Exercises in Hydrodynamics

1. A protein has a molecular weight of 50,000 Da and a v_p of 0.75 $cm^3 g^{-1}$. Calculate the following:
   (a) V_o (the unhydrated volume)
   (b) r_o (radius of ideal unhydrated spherical protein)
   (c) f_o ($\eta_s = 0.01$ g cm^{-1} sec^{-1})
   (d) V_h if $\delta_w = 0.25$ ($\rho_w = 1.00$ g cm^{-3})
   (e) r_h if $\delta_w = 0.25$
   (f) f if $\delta_w = 0.25$ and the protein is spherical
   (g) $D_{20,w}$ if $\delta_w = 0.25$

   Worked examples and/or answers:
   (a) $V_o = (M_{prot}/N_o) \cdot v_p$
   $= (5 \times 10^4 \text{ g mol}^{-1})\cdot(0.75 \text{ cm}^3 \text{ g}^{-1})/(6.023 \times 10^{23} \text{ mol}^{-1})$
   $= \mathbf{62.26 \times 10^{-21} \text{ cm}^3}$

   (b) $r_o = (3V_o/4\pi)^{1/3}$
   $= 2.46 \times 10^{-7}$ cm (24.6Å)

   (c) $f_o = 6\pi\eta_s r_o$
   $= \mathbf{4.64 \times 10^{-8} \text{ g sec}^{-1}}$

   (d) $V_h = (M_{prot}/N_o) \cdot (v_p + (\delta_w/\rho_w))$
   $= (5 \times 10^4 \text{ g mol}^{-1})\cdot(0.75 \text{ cm}^3 \text{ g}^{-1} + (0.25/1.00 \text{ g cm}^{-3})/(6.023 \times 10^{23} \text{ mol}^{-1})$
   $= 0.8302 \times 10^{-19} \text{ cm}^3 = \mathbf{83.02 \times 10^{-21} \text{ cm}^3}$

   (e) $r_h = \mathbf{2.71 \times 10^{-7}}$ **cm (27.1Å)**

   (f) $f = 6\pi\eta_s r_h$
   $= \mathbf{5.11 \times 10^{-8} \text{ g sec}^{-1}}$

   (g) $D_{20,w} = (kT/f)$
   $= (1.38 \times 10^{-16} \text{ g cm}^2 \text{ sec}^{-2} (^oK)^{-1})\cdot(293.2^oK)/(5.11 \times 10^{-8} \text{ g sec}^{-1})$
   $= 79.13 \times 10^{-8} \text{ cm}^2 \text{ sec}^{-1}$
   $=$ **7.91 x 10-7** $\mathbf{cm^2 \text{ sec}^{-1}}$

(2) Repeat the above calculations for $M_{prot} = 200{,}000$

(3) A protein has a molecular weight of 20,000 and δ_w = 0.257, v_p = 0.70 cm^3 g^{-1}. At 4^{o}C, η_s = 0.016 g cm^{-1} sec^{-1} and we assume that ρ_w = 1.00 g cm^{-3}. **At 4^{o}C**:

(a) Calculate V_h

(b) Calculate f if the protein is spherical

(c) Calculate f_o (if δ_w were zero) and (f/f_o) for the spherical, hydrated protein

(d) Suppose that the protein has δ_w = 0 but is a prolate ellipsoid with (b/a) = 3. What is f in this case? What is (f/f_o)?

(4) It is found by chemical analysis that a protein has a molecular weight of 70,000 and a diffusion coefficient of 6.94 x 10^{-7} cm^2 sec^{-1} at 25^{o}C. If v_p = 0.70 cm^3 g-1 and η_s = 0.01 g cm^{-1} sec^{-1}:

(a) Calculate f

(b) Calculate f_o

(c) Calculate δ_w on the assumption that the protein is spherical

(d) Calculate possible (b/a) values assuming that the protein is ellipsoidal and unhydrated

Answers to Exercises in Hydrodynamics

(2) (a) $V_o = 249 \times 10^{-21}$ cm^3 (Q. - Why do I like to express V_o and V_h values in terms of a power of ten divisible by three? - A.- Because it facilitates taking the cube root to determine r_o or r_h)

(b) $r_o = 3.90 \times 10^{-7}$ cm

(c) $f_o = 7.35 \times 10^{-8}$ g sec^{-1}

(d) $V_h = 332 \times 10^{-21}$ cm^3

(e) $r_h = 4.30 \times 10^{-7}$ cm

(f) $f = 8.10 \times 10^{-8}$ g sec^{-1}

(g) $D_{20,w} = 5.00 \times 10^{-7}$ cm^2 sec^{-1}

(3) (a) $V_h = 31.78 \times 10^{-21}$ cm^3

(b) $f = 5.93 \times 10^{-8}$ g sec^{-1}

(c) $f_o = 5.34 \times 10^{-8}$ g sec^{-1};
$(f/f_o) = 1.11$

(d) $f = 5.93 \times 10^{-8}$ g sec^{-1};
$(f/f_o) = 1.11$

(4) (a) $f = 5.93 \times 10^{-8}$ g sec^{-1}

(b) $f_o = 5.07 \times 10^{-8}$ g sec^{-1}

(c) $r_h = 3.15 \times 10^{-7}$ cm (spherical protein: F = 1.00)

$V_h = 131 \times 10^{-21}$ cm^3

$= (7 \times 10^4 \text{ g mol}^{-1})\cdot(0.70 \text{ cm}^3 \text{ g}^{-1} + \delta_w)/(6.023 \times 10^{23} \text{ mol}^{-1})$

(d) (f/fo) = F when $\delta_w = 0$. By interpolating in the table of Perrin factors in the 'Biochem/Chem. 404 Data Sheet,' we find that the calculated value of (f/fo) corresponds either to **an oblate ellipsoid of axial ratio (b/a) ≈ 4.1, or a prolate ellipsoid of axial ratio (b/a) ≈ 3.9**.