

Tuesdays, Thursdays: 13:05–14:35, Burnside, 1B39.

Course Page: WebCT

MICHAEL HALLETT

Office: Ferrier, 464.

E-mail: michael.hallett@mcgill.ca

Summary.

The purpose of this course is to act as an introduction to the philosophy of mathematics. There are many basic philosophical questions about mathematics. There are, first, matters of metaphysics: What is mathematics about? Does it have a subject matter, and if so, what is it? For instance, what are numbers, sets, points, lines, functions, and so on? Or is mathematics merely formal and about nothing in particular, like logic? Then there are semantic matters: What do mathematical statements mean, and can they be true? If so, what is the nature of mathematical truth? And there are also matters of epistemology: How is mathematics known? What is its methodology? Is observation involved, or is it a purely mental exercise? What is a proof? Are proofs absolutely certain, immune from rational doubt? And is proof the only way we can know mathematical truths? And a question which becomes important in the light of Gödel's Incompleteness Theorems: are there unknowable (unprovable?) mathematical truths?

The central aim of this course is to illustrate a few of these problems by focussing on some historically important original works. Among these might be Bishop Berkeley's *The Analyst*, Gauss's treatment of the so-called 'imaginary numbers', various development surrounding the emergence of non-Euclidean geometry, Dedekind's treatment of the irrational numbers, Cantor's introduction of the transfinite numbers, Frege's treatment of the natural numbers as 'logical objects', the Frege-Hilbert correspondence, and Gödel's philosophical conclusions drawn from his own famous incompleteness theorems. Many of these readings concern philosophical reaction to the *extension* of mathematics, perhaps best shown through various extensions of the number system. The course will be divided into at least 4 segments, The Development of the Calculus, Non-Euclidean Geometry, Numbers and the Extensions of the Number Systems, and Gödel's Realism. (*NB*: this list is provisional, and it will almost certainly change.) The papers we will read are intended as merely illustrative of problems that arose under these loose headings.

Prerequisites. Students **MUST** have done PHIL 310 (Intermediate Logic) or equivalent. Having done the course in the Mathematics Department on the History of Mathematics would also be an advantage.

Readings. The lectures will concentrate on close reading and discussion of the original texts and readings made available through the WebCT Website for the course. Many of these will be taken from the book

- William Ewald: *From Kant to Hilbert: Readings in the Foundations of Mathematics, Two Volumes*, 1996, Clarendon Press, Oxford.

These readings will be *essential*.

Requirements & grading. Students will be required to attend and participate in class, do the assigned readings, and be prepared to discuss them. The final grade depends on a final class paper (70%), on participation in class (5%), and also on submission of a 'notebook' on the course. This will be a fairly straightforward exercise of stating four philosophical questions which 4 of the various segments of the course raise, and providing two paragraph summaries of answers to 2 of these questions. This 'notebook' will be worth 25%; the notebook will not be assessed for content; however, the final paper might well be an expansion of one of your 'notebook' entries. *NB*: I require that all material (notebooks and final paper) be submitted to me as electronic files in PDF form.

McGill Policies

1. McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see www.mcgill.ca/integrity for more information). *NB*
2. In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.
3. Students have the right to submit work in French..