S}\sum_{i=A_s}[y_i - f_{NN}(x_i/w_s)]^2}{S-1} \quad (8)$$

Performance Indices

The performance of the developed ANN, BANN, WANN, and WBANN models were evaluated using four performance indices, namely, coefficient of determination (R^2), root mean square error (RMSE), mean absolute error (MAE), and peak percentage deviation (P_{dv}). These performance indices are defined as follows.

1. The coefficient of determination (R^2) is expressed as

$$R^2 = \left(\frac{\sum_{i=1}^{n}(O_i - \bar{O})(P_i - \bar{P})}{\sqrt{\sum_{i=1}^{n}(O_i - \bar{O})^2\sum_{i=1}^{n}(P_i - \bar{P})^2}}\right)^2 \quad (9)$$

where O_i and P_i = observed and forecasted water demand, respectively, $\bar{O}$ and $\bar{P}$ = means of the observed and forecasted water demand, respectively, and n = number of data points.

Values for R^2 range from 0 to 1, with 1 showing perfect forecasting ability.

2. Root mean square error (RMSE)

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(O_i - P_i)^2} \tag{10}$$

where O_i and P_i = observed and forecasted water demand, respectively, and n = number of data points. RMSE is always greater than 0 (0 occurs when the model fits the data perfectly).

3. Mean absolute error (MAE)

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n}|O_i - P_i| \tag{11}$$

where O_i and P_i = observed and forecasted water demand, respectively, and n = number of data points. MAE is always a positive number, with 0 representing a perfect model fitting observed values.

4. Percentage deviation in peak (P_{dv})

$$P_{dv} = \frac{P_p - O_p}{O_p}100 \tag{12}$$

where O_p and P_p = peaks of observed and forecasted water demand, respectively.

Study Area and Data

Calgary is among the largest cities in Canada, with a population of approximately 1.1 million people (City of Calgary 2011). The Bearspaw Plant treats water from the Bow River to primarily supply the northern half of the city, and the Glenmore Plant treats water from the Elbow River and supplies the south of the city (City of Calgary 2011). Each plant supplies approximately half of Calgary's total drinking water needs, and the 4,600 km distribution system is interconnected through transmission mains. Since 1980, the city has invested in the maintenance of the network by replacing corroded pipes with Polyvinyl Chloride (PVC) and by adding cathodic protection on pipes to reduce the rate of corrosion.

The average summer high in Calgary is 20°C, with a historic extreme high of 36°C, and the average winter low is −13°C, with a historic extreme low of −45°C. Annual rainfall in Calgary is approximately 320 mm, with a recorded extreme daily rainfall of 95.3 mm. Annual snowfall averages approximately 125 cm with an extreme daily snowfall of 48.4 cm (Environment Canada 2010).

The data used in this study were obtained for the City of Calgary from Environment Canada (2010) and consisted of maximum temperature (MaxT), total precipitation (TotP), and total water demand (WatDmand) from March 25, 2004 to December 31, 2006. Additional data were not available. For the development of the models, the data were divided into three sets: one for training the models, one for cross-validation to check that the models do not overfit, and one for testing the performance of the developed models. Table 1 shows the details of the data partitioning.

Out of less than three years of data, the first priority was given to having at least one calendar year data for training and another calendar year data for cross-validation to develop robust ANN models, and the remaining data (less than one year) were used to test the developed models. Including a lower amount of data for training and cross-validation would not have been better; instead, the preference was a complete calendar year of data for testing the developed models for one complete calendar year. The reason for this preference is that one of the most important steps in ANN modeling is training the model; if the ANN model is not trained appropriately, it will not provide good results from the testing dataset. Therefore, in this study, the data were divided by considering one calender year data each for training and cross validation, and the remaining 282 days of data were used to test the model (Table 1). Further, in addition to considering different performance indices to evaluate the model's performance, an important criterion was to evaluate the performance of the trained model for higher/peak water demand values. In this study, even though the length of the testing data was slightly shorter than a complete calendar year, the data contained days with more important higher water demand values and appropriately represented lower and medium water demand values, and these values were considered sufficient for comparing and assessing the performance of the trained models. Overall, in this study, despite the short length of the dataset, models were trained using a complete calendar year of data and were tested using a dataset that is representative of lower, medium, and higher water demand values.

Similarly, in the case of monthly water demand forecasting, 13-, 13-, and 10-month datasets were used for training, cross-validation, and testing, respectively, instead of 12-12-10. Priority was given to having 12-12 months of data each for training and cross-validation, but because the first month (March) has only seven values, and the first month average was calculated using only these seven values, average monthly data for the month of March were also included. In this way, 13 months of data were considered for training and cross-validation and the remaining 10 months of data were considered for testing. Referring to weekly water demand forecasting, the aim was to forecast the water demand for exactly the same day one week ahead (i.e., for $t + 7$ days' lead time).

