A Class of Performance-Based Subsidy Rules

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Abstract

We consider a benchmark static incentive scheme, i.e. a per unit subsidy, that induces a monopoly to produce a target output level. We show that the same output level can be achieved by a continuum of dynamic subsidy rules based on a performance indicator. The subsidy rules require only local information. The present value of the subsidies paid under anyone of our dynamic schemes is smaller than the amount paid under the static subsidy. Moreover, each of the dynamic subsidy rules results, *at each moment,* in a lower per unit subsidy than the static subsidy. The subsidy rate depends on a state variable that reflects the monopolist's history of performance. This variable depreciates over time, therefore requiring a permanent effort of the monopolist to maintain it at an optimal level. In an example with a linear demand, the subsidy costs of inducing efficiency are reduced by almost fifty per cent.

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1. Introduction

The output of a monopoly is below the socially optimal level. When information about the cost structure is available to the regulator a straightforward policy prescription is to require the monopolist to charge a price equal to the marginal cost (a lump-sum subsidy may be necessary to satisfy the participation constraint). In the absence of the information about cost, one can use alternative mechanisms based on the demand side. For example when the regulator's only information is the demand schedule, Loeb and Magat (1979) suggest that the regulator can offer the monopoly a subsidy that corresponds to the consumer surplus achieved at the price the monopoly decides to charge. This approach has been refined by Sappington and Sibley (1989, 1990) and Vogelsang and Finsinger (1979) who propose incremental mechanisms to determine the amounts transferred to the monopoly in order to induce efficiency. These approaches generally require knowledge of the whole demand schedule[1].

When only local information is available neither the marginal cost pricing coupled with a lump-sum transfer nor the Loeb and Magat (1979) mechanism can be used. In this paper we show that, in the case where only local information is available, there exists a family of linear-in-output subsidy rules that would (i) induce the monopolist to produce at the socially optimal level *at all time*[2], and (ii) economize on the payments to the firm. The trick is to make the subsidy rate (per unit of output) dependent on an index of the history of the monopolist's past performance.

Our approach is applicable also to situations other than regulation of monopoly. More generally, the main contribution of this paper is to build, starting from a benchmark static incentive scheme (e.g. a per unit subsidy), that induces an agent (e.g. a monopoly) to rationally choose a given action (e.g. a production level), a continuum of dynamic incentive schemes that induce the agent to choose the action that he would choose under the static benchmark scheme. Each member of our family of dynamic incentive schemes results in a different distribution of surplus between the firm and the regulator. Thus the family of schemes gives the regulator more freedom to pursue other objectives without compromising the ability to achieve the targeted level of production.

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[1] An exception is the incremental scheme proposed by Vogelsang and Finsinger (1979) that requires only partial knowledge of the demand schedule. However their scheme can be strategically manipulated by the monopolist (see Sappington (1980)).

[2] In Section 3.5 below, we compare our scheme with that of Laffont and Tirole (1993), where the monopolist is induced to produce the optimal quantity in the long run.

Our family of mechanisms works as follows. The regulator creates a "performance index" that summarizes the monopolist's past behavior. A high value of the index indicates a history of good performance. This index is continuously updated as the monopolist's output is observed. The subsidy rate (per unit of output) paid to the monopolist at each moment is specified as an increasing function (with parameters chosen by the regulator) of the current level of the index. The formula (including the parameters) for updating the index is announced from the outset, and thus the monopolist can optimally plan to build up the level of the index through its production. This index may be regarded as an intangible asset. Technically, it is a state variable in a well defined optimal control problem. The index may also depreciate over time[3]. The regulator decides on the depreciation rule to be applied to the index. Because the subsidy rate per unit of output is made dependent on the index, the production of the monopolist at any given moment affects both (a) its profit at that moment, and (b) the future subsidy rates and thus future profits.

We show that there exists a continuum of such subsidy schemes that induce the monopolist to achieve the socially desirable level of production at each moment. Each subsidy scheme in this continuum generates an infinite horizon dynamic optimization problem for the monopolist, and his optimal production path is shown to be a constant path that coincides with the targeted output level. It is not surprising that by making use of an additional dimension (i.e. time) one has more options to achieve a given objective (e.g. a targeted production level). For example, suppose a constant rate of subsidy of one dollar per unit of output is sufficient to achieve a target of production, it is intuitively plausible that if the regulator can vary the subsidy rate with time, he can achieve the same targeted production level. This is because under a dynamic subsidy scheme that gives a little less than a dollar of subsidy per unit for an initial period of time (the stick phase) the regulator could give a little more than a dollar for at least another period of time (the carrot phase) to make the monopoly produce at each moment the targeted level of output. A well planned combination of stick and carrot phases could achieve the same objective as a constant incentive scheme. This intuitive idea turns out to be not quite correct. Consider a constant subsidy rate that induces the monopoly to choose a targeted production level, what we show in this paper is that dynamic subsidy rules that require a smaller per unit subsidy at each moment can induce the monopoly to choose at each

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[3] In fact, it is as if we have turned the static monopoly into a producer of a durable good that depreciates over time. (See Malueg and Solow (1989) and Karp (1996) on durable good monopoly).

moment the given targeted level of output: the carrot phase is unnecessary.

