# Charles Babbage's Principles for Proper Notation

McGill
Project supported by the McGill Arts Research Internship Award (ARIA) program

By: Jonah Dutz
Supervisor: Prof. Dirk Schlimm
Department of Philosophy

## Background

Systems of notation serve as powerful problem-solving tools; they enable situations to be represented, extended, and manipulated in relatively straightforward yet robust ways. Charles Babbage (1791-1871) was a British mathematician, engineer, and inventor, most recognized for designing a machine capable of solving any analytical function (the Analytical Engine; a programmable mechanical computer). Early in his career, Babbage examined the power and influence of signs in mathematical reasoning; in so doing, he identified some general principles which make for well-adapted mathematical notations. This project sought to organize his thoughts on the matter in a comprehensive, coherent framework. Whether these principles could be extended to optimize formalized systems of notation in other domains was also explored.

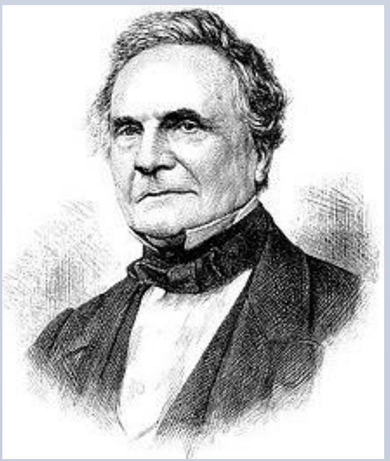


Fig. 1: Charles Babbage

## Importance of Thinking about Notation

- Though notational preferences may seem 'intuitive' or 'aesthetic', Babbage is convinced that they are guided by undisclosed principles
  - ➢ Bringing these to light encourages a more rational approach to notational choices; promotes reasoned discussion over subjective appeals
- "Good notation leads to mathematical progress and poor symbolism to stagnation." (Dubbey 1978, p.154)
  - ➢ Though differences in notation may seem apparently trivial, "the convenience or inconvenience of notation frequently depends on differences as trifling." (Babbage 1827, p. 398)
  - ➢ Well-adapted notation can suggest certain lines of reasoning/new avenues for exploration (see: Principle of Separability)
  - ➢ Poorly suited notations can multiply difficulties and restrict investigations (e.g. Newton's dot-notation for differential calculus)
- Important to avoid a heterogenous mass of signs
- Relatively simply issue to address, which promises some benefit

## Goal of Notation

"The great object of notation is to convey to the mind, as **speedily** as possible, a **complete** idea of operations which are to be, or have been, executed."
(Babbage 1830, p.412)

## Stages of Analysis

| | |
|---|---|
| **Stage I** | The first stage consists in translating the proposed question into the language of analysis [i.e. algebra]. |
| **Stage II** | The second, comprehends the system of operations necessary to be performed, in order to resolve that analytical question into which the first stage had transformed the proposed one. |
| **Stage III** | The third and last stage consists in retranslating the result of the analytical process into ordinary language. |

(Babbage 1827)

## Principles for Mathematical Notation

In his writings, Babbage introduces a series of *principles for mathematical notation.* These, in some way, further the *great object of notation* – that is, they offer an improvement in the speed and/or completeness of understanding afforded by a notation – and can be localized to one of the three distinct *stages of analysis*, stated above.

Babbage's principles for mathematical notation can be categorized as mainly *syntactic* (structure/features of representation) or *semantic* (relating to meaning).