Model Development

ANN Structure Identification

The selection of the most significant input variables is one of the most important steps in the ANN model development process. Whereas Cutore et al. (2008) could not observe any improvement by using climatic data in urban water demand forecasting, Jain et al. (2001), Bougadis et al. (2005), and Adamowski et al. (2012)

Table 1. Paritioning of Data for ANN Model Development

	Weekly forecasting		Monthly forecasting	
Dataset	Period	Number of data patterns for weekly water demand forecasting	Period	Number of data patterns for monthly water demand forecasting
Training	25/03/2004 to 24/03/2005	365	March, 2004 to March, 2005	13
Cross-validation	25/03/2005 to 24/03/2006	365	March, 2005 to March, 2006	13
Testing	25/03/2006 to 31/12/2006	282	March, 2006 to December, 2006	10

determined that maximum temperature, total precipitation, and water demand were very important input variables for water demand forecasting. Therefore, in this study, these three input parameters were considered for optimum model development.

No direct method exists to select the optimum number of inputs for ANN models. A trial-and-error procedure was adopted by taking the lagged or delayed information of all the input variables for one week and for one- and two-month lead-time water demand forecasts. In this way, for the development of ANN models, each variable was used by continuously increasing the lagged information until the performance of the ANN models started to deteriorate. Following this approach, the optimal number of hidden neurons was selected through a trial-and-error procedure for each lead time: the model architecture of an ANN was varied by testing 1–10 hidden neurons, and the number of hidden neurons that produced the lowest generalization error was determined to be the optimal structure (Jia and Culver 2006).

BANN, WANN, and WBANN Model Development

In addition to ANN models, three other ANN-based models were developed in this study: BANN, WANN, and WBANN models. The methodology is presented in this study. An ANN model was initially developed using the significant inputs identified after the data were log-transformed and linearly scaled to the range (0, 1). To train the ANN models, a second-order training method, the Levenberg–Marquardt method, was used to minimize the mean squared error between the forecasted and observed water demand values. All of the models were developed with Matlab codes using *MATLAB* (v.7.10.0), apart from the bootstrap resamples, which were generated using an Excel add-in (Bootstrap.xla) (Barreto and Howland 2006). To develop BANN forecasts, 100 ANN models were developed for each bootstrap resample dataset, and then all 100 forecasts were used to obtain an ensemble of forecasts. The WANN model was developed by inputting the wavelet sub-time series generated using the discrete wavelet components (DWCs). Out of several wavelet functions [i.e., Haar, Daubechies (i.e., db2, db3, db4, db5, db6), Sym3, and Coif1] tested in different water resources studies (Nourani et al. 2009; Wu et al. 2009; Kisi 2011; Tiwari and Adamowski 2013) that were also applied in this study at two to five decomposition levels, the Daubechies wavelet function db5 with three decomposition levels was found to be the most suitable for weekly and monthly water demand forecasting. Therefore, in this study, the db5 wavelet with three decomposition levels was used to decompose the time series data for the development of wavelet-based ANN models.

All input variables were decomposed into approximation ($A3$) and details ($d1$, $d2$, and $d3$) for each component, and these components served as the new time series for model development. Fig. 1 shows the wavelet sub-time series of the average monthly maximum temperature of Calgary from March 2004 to December 2006. Table 2 presents the correlation between different wavelet components and corresponding original daily and monthly water demand time series. In previous studies (Tiwari and Chatterjee 2010a;