Our method of creating a multiplicity of dynamic schemes that induce the same production as a benchmark static scheme can be easily applied to other static incentive schemes (such as the mechanism suggested in Loeb and Magat (1979)). Moreover, it is clear that the use of incentive schemes that are functions of an index reflecting the past performance of the monopoly can also be extended to models that incorporate the cost of public funds and the economic distortions created when raising non-lump sum taxes[4].

2. The basic model

2.1. Notation and basic assumptions

Let us begin by considering a monopolist in a static environment. The inverse demand function is $p = p(q)$ and the total cost function is $c(q)$, where q is the firm's output. We assume that

$$c(0) = 0, \qquad c'(q) > 0, \qquad c'(0) < p(0), \qquad p'(q) < 0 \tag{1}$$

and marginal revenue, $p(q) + qp'(q)$, falls as q rises. We make no restriction on the sign of $c''(q)$. Thus marginal cost can be falling, or rising. We only require that if the marginal cost is falling, its slope at the point of intersection with the marginal revenue curve is less negative than the slope of the marginal revenue curve. In the absence of tax or subsidy, the profit function is

$$R(q) \equiv p(q)q - c(q) \tag{2}$$

We assume that $R(q)$ is strictly concave[5] and attains its maximum at some $q^L > 0$. (This does not imply that $c(q)$ is necessarily convex.) The assumption that $c(0) = 0$ implies that $R(0) = 0$, i.e., if the monopolist's output is zero, his profit is zero[6].

Let the social welfare be

$$W(q) = \int_0^q p(\widetilde{q})d\widetilde{q} - c(q)$$

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[4] Revenue raising by taxes (other than lump sum taxes) are distortionary. This realization has led to the concept of "marginal cost of public funds" (see, e.g., Browning, 1976, Ballard et al. 1985 for theory and empirical measures). We do not wish to incorporate this into our model, in the interest of simplicity.

[5] If the profit function is not strictly concave, the problem of designing optimal tax and subsidies can be very complicated. See, for example, Guesneries and Laffont (1978), also Laffont (1987, pp. 81-83).

[6] This assumption is made for simplicity, it is straightforward to generalize the conclusions of the paper to the case where $c(0) = f$ where f represents a positive fixed cost.

The socially optimal output level is denoted by q^{so}. At q^{so}, price is equal to marginal cost:

$$p(q^{so}) = c'(q^{so}) \tag{3}$$

We assume that (3) has a unique solution. (Clearly, $q^{so} > q^{L}$.)

In this paper, we assume the regulator only has local information about the cost function and the demand function[7] and her primary objective is to induce the monopoly to produce q^{so}. Let us assume that the regulator knows $q^{so}$and the numerical value of the slope of the demand curve at q^{so}, though she does not have exact information about the numerical value of the slope of the demand curve at other output levels (of course, to know q^{so} one must know $c'(q^{so})$ so the assumption amounts to that the regulator knows where the marginal cost equals the marginal benefit.)

In this case, a static mechanism that can be implemented to induce efficiency is to subsidize the monopolist's output at a *constant* rate s^* per unit of output, where

$$s^* = -p'(q^{so})q^{so} > 0 \tag{4}$$

(For a proof, see Appendix A)

In other words, s^* is the difference between the price $p(q^{so})$ and the marginal revenue $p(q^{so}) + p'(q^{so})q^{so}$ (both evaluated at the socially optimal output level).

If the monopolist produces the quantity q^{so}, his profit (including the subsidy) is

$$\pi(q^{so}) = R(q^{so}) + s^* q^{so}$$

At q^{so}, we have

$$R'(q^{so}) + s^* = 0$$

We need to make the following additional assumption.

Assumption A1 : $R(q^{so}) + s^* q^{so} > 0$

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[7] To obtain the demand schedule a regulator can poll consumers and have them report their willingness to pay for an additional unit of good at different quantities. It is a very difficult task to obtain an accurate value of people's willingness to pay because the question requires individuals to report an unprecise value in very hypothetical situations. In general, a consumer can give a good or reliable approximation of her willingness to pay for the last units consumed or next additional units that she would consume. However it becomes much difficult to get an accurate estimate of the willingness to pay in farther ranges of consumption levels, e.g. the willingness to pay for the first unit consumed.

This assumption ensures that the total costs of producing q^{so} are covered and therefore the firm will choose to produce q^{so} rather than zero, if it faces the subsidy rate s^*.

For our dynamic mechanisms, we will make use of the following fact.

Fact 1: there exists a subsidy rate $\bar{s} < s^*$that induces an output level $\overline{q} \in (0, q^{so})$ with $R(\overline{q}) + \overline{s}\overline{q} > 0$.

This fact follows from Assumption A1 and the continuity and strict concavity of the function $R(q)$.

We assume the following:

Assumption A2: The regulator knows one such value of $\overline{q}$ and $\bar{s}$.

An interesting special case of $(\overline{q},\ \bar{s})$ is the pair $\left(q^L, 0\right)$.

To summarize, in a stationary environment, the regulator can ensure efficiency by offering the monopolist a *linear subsidy schedule*

$$S(q) = s^* q \tag{5}$$

where s^* is given by (4). The monopolist, taking this schedule as given, will choose the output level q^{so}. This subsidy scheme only requires the regulator to have local information about the demand function and marginal cost function (around the social optimum).