### Syntactic Principles

#### Clarity

- Condense meaning in an unambiguous way
- Encompasses two coincident notions: *simplicity* and *brevity*
- Harkens back to origin of mathematical notation
- The clearer each step, the more intelligible the overall succession of ideas
- Facilitates the immediate working of a problem (Stage II of analytical problem-solving process)

#### Readability

- Configure notations such that they are more readable, i.e. able to be processed more fluidly
- Extremely practical; explored different text-paper colour printing combinations (Babbage 1831)
- Example rule: *"parentheses may be omitted, if it can be done without introducing ambiguity"* (Babbage 1830, p.421)
- By definition, affords more rapid understanding of mathematical expressions
- Facilitates the immediate working of a problem (Stage II)

#### Symmetry

- Two species:
  1. *Featural* – use systems of characters that express relations which exist between the data
  2. *Structural* – reformulate expressions to suggest the course or meaning of the operations to be performed
- (1) discloses meaning of quantities, and (2) of operations, in an expression
- Both facilitate translation of analytical results into ordinary language (Stage III)

#### Consistency

- Two types:
  1. *Universality of use* – adopt universal conventions (e.g. establish international congress)
  2. *Immutability of meaning* – maintain one-to-one correspondence between signs and concepts/objects
- Both reduce ambiguity
- (1) facilitates translation of problems into analytical language (Stage I)
- (2) allows attention to be focused on relations between signs
- (2) acts on all stages

### Semantic Principles

#### Suggestiveness

- Indicate important features of the operation or quantity represented
- Examples include:
  - ➢ Mnemonic aids: *v* for velocity, etc.
  - ➢ Iconic aids: <, >, =, Carnot's translation of geometry (e.g. $\overline{AB}$ for line from A to B), etc.
  - ➢ Leave trace of operations to be performed instead of computing value: $\sqrt{x}$, etc.
- Serves to immediately express meaning, reduce ambiguity, reveal process by which result is obtained
- Results easier to interpret, and thus translate back into ordinary language (Stage III)

#### Similarity

- When investigating a new domain, draw, whenever applicable, from established notations
- More familiar to the reader, therefore easier to understand
- Makes link between related operations explicit
  - ➢ Rule for inverses: denote by affixing the index '–1' (Babbage 1830)
- Can disclose characteristics about the new object of study
  - ➢ Reasoning by analogy; seen when extending exponentiation to calculus of functions (Babbage 1822)
- Facilitates Stage I

#### Necessity

- Only introduce new signs with good reason – are in no way superfluous and instead produce some tangible benefit, including:
  - ➢ Expressing distinct nature of quantities/operations (e.g. algebraic quantity, *x*, vs. pi, π)
  - ➢ Frequently occurring, unusual combinations of existing operations (e.g. iterative multiplication as exponentiation)
  - ➢ Demonstration of new properties
- No direct impact on speed or completeness of understanding
- Not easily localized to any one stage of analysis

#### Separability

- Contrive notation such that its parts can be employed separately
- Allows different components of an expression to be treated and interpreted independently, building to an understanding of the whole
- Turns solving compound expressions into a mechanical process
- Brings certain robustness to solution; step-by-step simplifications which can be examined in isolation
- Facilitates the immediate working of a problem (Stage II)