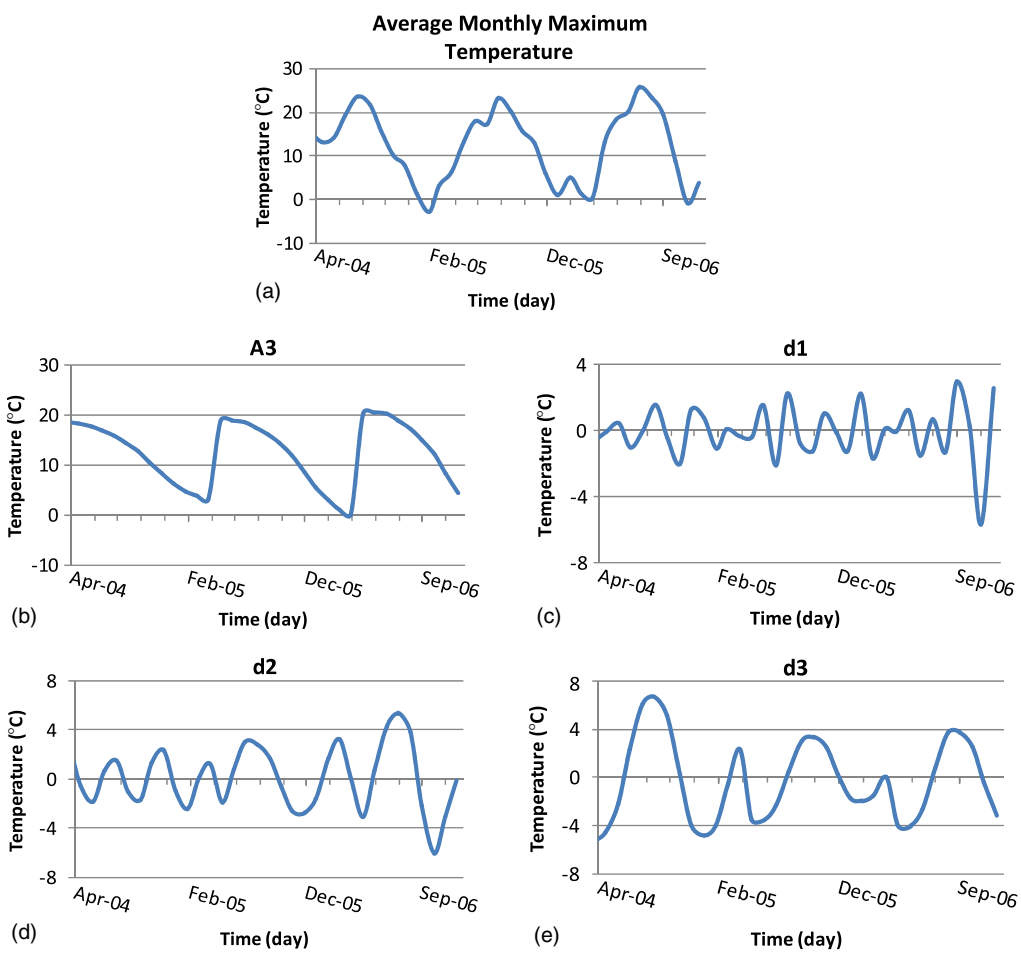

Fig. 1. Wavelet sub-time series: (a) original time series; (b) $A3$ component; (c) $d1$ component; (d) $d2$ component of average monthly maximum temperature in Calgary from March 2004 to December 2006

Table 2. Correlations between Different Wavelet Sub Time Series and the Original Time Series

Wavelet sub time series	Daily			Monthly		
	Water demand	Maximum temperature	Total precipitation	Water demand	Maximum temperature	Total precipitation
$A3$	0.40	0.32	0.01	0.74	0.73	0.58
$d1$	0.10	0.02	−0.06	0.33	0.23	−0.25
$d2$	0.15	0.06	−0.13	0.51	0.40	0.18
$d3$	0.16	0.06	−0.10	0.32	0.37	0.42
Original water demand	1.00	0.32	−0.13	1.00	0.89	0.32

Kisi 2010; Adamowski and Sun 2010; Tiwari and Chatterjee 2011), the significant wavelet sub-time series of a particular time series on the basis of the cross-correlation were added and used, which became the new inputs to develop the WANN model. In this study, all of the components of each variable were used to develop WANN models because all of the components play a different role in the original time series. The WANN models were developed using all of the components separately as inputs. The WBANN models are the combination of 100 WANN models developed using 100 resamples of the wavelet sub-time series dataset.

Results and Discussion

Weekly Water Demand Forecasting Using ANN, BANN, WANN, and WBANN Models

Table 3 shows the structure and performance of the best ANN, BANN, WANN, and WBANN models for water demand forecasting in Calgary for one-week lead-time forecasts. Fig. 2 shows the observed and forecasted water demand values in Calgary. For the development of the ANN models, present and past values of MaxT, TotP, and WatDmand were considered and, after a trial-and-error process, the significant input variables for water demand at a weekly lead-time step ($t + 7$ day) were identified as WatDmand(t)) and TotP(t), with 6 hidden neurons. This best ANN structure was also used to develop the BANN models. After a trial-and-error procedure (previously described), the best WANN model was found to have the following inputs: $A3(t)$, $d1(t)$, $d2(t)$, $d3(t)$ of WatDmand; $A3(t)$ and $d3(t)$ of MaxT; and $A3(t)$ of TotP with one, two, and three lag time variables. The optimum number of hidden neurons was found to be 4.