For our family of dynamic subsidy schemes, the regulator will also need to know another "viable" production level (our $\overline{q}$) and a corresponding subsidy rate $\bar{s}$. A major problem with the static subsidy rule (5) is that it does not seem “fair” : the subsidy makes the monopolist even richer[8]. In the next subsection, we propose a class of simple linear subsidy rules that do not enrich the monopolist as much and still ensure that the efficient output level q^{so} is produced.

2.2. Performance-related linear subsidy rules

In this section we introduce an index that represents a measure of the cumulative performance of the monopolist. Let the “state variable” $X(t)$ be the value taken by that index at time t, where $X(0) = X_0 > 0$. Here, X_0 is to be chosen by the government at the beginning of the game. We suppose the government announces the following rule of “updating” $X(t)$:

$$\dot{X}(t) = q(t) - \delta X(t) \tag{6}$$

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[8] When feasible the government could announce a per unit subsidy rate s^* and auction the right to supply the market to a single producer (the monopoly). See Remark 4 below.

where $q(t)$ is the firm's output at t and $\delta > 0$ is the rate of depreciation of the index (also to be chosen by the government). The government informs the monopolist, at the beginning of the game, the differential equation (6), and its chosen constants δ and X_0. The government makes the binding commitment that after time $t_0 = 0$, it will not change the chosen constants. Note that since $q(t) \geq 0$ and $X_0 > 0$, the index $X(t)$ is always positive at any finite t.

A possible interpretation of (6) is as follows. Suppose the government sets $X(0) = \frac{q^{so}}{\delta}$, then as soon as the monopolist's output level falls short of q^{so}, the index $X(t)$ will decrease, indicating that the firm has "misbehaved", and, with a well chosen subsidy rule, such misdeed will entail a decrease in the subsidy rate. As we will show below, if the subsidy rule is well designed the implicit threat of a decrease in the subsidy rate will induce the firm's optimal path to coincide with the socially desirable production path. Alternatively, the government may set $X(0) < \frac{q^{so}}{\delta}$. Then if the firm chooses $q(0) = q^{so}$ it will raise the index $X(t)$ upon which the subsidy rule is tuned. This behavior should be encouraged by an anticipation of a higher future subsidy rate, as prescribed by a subsidy formula. If the prospect of a more favorable subsidy rate is fine-tuned by the government, the firm's optimal path will again coincide with the socially desirable production path.

The government announces at the outset a subsidy rule $S(X, q) = \sigma(X)q$, where $\sigma(X)$ is a function defined for all $X \geq 0$. The rule $S(X, q) = \sigma(X)q$ is linear in q and non-linear in X. At time t, the monopolist will receive a net subsidy rate $\sigma(X(t))$ per unit of output, i.e., he is given the subsidy amount $\sigma(X(t))q(t)$ if he produces $q(t)$ when the index of his past behavior takes the value $X(t)$. Note that $S(X, q)$ does not depend explicitly on time. In the next section, a concrete form of the function $\sigma(X(t))$ will be proposed, and its efficiency implication investigated.

3. Achieving efficiency by rules based on a performance index

3.1. Designing a class of efficiency-inducing subsidy rules

We now offer a set of performance-related per unit subsidy rules $\sigma(X)$ that induce the monopoly to choose at each moment the production level chosen under the constant per unit subsidy s^*, i.e. q^{so}. The representative member of our set of subsidy rule is

$$\sigma(X(t)) = s^* - KX(t)^{-\beta} \text{ and } X(0) = X_0 > 0 \tag{7}$$

where K is a positive number, r is the interest rate, $\beta \equiv \frac{r}{\delta}+1>1$, and s^* is given by (4). We will show[9] that efficient production can be ensured by appropriate choices of δ,K, and X_0. The value of K has to be within a certain range to ensure that the monopolist earns positive profits, while δ and X_0 must be chosen to ensure that the monopolist does not exit the market in finite time; this will become clear in what follows.

The class of rules specified by equation (7) has the property that if the monopolist builds up the level of the index, he will get a higher subsidy rate, i.e.,

$$\frac{d\sigma(X)}{dX}=\beta K X^{-\beta-1}>0 \tag{8}$$

Clearly, for any given positive K the subsidy rule (7) results in a negative subsidy rate (i.e., a tax) when the index variable $X\left(t\right)$ is below the threshold $\bar{X}\equiv\left(\frac{K}{s^*}\right)^{\frac{1}{\beta}}$. Thus if the firm adopts a production path that drives the index $X\left(t\right)$ to a level below $\bar{X}$ the subsidy changes into a tax.

