Phil 310 *Intermediate Logic* Winter 2012

Tuesdays, Thursdays: 13:05–14:35, Burnside, 1B39.

Course Page: WebCT

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Summary. In a first course on logic, the emphasis is on setting out the standard logical system, and seeing how it works from *within*; in this course, however, we concentrate much more on proving theorems *about* the standard logical systems, though this requires a deeper analysis of how the system works. Because of this, this course is quite different in character from an introductory logic course: it is much more *mathematical* in nature.

We will begin with a brief introduction to types of proofs, and then some basic set theory. This will be followed by a discussion of the Completeness Theorem for *propositional logic*. After this we shall concentrate our studies on *classical first-order predicate logic*. In particular our focus will revolve around two major results: (1) the *Completeness Theorem for first-order logic*, and (2) *Gödel's First Incompleteness Theorem*.

Surrounding (1), we shall study Henkin's proof of the Completeness Theorem itself, the Compactness Theorem, and the Löwenheim-Skolem Theorem, which implies the non-categoricity of the first-order axioms for arithmetic and the existence of non-standard models (Skolem's Theorem). This will provide an important bridge to the material concerning (2). Surrounding (2), we will prove Gödel's Theorem itself, and leading up to this, we will present important elements of recursive function theory, Gödel numbering and representability; we will also give accounts of Church's Theorem on the undecidability of first-order logic, Tarski's Theorem on the undefinability of truth, and Gödel's Second Incompleteness Theorem.

By studying some of the most important theorems of 20th century logic we will learn about the power and the limitations of first-order logic. These technical results have far-reaching philosophical implications (e.g., computational theory of mind, the theory of truth, the nature of mathematical and scientific theories, Hilbert's programme), not just for the study of the philosophy of mathematics, but for philosophy generally.

Prerequisites. Introduction to Deductive Logic (Philosophy 210A), or equivalent. Not open to students who have taken MATH-498.

Textbook. The lectures will follow closely the development in the second half of

- Moshé Machover: *Set Theory, Logic and Their Limitations* (Cambridge, 1996).

The book will be available at The Word Bookstore, 469 Milton Street (5 mins. from the University Street Gates). This text is essential.

Requirements & grading. Students will be required to attend and participate in class, do the assigned readings, complete weekly homeworks, and take a final exam. The final grade depends on homeworks (70%), final exam (25%), and participation in class (5%). Failure to hand in the homeworks on time will result in the loss of marks.

Supplementary Reading As interesting informal background reading, I highly recommend the small book *Gödel's Proof* by Ernest Nagel and James Newman (Routledge, 1958; second edition 2001 by NYU Press). This presents the very important background which culminates in Gödel's Incompleteness Theorem, and has an informal presentation of the result itself. *Understanding the material in this book is essential for understanding many of the developments described in this course.* There are many other good books on the material covered here. Four of these are: (a) Hubert Enderton: *A Mathematical Introduction to Logic;* (b) Elliot Mendelson: *Introduction to Mathematical Logic*; (c) George Boolos, John Burgess and Richard Jeffrey: *Computability and Logic* (5th edition, Cambridge University Press, 2007); and (d) Peter Smith *An Introduction to Gödel's Theorems* (Cambridge University Press, 2007). A very nice book (which, however, doesn't go as far as we will) is Dirk van Dalen: *Logic and Structure* (3rd edition, Springer-Verlag). The best book in French is S. C. Kleene, *Logique mathématique* (Paris, Armand Colin, 1971, a translation of Kleene?s *Mathematical Logic*). Extra reading on specific topics will be made available as the course progresses.

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1. *McGill University values academic integrity. Therefore all students must understand the meaning and consequences of cheating, plagiarism and other academic offences under the Code of Student Conduct and Disciplinary Procedures (see* `www.mcgill.ca/integrity` *for more information).* NB

2. *In the event of extraordinary circumstances beyond the University's control, the content and/or evaluation scheme in this course is subject to change.*

3. *Students have the right to submit work in French..*