The performance of the best ANN model was observed to not be satisfactory because the hydrograph of forecasted water demand does not show the general behavior of the observed values. The performance is considered satisfactory for low and medium water demand values; however, the ANN model is unable to simulate high water demand values. This finding may be the result of the very small length of the training dataset (i.e., one year) that also contained a small number of higher water demand values, thereby making it difficult for the ANN model to simulate the values satisfactorily. Another reason for the weakness of the ANN model in modeling high water demand values could be the inability of the model to capture nonstationarity in the dataset. The performance of the BANN models is comparable with the ANN models. Although the performance of the BANN model was slightly less accurate than that of the ANN model, the BANN model may be considered more consistent and reliable because it was developed considering different realizations of the training datasets. As such, even if the nature of the datasets changes in the future, the BANN model is expected to perform consistently.

The performance of the WANN model in terms of R^2, RMSE, Pdv, and MAE is significantly better compared to the best ANN and BANN models. The results of the WBANN model in terms of R^2, RMSE, and MAE performance indices for the testing period is better compared to that of the ANN, BANN, and WANN models, indicating that, whereas wavelet analysis extracts nonstationarity from the training dataset, bootstrap analysis averages over the error. The WBANN models are an ensemble of WANN models trained using 100 realizations of the training dataset. Thus, even if the nature of the training dataset changes, the forecasts are expected to be more reliable and accurate.

Monthly Water Demand Forecasting Using ANN, BANN, WANN, and WBANN Models

The significant input variables for water demand at a one-month lead time were WatDmand(t) and TotP(t), and the significant inputs were WatDmand(t), TotP(t), and MaxT(t) for a two-month lead time. The optimum numbers of hidden neurons were found to be 10 and 4 for one-month and two-month lead times, respectively. For WANN model development, the significant input variables were all of the wavelet components ($A3$, $d1$, $d2$, and $d3$) of water demand (i.e., WatDmand), maximum temperature (i.e., MaxT), and

Table 3. Performance of the Best Models for the Testing Dataset Using ANN, BANN, WANN and WBANN Models for 1 Week Lead Time Water Demand Forecasting

Model	Best model structure		Performance indices			
	Input variables	Hidden neurons (HN)	R^2	RMSE (MGI/day)	P_{dv} (%)	MAE (MGI/day)
ANN	WatDmand(t), TotP(t),	6	0.59	60.05	27.38	44.39
BANN	Same as NN	6	0.56	58.63	17.88	42.53
WANN	$A3(t)$, $d1(t)$, $d2(t)$, $d3(t)$ of WatDmand(t); $A3(t)$ and $d3(t)$ of MaxT(t) and $A3(t)$ of TotP(t) with 1, 2 and 3 lag time variables	4	0.73	45.59	10.85	34.18
WBANN	Same as WNN	4	0.80	40.00	13.68	29.24

Note: WatDmand(t), TotP(t) and MaxT(t) = Urban water demand, total precipitation and maximum temperature at time t, respectively. t = daily time step.

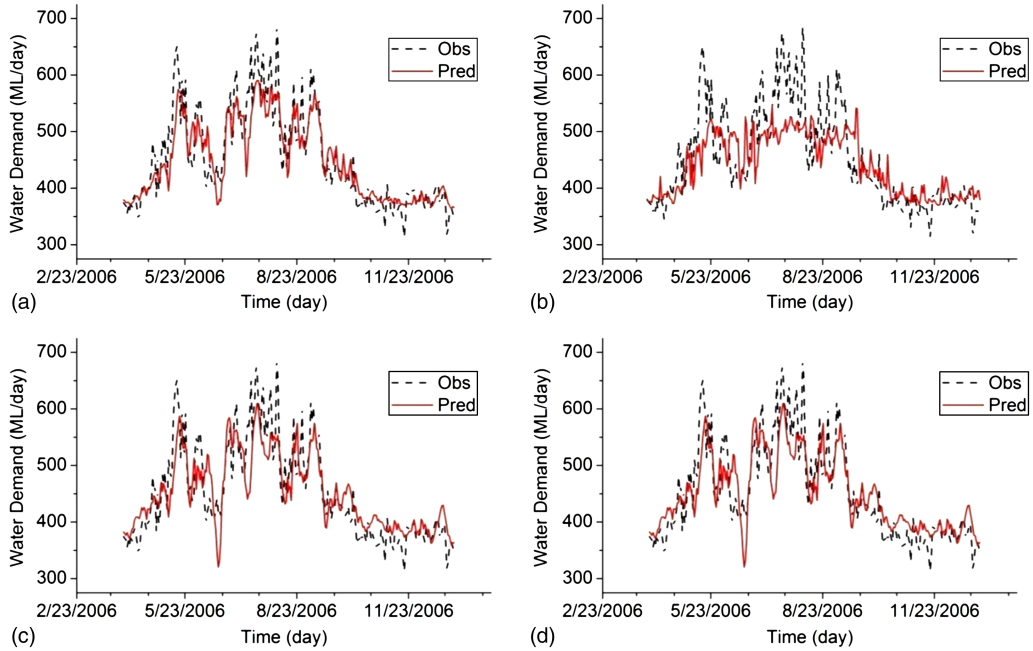

Fig. 2. Hydrographs of observed and predicted water demand in Calgary for one-week lead-time forecasts for the testing dataset using (a) ANN; (b) BANN; (c) WANN; (d) WBANN