The firm takes as given (i) the updating rule (6) and the depreciation rate δ, (ii) the subsidy rule (7) and the constant K, and (iii) the initial value X_0. The firm's profit function, inclusive of the subsidies, is

$$\pi(q,s)=p(q)q-c(q)+\sigma(X)q\equiv R(q)+\sigma(X)q\equiv\widehat{\pi}(q,X)$$

The firm chooses a time path $q(t)\geq 0$ and a terminal time $T\geq 0$ to maximize the discounted stream of profit:

$$\max_{q\geq 0}\int_0^T\widehat{\pi}(q,X)e^{-rt}dt \tag{9}$$

subject to $\dot{X}=q-\delta X$, $q\geq 0$, and $X(0)=X_0$. Note that the firm can ensure that the integral is non-negative: by choosing $q(t)=0$, profit is zero at time t. Also, if the subsidy rule is not well designed, the firm may choose to make a quick profit over some finite time interval $[0,T]$ and exit the market at time T (that is, $q(t)=0$ for $t>T$) to avoid future taxes. We call such a strategy the "*hit and run*" strategy. In what follows we show that such a strategy will not be chosen by the firm if the parameters K,δ, and X_0 are well chosen.

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[9] It can also be shown that given the transition equation of the index variable X,(6), any stationary markovian efficiency inducing subsidy rule is of the form (7).

3.2. The main results

We now state and prove our main results, Propositions 1 and 2.

Proposition 1: Assume the monopolist does not choose the *"hit and run"* strategy. The per unit subsidy rate $\sigma(X(t))$ given by (7), where $K > 0$ and where $X(t)$ satisfies the differential equation (6) ensures that the monopolist will always produce the socially optimal output level q^{so}.

Proof: *We establish that there is a saddle-path that leads to a unique steady state.*

Let $\psi(t)$ denote the co-state variable. The Hamiltonian associated with the monopolist's problem is

$$H = R(q) + s^{*}q - KX^{-\beta}q + \psi\left[q - \delta X\right]$$

where

$$R(q) = p(q)q - c(q)$$

The Hamiltonian is concave in the state variable X because $K \geq 0$. It is strictly concave in the control variable q. Note that by definition of s^{*},

$$R'(q^{so}) = -s^{*} \tag{10}$$

The maximum principle gives

(i) the maximality condition: given $\psi(t)$ and $X(t)$, the monopolist's control variable $q(t)$ maximizes the Hamiltonian, thus

$$\frac{\partial H}{\partial q} = R'(q) + s^{*} + \psi - KX^{-\beta} \leq 0,\ q \geq 0, \text{ and } q\frac{\partial H}{\partial q} = 0 \tag{11}$$

(ii) the adjoint equation:

$$\dot{\psi} = (r + \delta)\psi - \beta KX^{-\beta-1}q \tag{12}$$

(iii) the transition equation:

$$\dot{X} = q - \delta X \tag{13}$$

In addition, the transversality conditions are as follows. If $T = \infty$, we have

$$\lim_{t\to\infty} e^{-rt}\psi(t)X(t) = 0 \tag{14}$$

$$\lim_{t\to\infty} e^{-rt}\psi(t) \geq 0 \tag{15}$$

and if T is finite, we have

$$\lim_{t\to T} \psi(t) = 0 \tag{16}$$

(since $X(T)$ is free, and is positive given that $X_0 > 0$), and

$$\lim_{t\to T} H(t) = 0. \tag{17}$$

We now show that, given any $X_0 > 0$, we can construct the time path of the triplet (X, ψ, q) that satisfies conditions (11)-(15) above. Let

$$X(t) = \left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} + \frac{q^{so}}{\delta} \tag{18}$$

$$\psi(t) = KX(t)^{-\beta} = K\left[\left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} + \frac{q^{so}}{\delta}\right]^{-\beta} \tag{19}$$

$$q(t) = q^{so} \tag{20}$$

Clearly, using (18), we can verify that (13) is satisfied:

$$q^{so} - \delta X = q^{so} - \delta\left[\left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} + \frac{q^{so}}{\delta}\right] = \delta\left(X_0 - \frac{q^{so}}{\delta}\right) e^{-\delta t} = \dot{X}$$

Using (19), we can verify that (12) is satisfied:

$$\dot{\psi} = -\beta K X^{\beta-1}\dot{X} = -\beta K X^{\beta-1}\left[q^{so} - \delta X\right] = \beta\delta K X^{-\beta} - \beta K X^{\beta-1} q^{so}$$

$$= \beta\delta\psi - \beta K X^{\beta-1} q^{so}$$

$$= (r+\delta)\psi - \beta K X^{-\beta-1} q$$

Finally, using $\psi(t) = KX(t)^{-\beta}$ it is easy to see that q^{so} is the solution of

$$\max_q R(q) + (s^* - KX^{-\beta})q + \psi(q - \delta X)$$

or of

$$\max_q R(q) + (s^* - KX^{-\beta} + KX^{-\beta})q - \psi\delta X$$

in view of (10).

The time path of the triplet (X, ψ, q) constructed above converges to the steady-state triplet

$$(X_\infty, \psi_\infty, q_\infty) = \left(\frac{q^{so}}{\delta}, K\left(\frac{q^{so}}{\delta}\right)^\beta, q^{so}\right) > (0,0,0)$$

In fact, it can be shown that, in the space (ψ, X), the constructed path is the unique path that leads to the steady state pair (ψ_∞, X_∞)

$$(\psi_\infty, X_\infty) = \left(\frac{q^{so}}{\delta}, K\left(\frac{q^{so}}{\delta}\right)^\beta\right)$$

This is done by showing that the phase-diagram displays the saddle-point property (please see Appendix B) ■

Unfortunately, the result in Proposition 1 is only "half" a good news to the regulator. The possibility that the monopolist might choose the "hit and run" strategy is real. This latter strategy will be increasingly attractive when the subsidy rule offered by the regulator leaves the monopoly with a very small subsidy rate in the long-run. This is illustrated with the following example.