## Mechanical Notation

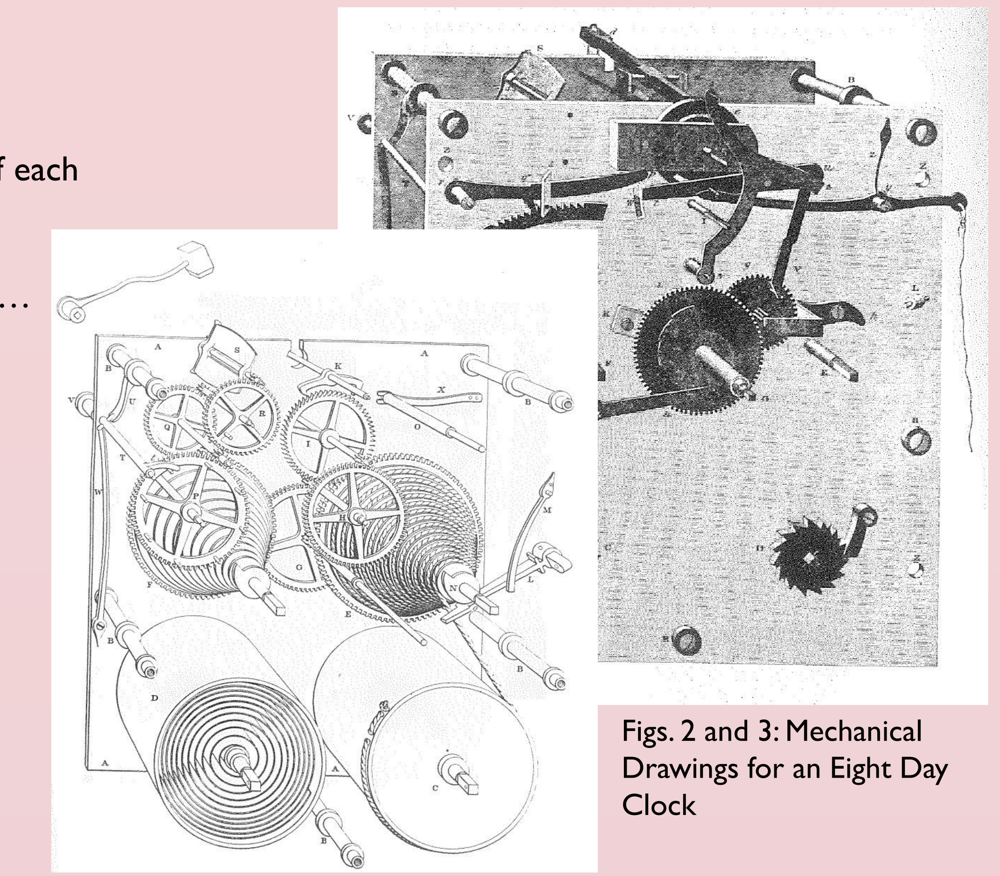
Babbage was best known for his calculating machines, the Difference Engine and the Analytical Engine. In designing these, he developed the *Mechanical Notation*. This system of signs demonstrates how the principles for mathematical notation can be extended to other domains.

The Mechanical Notation conveys information about three components of a machine:

1. **Its structure**

Involves…
- Mechanical drawings of each working layer
- Labeling scheme

Influenced by principles of…
- Symmetry
- Consistency
- Similarity



Figs. 2 and 3: Mechanical Drawings for an Eight Day Clock

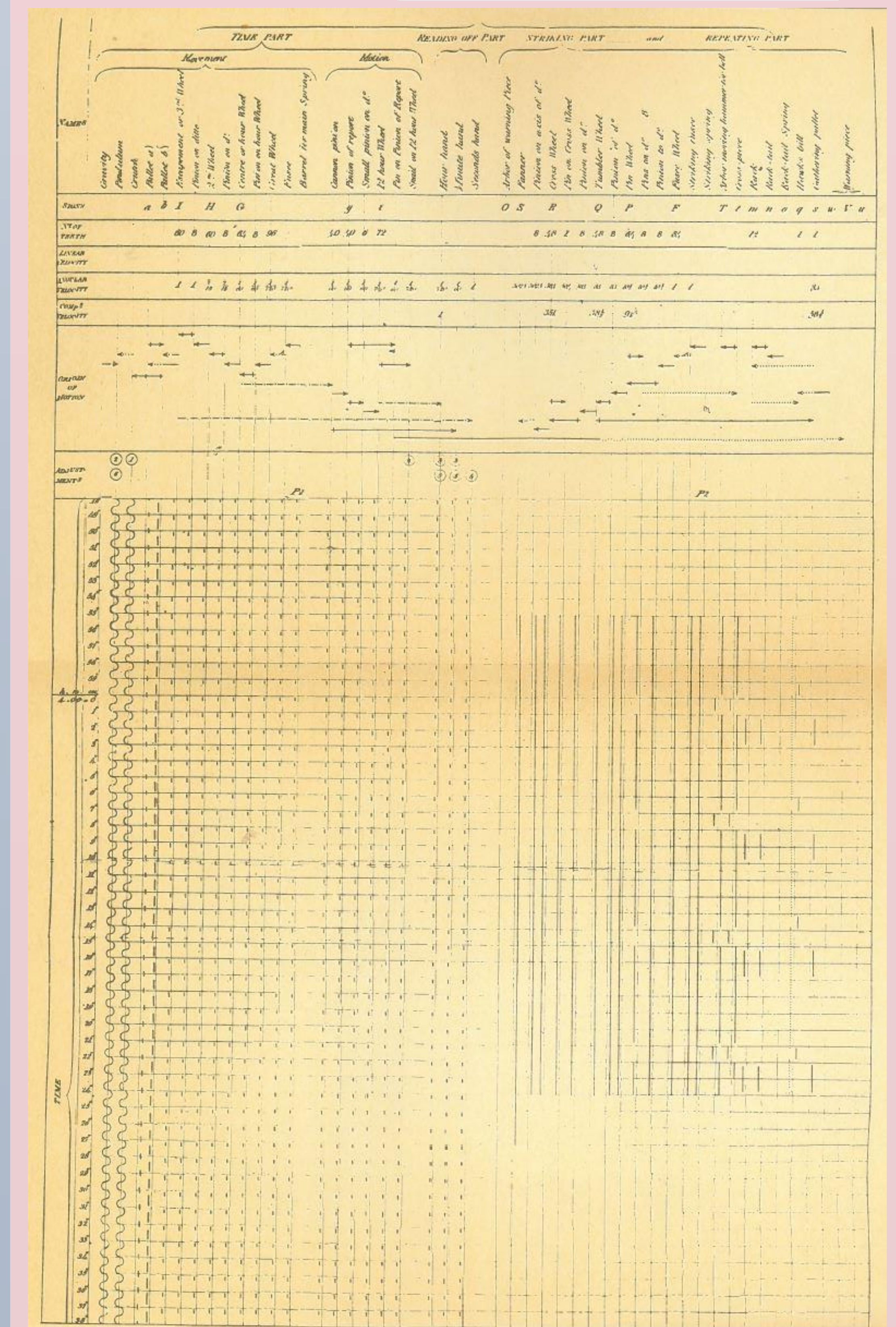


Fig. 4: Logic and Timing Diagrams for an Eight Day Clock

2. **Its logic**

Involves…
- Information about parts (velocities, adjustments, etc.)
- Mechanistic chains of cause and effect
- Origins of motion diagram

Influenced by principle of…
- Suggestiveness

3. **Its timing**

Involves…
- The time and species of every motion throughout the action of the machine

Influenced by principle of…
- Suggestiveness

(Babbage 1826)

**References**:

Babbage, C. (1822). Observations on the notation employed in the calculus of functions. *Transactions of the Cambridge Philosophical Society*, 1:63-76. Reprinted in (Campbell-Kelly 1989a, p. 344-354)

Babbage, C. (1826). On a method of expressing by signs the action of machinery. *Philosophical Transactions of the Royal Society*, 116:250-265. Reprinted in (Campbell-Kelly 1989c, p. 209-223).

Babbage, C. (1827). *On the inuence of signs in mathematical reasoning. Transactions of the Cambridge Philosophical Society*, 2:325-377. Reprinted in (Campbell-Kelly1989a, p. 371-408).

Babbage, C. (1830). Notation. *The Edinburgh Encyclopaedia*, 15:394-399. Reprinted in (Campbell-Kelly 1989a, p. 409-424).

Babbage, C. (1831). Preface to specimen of logarithmic tables printed with different coloured inks on variously coloured papers. *Edinburgh Journal of Science*, New Series, 6:144-150. Reprinted in (Campbell-Kelly 1989b, p. 115-117).

Campbell-Kelly, M. (1989a). *The works of Charles Babbage; Volume 1; Mathematical Papers*. Pickering, London.

Campbell-Kelly, M. (1989b). *The works of Charles Babbage; Volume 2; Difference Engine and Table Making*. Pickering, London.

Campbell-Kelly, M. (1989c). *The works of Charles Babbage; Volume 3; Analytical Engine and Mechanical Notation*. Pickering, London.

Dubbey, J. M. (1978). *The mathematical work of Charles Babbage*. Cambridge University Press, Cambridge, England.