total precipitation (i.e., TotP) at monthly time steps. The optimum number of hidden neurons was found to be 2 and 8 for one- and two-month water demand forecasts, respectively. This best WANN structure was also applied for further development of the WBANN models for water demand forecasting. WBANN models were also used to forecast water demand in Calgary. Table 4 shows the best structure of the developed models and their performance. Fig. 3 shows the observed and forecasted water demand values.

With an RMSE value of 45.20 ML/month and an MAE of 28.12 ML/month, the performance of the ANN model may be considered satisfactory for one-month lead-time forecasts. Further, for one-month lead-time forecasts, performance is good for low discharge profiles but deteriorates significantly for medium and higher discharge profiles. This phenomenon is more prominent for two-month lead times. The performance of BANN models with an RMSE value of 59.05 ML/month and an MAE of 46.26 ML/month is considered satisfactory for one-month lead-time forecasts, respectively. Similar to ANN models, the performance of the BANN model for low discharge values is good but deteriorates significantly for medium and higher discharge values—and this phenomenon dominates for longer lead times (i.e., two months). The weakness of the ANN and BANN models for forecasting higher water demand values indicates the weakness of the ANN model structure to capture nonstationarity in the training dataset.

The performance of the best WANN model with an RMSE value of 32.32 ML/month and an MAE of 27.14 MLI/month for

Table 4. Performance of the Best Models for the Testing Dataset Using ANNs for 1 and 2 Months Lead Time Water Demand Forecasting

	Best model structure		Performance indices			
Lead time (month)	Input variables	Hidden neurons	R^2	RMSE (MGI/month)	P_{dv} (%)	MAE (MGI/month)
ANN						
1	WatDmand(t), MaxT(t), TotP(t)	10	0.66	45.20	9.77	28.12
2	WatDmand(t), MaxT(t), TotP(t), WatDmand($t-1$), MaxT($t-1$), TotP($t-1$)	4	0.76	55.96	14.24	46.90
BANN						
1	Same as ANN	10	0.54	59.05	7.18	46.26
2	Same as ANN	4	0.14	92.40	15.93	79.11
WANN						
1	$A3$, $d1$, $d2$, $d3$ components of WatDmand(t), MaxT(t) and TotP(t)	2	0.83	32.32	11.80	27.14
2	$A3$, $d1$, $d2$, $d3$ components of WatDmand(t), MaxT(t) and TotP(t)	8	0.75	43.88	10.38	33.60
WBANN						
1	Same as WANN	2	0.85	32.40	6.48	26.03
2	Same as WANN	8	0.70	44.80	12.53	37.20

Note: t = monthly time step.