Example (Hit and run):

Let $p(q) = 1 - q$, $c(q) = 0$ and $r = 0.1$. Then $q^{so} = 1$ and $s^* = 1$ moreover $R(q^{so}) = 0$ and $\pi(q^{so}, s^*) = 1$. Consider the following subsidy rule

$$\sigma(X) = s^* - KX^{-\beta} = 1 - KX^{-2}$$

where $\delta = 0.1$ (which yields $\beta = 2$), and X is an index variable that follows (6) with $X(0) = X_\infty \equiv \frac{q^{so}}{\delta} = 10$. Such a subsidy rule belongs to the family of subsidy rules (7). Moreover given the choice of the initial value of the index variable the subsidy rate will be constant over time if the monopolist produces the socially desirable production rate. Let $\widehat{s}$ denote the corresponding subsidy rate. The value of $\widehat{s}$ will be determined by the choice of the parameter K :

$$\widehat{s} = 1 - \frac{K}{(10)^2}$$

The value of $\widehat{s}$ can be made arbitrarily small (close to zero) by choosing K close to 100. In the limit case where the parameter K is set at 100 we have $\widehat{s} = 0$ and choosing $q(t) = q^{so} = 1$ yields zero profits to the monopolist for all $t \geq 0$. But clearly, the monopolist can do better by setting $q(t) = 1/2$, thus earning positive profit at time $t = 0$. The next instant, $X(t)$ will fall, thus $(1 - 100X(t)^{-2})q(t) < 0$, i.e. he will have to pay a tax. But since $X(t)$ remains close to X_0 for $t \in (0, \varepsilon)$ for some small ε, he continues to earn positive profit by choosing $q(t)$ close to 1/2. After a while, he quits the industry, and his accumulated profit is positive.

In the following proposition, we give a sufficient condition (on the parameters of the subsidy rule) that guarantees that the monopolist will never choose the "hit and run" strategy and will produce the socially desirable output level.

Proposition 2: The per unit subsidy rate $\sigma(X(t))$ given by (7), where $K > 0$ and where $X(t)$ satisfies the differential equation (6) ensures that the monopolist will always produce the socially optimal output level q^{so}, provided that (K, δ, X_0) satisfies the following conditions:

$$\sigma(X_0) = s^* - KX_0^{-\beta} > \bar{s} \equiv -R'(\bar{q}) \tag{21}$$

and

$$\delta X_0 < \bar{q}. \tag{22}$$

Proof:

Part 1: Ensuring that the monopolist does not adopt the "hit and run" strategy.

We now show that when conditions (21) and (22) hold the hit and run strategy will not be chosen. To do this, we proceed in two steps:

(i): Show that the following condition is sufficient to ensure that the hit and run strategy will not be chosen:

$$R'(0) + s^* > KX(t)^{-\beta} \tag{23}$$

where $X(t)$ is the path of the index variable along the monopolist's optimal production path.

(ii) Show that the satisfaction of both conditions (21) and (22) is sufficient for the condition (23) to hold for all $t > 0$.

Step (i) If the monopolist finds it optimal to adopt the "hit and run" strategy, then at the "exit time" T, we have, from (16) and (17),

$$\lim_{t \to T} q(t) = q(T) = 0 \tag{24}$$

This condition in turn implies, via (11),

$$R'(0) + s^* - KX(T)^{-\beta} \leq 0 \tag{25}$$

In order for the monopolist not to adopt the hit and run strategy, we want to ensure that along the monopolist's optimal production path, we *never* have

$$R'(0) + s^* - KX(t)^{-\beta} \leq 0 \tag{26}$$

Clearly, a sufficient condition for this non-occurrence is (23), where $X(t)$ is the path of the index variable along the monopolist's optimal production path. This completes (i).

Step (ii) We now show that if both conditions (21) and (22) are statisfied then inequality (23) holds.

We will first establish that conditions (21) and (22) will guarantee that along the production path chosen by the monopolist the index variable $X\left(t\right)$ will remain above X_0. To do this, it is convenient to introduce the concept of a myopic monopolist. A myopic monopolist only cares about the instantaneous profit.

We note that the condition (21) $\sigma(X_0) = s^* - KX_0^{-\beta} > \bar{s}$ along with FACT 1 imply that a myopic monopolist (who by definition chooses an output level that maximizes $R(q) + \sigma(X_0)q$) will choose to produce at time $t = 0$ a positive quantity q^m that satisfies

$$R'(q^m) = -s^* + KX_0^{-\beta}$$

Here the superscript m stands for "myopic".

This first order condition determines q^m as function of X_0 (for given K, δ):

$$q^m = (R')^{-1}\left[-s^* + KX_0^{-\beta}\right] = q^m(X_0; K, \delta)$$

Clearly, a non-myopic monopolist never produces at time $t = 0$ an output level less than $q^m(X_0; K, \delta)$. The reason is that if he does, his current profit will be lower than $\pi^m \equiv R(q^m(X_0; K, \delta)) + \sigma(X_0)q^m(X_0; K, \delta)$, and his future profit will also be lower than his future profits if $q^m(X_0; K, \delta)$ was produced because, the lower output will result in a lower stock X in the future, and hence a lower future subsidy rate.

