PHIL 510: *Seminar on Logic* Autumn 2011

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Fridays: 09:35–11:25 McTavish 3438, Room 03
Course web site: **WebCT**

COURSE DESCRIPTION

Summary of Subject Matter. This seminar will be on *mathematical aspects of infinity and its paradoxes*. The aim is to see how modern mathematics and modern logic deals with the notion of infinity, both the *infinitely large* and the *infinitely small*. The mathematician Hilbert wrote in 1925: 'The infinite has always stirred the emotions of mankind more deeply than any other question; the infinite has stimulated and fertilised reason as few other ideas have; but also the infinite, more than any other notion, is in need of clarification'. Problems with infinity have recurred throughout the history of thought, especially in two of the most fundamental realms of knowledge, mathematics and our knowledge of the physical world, particularly the way that mathematics is used to represent the physical world. Two striking places where such paradoxes emerge are in Zeno's various paradoxes of motion and extent, and Kant's First Antinomy. These concern the way in which mathematics can be used to represent (resp.) ordinary, finite regions of space, and the way we present the unlimited nature of the cosmos. Many of these problems are resolved (if not finally solved) by developments in mathematics, especially in the later 19th century the explanation of the notion of a limit using the the $\varepsilon - \delta$ method, and Cantor's development of the theory of infinite sets. Central here is the recognition by Cantor and Dedekind that the most fundamental (perhaps defining) property of an infinite collection is that it contains a proper subset of itself with which it is in complete one-to-one correspondence. Most of the problems with the supposition that there are infinitely large or small things in nature are then solved by 5 fundamental distinctions: that between *actual* infinity and *potential* infinity; that between *cardinal* and *ordinal* numbers; that between *denumerable* (or countable) infinities and *non-denumerable* (uncountable) infinities; that between the *continuous* and merely *densely ordered* and between the *densely ordered* and the *discrete*; and that between *size* and *measure*. We will discuss some well-known examples of some of the problems and of their resolution. However, the problems do not end with such resolutions, for the issue then becomes one, not so much one of how mathematics can represent the way the infinite is used to analyse the physical world, but of how *logical frameworks* can represent the *mathematical theories* themselves. The paradoxes of the infinite give place, first to the *paradoxes of logic and set theory*, and then (assuming that these can be dealt with in a reasonably satisfactory way), paradoxes of the representation of mathematical theories in logical frameworks, most remarkably, the theory of the ordinary, finite, whole numbers.

Aim. Part of the point of this choice of material is to introduce students to some of the major developments in the philosophy of mathematics at the hands of Cantor, Dedekind, Hilbert and others.

Requirements. Students must be comfortable with logic and formal techniques; having completed PHIL 310 (Intermediate Logic) or the equivalent, is an *absolute* requirement, and students will preferably have done at least one other course in Analytic Philosophy. An aptitude, and enthusiasm, for mathematics is also important. The course will be a balance between technical material and philosophical analysis of it.

Reading Matter. The reading material will be assigned as the course progresses, and made available through WebCT or by download from McGill Library. Some of the readings will be original sources, e.g., Berkeley's famous criticism of the infinitesimal calculus *The Analyst* (1734), some selections from Bolzano's *Paradoxes of the Infinite* (1858) the paper where Cantor first (1883) explained his modern theory of the 'actual infinite', and set out the fundamental distinction between ordinal and cardinal numbers, and Hilbert's famous paper from 1925 (quoted from above) 'On the infinite'. But since we will also survey a good deal more material, we will also use selections from survey sources (e.g., Graham Oppy's book *Philosophical Perspectives on Infinity*, Russell's *Our Knowledge of the External World*, and perhaps Grünbaum's *Modern Science and Zeno's Paradoxes*).

Marking and Assessment Students will be expected to read the texts assigned for the week, and be prepared to initiate and/or pursue the discussion if assigned to do so. Students will be required to write a standard term paper for a seminar of around 5000 words, due on the final day of the semester. **NB. Please keep copies of work submitted.** *Extensions to deadlines set will be granted only in **very exceptional** circumstances, usually only for medical reasons and with a medical note (or for other, similar emergencies, appropriately documented).*

Policy for Late Work Late papers will be penalised at the rate of a third of a grade per day overdue. (Thus a paper adjudged to be worth a B+ will be assigned a B if it's one day late, a B− if late two days, and so on.)

NB Note that the seminar will NOT meet on Friday, 2nd September; it will meet for the first time on Friday, 9th September.

Statements of McGill Policy:

- *Plagiarism* McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarismand other academic offences under the Code of Student Conduct and Disciplinary Procedures (see `www.mcgill.ca/integrity` for more information).
- *Language* Students have the right to submit work in French.
- *Warning* In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.

Notes:

1. The infinite conceived of as something endless, unlimited, perhaps unsurveyable, and therefore beyond measure. Absolute, perfect, total or complete.

the Absolute then as an end towards which the finite strives. The infinite therefore does have an end — in the total, the Absolute.

2. What we perceive and have knowledge of is finite; this gives rise to A's idea of the potential infinite.