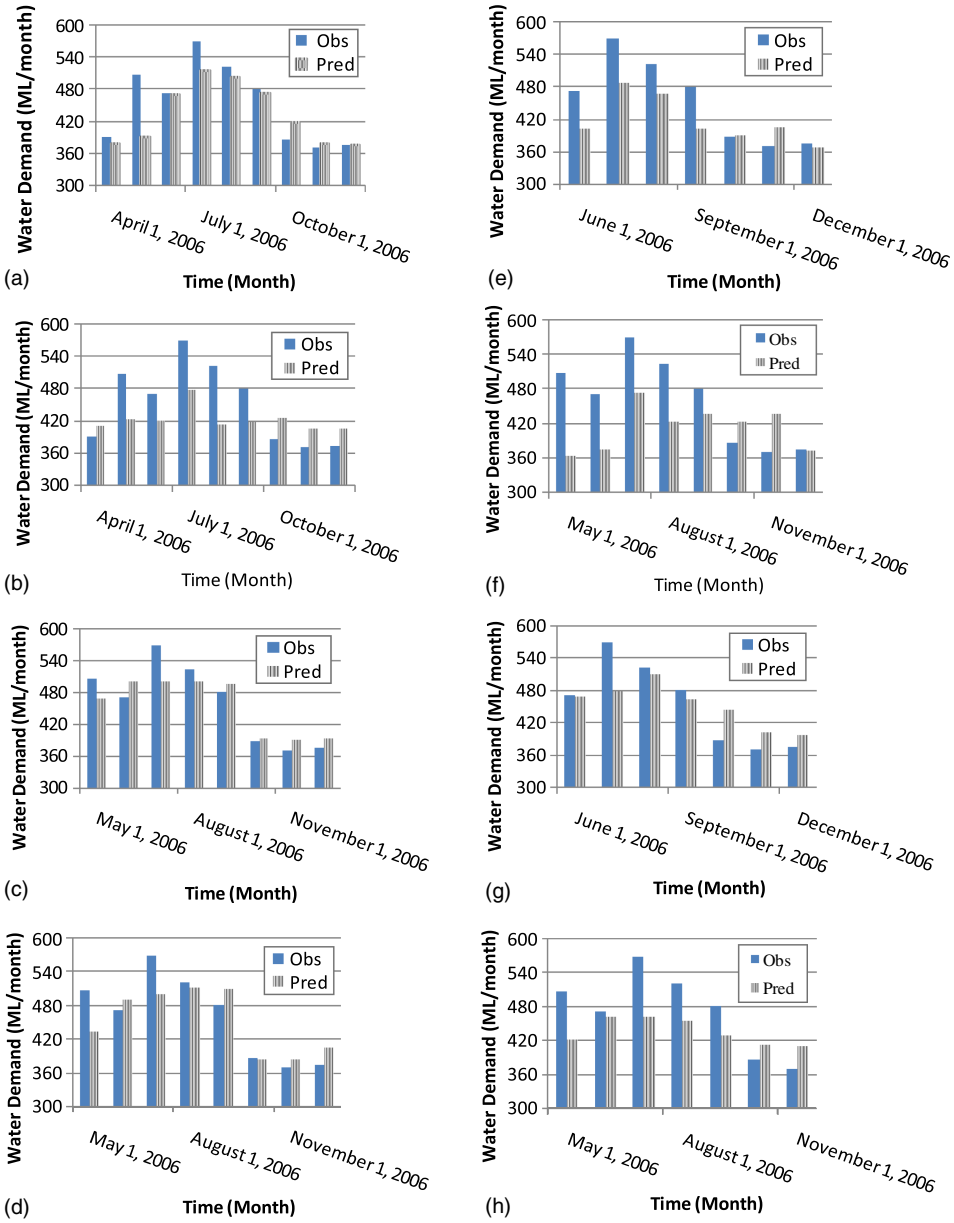

Fig. 3. Observed and predicted water demand forecasts for one month using (a) ANN; (b) BANN; (c) WANN; (d) WBANN; and for two-month lead times using (e) ANN; (f) BANN; (g) WANN; (h) WBANN

the one-month lead time, and an RMSE value of 43.88 ML/month and an MAE of 33.60 ML/month for the two-month lead time, is considered very good. The performance of the WANN model is much better compared with the ANN and BANN models, indicating the robustness of the WANN model and demonstrating the ability of wavelet analysis to capture useful information from different periodic components (i.e., wavelet sub-time series).

On the basis of the performance of the WBANN model in terms of the R^2, RMSE, Pdv, and MAE performance indices, along with observed and forecasted water demand values, the performance of the best WANN and WBANN models were observed to be better compared with those of the best ANN and BANN models. The performance of the WBANN model was found to be better compared with the WANN model for a one-month lead time, whereas the performance of the WANN model was found to be better compared with the WBANN model for a two-month lead time. The performance of WANN and WBANN models was observed to be very similar; however, the overall performance of the WBANN model is considered to be better because WBANN models are ensembles of 100 bootstrapped WANN models that consider the uncertainty attributable to the variability in the training data set.

Uncertainty Assessment Using BANN and WBANN Models

The BANN and WBANN models were developed as an ensemble of forecasts using ANN and WANN models trained through 100 realizations of the training dataset without removing any of the low performing model forecasts. These forecasts were applied to assess the predictive uncertainty associated with the forecasts in terms of confidence bands.