Using (21),

$$-R'(q^m) = s^* - KX_0^{-\beta} > \overline{s} \equiv -R'(\overline{q})$$

which implies that $q^m > \overline{q}$ (since $R'(q)$ is a decreasing function).

It follows that when the conditions (21) and (22) are satisfied the output of the non-myopic monopolist at time t, denoted by $\widetilde{q}(0)$, satisfies

$$\widetilde{q}(0) \geq q^m(X_0; K, \delta) \geq \overline{q} > \delta X_0 \tag{27}$$

The conditions (21) and (22) thus ensure that $\dot{X}$ will be positive when $X = X_0$. It follows that if the vector (K, δ, X_0) satisfies condition (27), the (non-myopic) monopolist will never run X to a level below X_0, and thus $X(t) \geq X_0$ always.

We now show that conditions (21) and (22) ensure that condition (23) is satisfied.

Since $X(t) \geq X_0$ for all $t > 0$, and $R'(q)$ is a decreasing function,

$$R'(0) + s^* - KX(t)^{-\beta} \geq R'(0) + s^* - KX_0^{-\beta}$$

$$> R'(\delta X_0) + s^* - KX_0^{-\beta} > R'(\delta X_0) + \overline{s} > R'(\overline{q}) + \overline{s} = 0$$

where we have made use of (21) and (22).Thus inequality (23) holds, which is a sufficient condition for the hit-and-run strategy to be non-optimal for the monopolist.

This completes Part 1.

Part 2: Part 1 and Proposition 1 complete the proof ■

Remark 1: Note that condition (21) implies that a myopic monopolist chooses a positive production (not less than $\bar{q}$) and by the same token guarantees the participation of the monopolist to the program.

Remark 2: The set of triplets (K, δ, X_0) that satisfy conditions (21) and (22) with $K > 0$ is non-empty. In particular for any $X_0 < \frac{q^{so}}{\delta}$ condition (21) gives $0 < K < \bar{K}$

$$0 < K < \bar{K} \tag{28}$$

where

$$\bar{K} \equiv (s^* - \bar{s})\, X_0^{\beta} > 0.$$

Remark 3: Conditions (21) and (22) in Proposition 2 specify a limit on the savings on the subsidy that can be achieved. These savings are constrained by the information the regulator has. The lower the value $\bar{s}$ that the regulator knows (which yields a "viable" situation to the monopoly) the larger the set of parameters (K, δ, X_0) that ensure efficiency and the larger the savings the regulator can achieve.

3.3. Interpretations

Condition (21) can be interpreted as follows. For a given pair (δ,X_0) the variable K controls for the difference between the actual subsidy rate $s\left(t\right)=\sigma\left(X\left(t\right)\right)$ and standard constant subsidy rate s^*. For a given pair (δ,X_0), the higher the value of K the greater are the savings achieved by the regulator by using the subsidy rate $s\left(t\right)$ instead of the constant subsidy rate s^*.

Let s_0 denotes the initial subsidy rate at time $t=0$ and s_∞ the subsidy rate at the steady state, i.e.,

$$s_0=s^*-KX_0^{-\beta} \text{ and } s_\infty=s^*-KX_\infty^{-\beta}$$

where $X_\infty=(1/\delta)q^{so}$. We show in appendix C that from (21) and (22) we have :

$$s^*-\left(\frac{\bar{q}}{q^{so}}\right)^{\left(\frac{r}{\delta}+1\right)}\left(s^*-s_0\right)<s_\infty \tag{29}$$

Condition (29) stipulates that for a given δ and initial subsidy rate s_0 there is a minimum subsidy rate that the regulator must provide at the steady state in order to ensure that the monopoly will not choose the hit and run strategy.

Remark 4: When feasible, the government could auction the monopoly right. The amount M that the government can collect depends on the per unit subsidy it announces. If the auction is competitive (the number of bidders is very large) and the government announces a per unit subsidy s^* the maximum bid would equal $M_{s^*}=\int_0^\infty\left(R(q^{so})+s^*q^{so}\right)e^{-rt}dt$. If the government announces a subsidy rule $\sigma(X(t))$ given by (7), where $K>0$ and where $X(t)$ satisfies the differential equation (6) and $0<K<\bar{K}$ then the maximum bid would be $M_\sigma=M_{s^*}-\int_0^\infty\left(KX^{-\beta}q^{so}\right)e^{-rt}dt<M_{s^*}$. Using such a subsidy rule reduces the amount of transfers from the bidder to the government (at the auction stage) and also reduces the transfers (production subsidies) from the government to the monopoly. This reduction in the amounts of money collected and distributed by a government may be desirable for two reasons. First, the reduction of M may increase the number of bidders, which may result in more competitive bidding (since the number of bidders who have the financial resources to bid a value M_σ is larger than the number

of bidders who can afford bidding $M_{s^*} > M_\sigma$). Second, the transfer of money from and to the government is costly due to possible leakages in the management of public funds.

3.4. A numerical example

Let $P(q) = 1 - q$ and $c(q) = 0$. Then $q^{so} = 1$ and $s^* = 1$. Furthermore any constant per unit subsidy rate larger than -1, (i.e. a per unit tax of one dollar) will induce a positive production level. For this example we assume that $\bar{s} = -0.1$ and $\bar{q} = \frac{9}{2}$. Let $r = 0.1$. If the regulator announces a subsidy scheme

$$\sigma(X) = s^* - KX^{-2}$$

where $\delta = 0.1$, $K = 20$ and $X(0) = X_0 = 2\sqrt{5}$ then the monopolist will produce at each moment the socially desirable output $q^{so} = 1$. The amount of subsidy per unit of output will monotonically increase from 0 (at time $t_0 = 0$) to 0.80 at the steady state. It can be checked that the conditions (21) and (22) are satisfied. Along the optimal production path the variable $X(t)$ is given by

$$X(t) = \left(2\sqrt{5} - 10\right)e^{-0.1t} + 10$$

and the subsidy rate is

$$s(t) = 1 - \frac{20}{\left(\left(2\sqrt{5} - 10\right)e^{-0.1t} + 10\right)^2}$$

The steady state subsidy rate with this subsidy rule is 20% below the standard constant subsidy rate s^*. The present value of the overall savings is

$$\int_0^\infty \frac{20}{\left(\left(2\sqrt{5} - 10\right)e^{-0.1t} + 10\right)^2} e^{-0.1t} dt = 4.472\,1$$

If the static subsidy rate was used the present value of the stream of subsidies is

$$\int_0^\infty e^{-0.1t} dt = 10$$

Using the dynamic subsidy rule above results in a decrease by more than 44% in the present value of the subsidies given to the monopolist.

3.5. *Some remarks about alternative schemes*

We note that other schemes can be used by the regulator to induce the monopoly to produce the socially optimal quantity[10]. For example, if the regulator has global (i.e. for all $q \geq 0$) information about consumers' willingness to pay and the production cost, then the regulator could induce the monopoly to produce q^{so} by setting a price cap where the monopoly is not allowed to charge a price higher than $p(q^{so})$ and supplement it with fixed transfers. The amount to be transferred depends on the objective of the regulator and the surplus she wishes to leave to the monopoly. Determining the amount to be transferred requires global information about the production cost. In our scheme the regulator is only required to have "local" information about the cost function, i.e. the marginal cost evaluated at the socially optimal quantity. Another alternative efficiency inducing scheme consists of giving a per-unit subsidy s^* to the monopoly, accompanied by a lump-sum tax $T = s^* q^{so}$. Such a scheme also requires only local information as the dynamic scheme we propose. However such a scheme does not guarantee the participation of the monopoly: under such a scheme the monopolist's profits could be negative[11]. Moreover when the management of money and programs by the government is costly due to possible leakages in the management of public funds, it can preferable to reduce as much as possible the amount of subsidies/taxes managed by the regulator.

We would like to note that there exists literature on the use of historical data to construct a price tax to implement when the pricing is delegated to the monopoly (see Laffont and Tirole (1993, Chapter 2) and the references therein[12]). Laffont and Tirole (1986, 1993) consider a scheme that depends on price changes that leads the monopoly to choose the socially optimal quantity in the long-run. In their scheme, even though the regulator need not know the socially optimal quantity to be produced, at the steady state the monopoly produces the efficient quantity. While our proposed schemes require knowledge by the regulator of the socially optimal quantity, they induce the monopoly to produce that quantity *at all time.*

4. Conclusion

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[10] We thank an anonymous referee for pointing out the schemes discussed in this section.

[11] We note that this problem does not arise in the simple numerical example we discuss, the main purpose of which is an illustration of the process of building the dynamic schemes we propose.

[12] The principal-agent approach to regulation originated from Baron and Myerson (1982), and Baron and Besanko (1982).

We have provided a continuum of subsidy rules that induce a monopoly to choose the socially optimal production level. These subsidy rules result, at each moment, in a reduction of the amount of subsidy transferred to the monopolist compared to the standard case where a constant subsidy rate is used. The subsidy rules depend on a state variable that reflects the monopolist's past performance and that depreciates over time, therefore requiring a permanent effort of the monopolist to maintain it at an optimal level. The use of such subsidy rules can achieve significant cost savings for the government compared to the use of a standard constant subsidy rate. We have provided a numerical example with a linear demand and no production cost, in which the subsidy costs of inducing efficiency are reduced by almost fifty percent.

The proposed class of subsidy rules can be used in different ways depending on the priorities of the regulator. A possible objective could be to achieve the socially optimal output level with the smallest amount of initial subsidy. This could be the case if the regulator initially faces severe budget constraints. Alternatively the objective could be achieving the efficient output level while meeting a target level of subsidy at the steady state, perhaps because a ceiling on long-run subsidies is imposed by some multilateral agreement.

We believe that our method of creating dynamic schemes that achieve the same production target that a benchmark static scheme achieves can be applied to other static incentive schemes. Moreover, we believe it is possible to extend our results to the case of oligopolists that play dynamic games as dynamic Cournot rivals; see Benchekroun and Long (1998) for a possible framework for a dynamic symmetric oligopoly[13]. Another worthwhile extension is to consider the case where the government cannot make long term commitment to the parameters of the subsidy rules. In such cases, a feedback equilibrium of a differential game would have to sought[14].