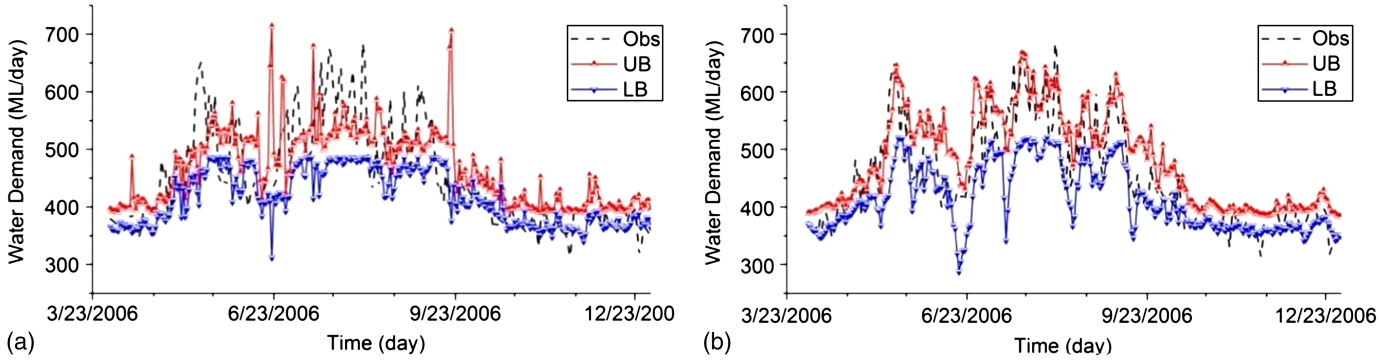

Fig. 4. Observed values along with upper and lower band for one-week lead time using (a) BANN; (b) WBANN

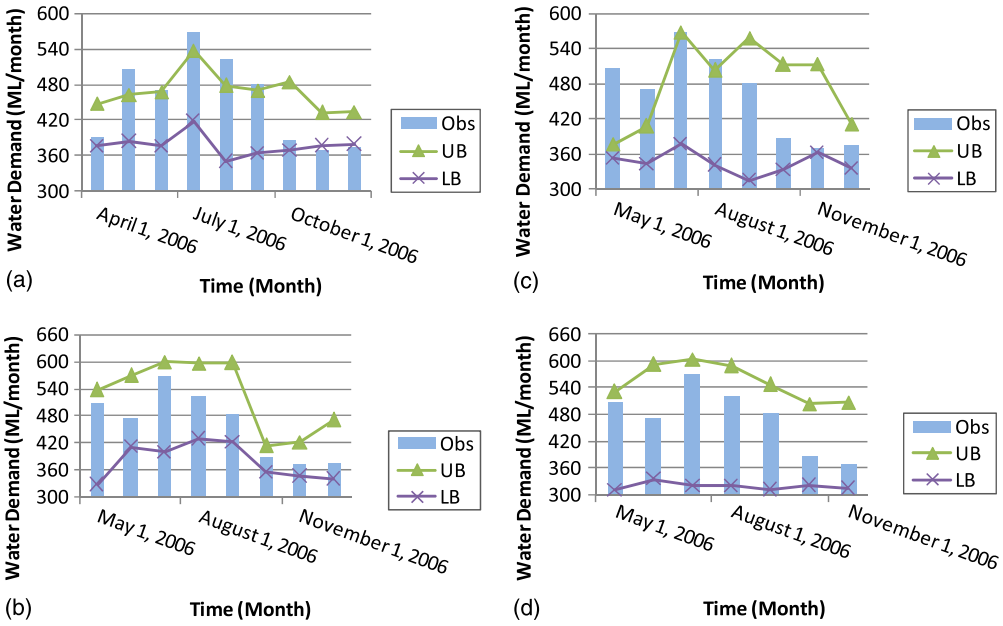

Fig. 5. Observed values along with upper and lower band for (a) one month using BANN; (b) one month using WBANN; (c) two months using BANN; (d) two months using WBANN models

Weekly Water Demand Forecasts

Fig. 4 shows the observed values along with predicted confidence bands developed using both the BANN and the WBANN models. The WBANN model is observed to assess uncertainty more accurately compared with the BANN model. Whereas the WBANN model satisfactorily simulates most of the higher water demand values, the BANN model could not satisfactorily assess uncertainty associated with higher water demand values.

Monthly Water Demand Forecasts

Fig. 5 shows the observed values along with upper and lower bands using the BANN and WBANN models for one-month and two-month lead times. The WBANN model is observed to simulate the peak values very well for one- and two-month lead-time water demand forecasts, whereas the BANN model shows its weakness in simulating and assessing the uncertainty associated with higher, medium, and lower water demand values. Again, this result shows the capability of WBANN models to accurately forecast water demand values and to provide uncertainty assessments associated with the forecasts.

Discussion of the Comparative Performance of the Models

Data-driven techniques such as neural networks, fuzzy inference systems, and genetic algorithms require longer time series data to train the models. In this study, a very short length of training data for weekly and monthly water demand forecasting was considered because only limited data are available in many real life situations. Thus, exploring the use of new data driven or machine learning methods in situations in which limited data exist to assess the appropriate methods, if any, is important. The poor performance of the ANN and BANN models, particularly for peak values, is the result of the very short length of the datasets available and used in this study. The BANN model did not simulate the peak values very well but showed more reliable results even when the pattern of the training dataset changed.

Inclusion of wavelet analysis in the ANN and BNN models by developing the BANN and WBANN models, respectively, significantly improved the performance of these models for weekly and monthly water demand forecasting and showed that they may be applied effectively even with very short dataset lengths. More

importantly, for the development of WANN and WBANN models, all wavelet components of the previously measured water demand values were found to be effective, which rendered the physical structure of the data (i.e., trends, sharp fluctuations, and seasonality) more obvious, and nonstationarity was simplified into different stationary detail components. Only the approximation component ($A3$) of both MaxT and TotP were noted as significant, showing that only the trend of these data affect water demand forecasts, whereas sharp changes or fluctuations in these data have no effect on forecasting water demand.

The comparable results of the WBANN and WANN models for weekly water demand forecasting indicated that the bootstrap method was able to produce stable results even if the pattern of the dataset changed. This indication becomes more important as 100 realizations were generated using a training dataset with a very small length, and one data resample differs significantly compared with another resample dataset. The better performance of the WBANN models compared with the BANN models demonstrates the individual strength of wavelet analysis and the bootstrap resampling technique to improve ANN model performance.

Similar to the weekly forecasting previously described, the poor performance of the ANN and BANN models for monthly forecasting was the result of the very short-length dataset (only 13 data patterns). The BANN model is not advisable in this situation because different realizations generated using bootstrap resampling are not able to develop a robust ANN model. Overall, for weekly and monthly water demand forecasting, the very good performance of the WBANN model for weekly and monthly water demand forecasting demonstrates its potential to be applied even in situations with short data availability.

The proposed model (WBANN) is observed as being able to address two important issues related to water demand forecasting: accuracy and uncertainty. The major advantage of the proposed WBANN approach is that it builds on the capabilities of: (1) wavelet analysis to improve ANN model forecasting; and (2) bootstrap resampling to help generate confidence bands for reliable forecasting. Even though one drawback exists that is related to bootstrap resampling (i.e., the computational time needed to generate different realizations, particularly for long-length datasets), the demonstrated very good performance of the WBANN model even with very short dataset lengths demonstrates that bootstrapping is an effective and computationally efficient approach when coupled with wavelet analysis.

Summary and Conclusions

Accurate and reliable urban water demand forecasting is necessary for effective and sustainable urban water resources planning and management. This study was carried out to develop an urban water demand forecasting model that is accurate and reliable for weekly and monthly urban water demand forecasting in situation with limited data availability.

For weekly water demand forecasting, that WANN and WBANN models were found to perform considerably better than the ANN and BANN models on the basis of four performance indices. The superior performance of the WANN model compared with the ANN model for one-week lead times demonstrates the usefulness of wavelet decomposition. The best ANN and BANN models for one-week water demand forecasting considered only water demand and total precipitation to be significant, whereas maximum temperature was not found to be useful in the models. In the case of WANN and WBANN models, after considering wavelet derived sub-time series, all variables were found to be significant and to considerably improve model accuracy. The use of wavelets improved the accuracy of the forecasts, whereas the use of bootstrapping ensured model robustness along with improved reliability by reducing variance.

For average monthly water demand forecasting, all three variables played an important role for one- and two-month lead times, and inclusion of different wavelet sub-time series of all these three variables significantly improved the performance of the WANN models. This study showed that very short lengths of data cannot provide robust ANN and BANN models, but when these models are coupled with bootstrap and wavelet analysis, performance is significantly improved. Moreover, since the performance of WBANN models is very similar to that of WANN models, this indicated the robustness of the WANN model and the ability of WBANN models to make accurate ensemble forecasts even if the nature of the dataset changes. The significantly superior performance of WANN and WBANN models compared with that of ANN and BANN models for one-week and one- to two-month water demand forecasting demonstrated the usefulness of wavelet analysis. This finding is supported by the fact that WBANN models performed well even though they were developed using realizations of very short lengths of training datasets, with these realizations being quite different in their nature from one another. Future research should focus on assessing the effectiveness of WBANN models using longer dataset lengths (this study focused on situations with limited data availability). The methodology proposed in this study is useful for making ensemble forecasts instead of relying on point forecasts.

This study found that WBANN models are capable of assessing uncertainty associated with forecasts and are helpful in operational water demand forecasting. In this study, the major objective was to develop a robust hybrid model that provides accurate and reliable water demand forecasts for weekly and monthly lead times; further studies may be undertaken to assess the effects of different wavelet functions, decomposition levels, and number of bootstrap samples on water demand forecasting results. The proposed WBANN model may also be tested for different applications in hydrology such as groundwater level forecasting, streamflow forecasting, modeling evapotranspiration, and rainfall forecasting.

Acknowledgments

This research was funded by an NSERC Discovery Grant, an FQRNT New Researcher Grant, and a CFI Grant held by Jan Adamowski. Data were obtained from the City of Calgary, whose help is gratefully acknowledged.

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