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[13] In an asymmetric oligopoly, complications may arise; see Long and Soubeyran (2003).

[14] For examples of feedback equilibrium see Shimomura (1991) and Dockner et al. (2000).

References

Ballard, C.L., J. B. Shoven, and J. Whalley, 1985, General Equilibrium Computation of Marginal Welfare Costs of Taxes in the United States, American Economic Review 75: 128-38.

Baron, David and David Besanko, 1984, Regulation, Asymmetric Information and Auditing, RAND Journal of Economics 15, 447-70

Baron, David and Roger Myerson, 1982, Regulating a Monopolist with Unknown Costs, Econometrica 50, 911-930.

Benchekroun, Hassan, and Ngo Van Long, 1998, Efficiency-Inducing Taxation for Polluting Oligopolists, 1998, Journal of Public Economics: 70, pp 325-342.

Bishop, M.K., J. Mayer and P. Colin, eds., The Regulatory Challenge, Oxford University Press.

Browning, Edgar K., 1976, The Marginal Cost of Public Funds, Journal of Political Economy 84: 283-98.

Dockner, Engelbert, Steffen Jorgensen, Ngo Van Long, and Gerhard Sorger, 2000, Differential Games in Economics and Management, Cambridge University Press.

Guesneries, R., and J.-J. Laffont, 1978, Taxing Price Makers, Journal of Economic Theory 19, 423-455.

Karp, Larry, 1996, Depreciation Erodes the Coase Conjecture, European Economic Review 40, 473-490.

Laffont, J.-J., 1987, Fundamentals of Public Economics, MIT Press, Cambridge, Mass.

Laffont, J.-J. and J.Tirole, 1986, Using Cost Observation to Regulate Firms, Journal of Political Economy 94, 614-641

Laffont, J.-J. and J.Tirole, 1993, A Theory of Incentives in Procurement and Regulation, Cambridge, MA: The MIT Press

Leonard, Daniel, and Ngo Van Long, 1992, Optimal Control Theory and Static Optimization in Economics, Cambridge University Press.

Loeb, Martin and Wesley A. Magat, 1979, A Decentralized Method for Utility Regulation, Journal of Law and Economics, Vol. 22, pp. 399-404.

Long, Ngo Van and Antoine Soubeyran, 2003, Selective Penalization of Polluters: an Inf-Convolution Approach, Economic Theory, to appear.

Malueg, David A. and John L. Solow, 1989, A Note on Welfare in a Durable Good Monopoly, Economica 56, 523-27.

Rees, Ray, and John Vickers, 1995, RPI-X Price Cap Regulations, in M.K Bishop, J. Mayer and P. Colin, eds., The Regulatory Challenge, Oxford University Press. pp 358-385.

Shimomura, Koji, 1991, The Feedback Equilibrium of a Differential Game of Capitalism, Journal of Economic Dynamics and Control 15(2), pp 317-338.

Sappington, David, 1980, Strategic Firm Behavior Under a Dynamic Regulatory Adjustment Process, Bell Journal of Economics, Vol. 11, No. 1, pp. 360-72.

Sappington, D and D.S. Sibley, 1988, Regulating without Cost Information: The Incremental Surplus Subsidy Scheme, International Economic Review, Vol. 29, No. 2, pp. 297-306.

Schwermer Sylvia, 1994, Regulating Oligopolistic Industries: A Generalized Incentive Scheme, Journal of Regulatory Economics, Vol. 6, No. 1, pp.97-108.

Vogelsang and Finsinger, 1979, A Regulatory Adjustment Process for Optimal Pricing by Multiproduct Monopoly Firms, Bell Journal of Economics, Vol. 10, No. 1, pp. 157-71.

Appendix A

To show that the monopolist who faces s^* would choose q^{so}, we note that, given any constant subsidy rate s, he chooses q to maximize the profit function

$$\pi(q, s) \equiv p(q)q - c(q) + sq \equiv R(q) + sq \tag{30}$$

Thus, with $s = s^*$, the first order condition of the monopolist's problem is

$$p(q) + qp'(q) - c'(q) + s^* = 0 \tag{31}$$

Clearly, q^{so}, as defined by (3), satisfies the monopolist's net-profit-maximizing condition (31) if s^* is given by (4). By the assumption of strict concavity of $R(q)$ we have

$$\pi(q^{so}, s^*) > \pi(q, s^*) \text{ for all } q \neq q^{so} \tag{32}$$

Appendix B

From (11), we know that

(i) If

$$\psi \leq KX^{-\beta} - R'(0) - s^* \tag{33}$$

then the optimal q is zero.

(ii) If

$$\psi > KX^{-\beta} - R'(0) - s^* \tag{34}$$

then the optimal q is positive and satisfies

$$R'(q) + s^* + \psi - KX^{-\beta} = 0$$

We construct a phase diagram in the space (X, ψ). Note that since $q \geq 0$ and $X_0 > 0$, X can never become negative. It follows that in the phase diagram in the space (X, ψ), there are two regions. Region A is the set of points (X, ψ) such that (33) holds, and region B is the set of points (X, ψ) such that (34) holds. The upper boundary of region A is a downward sloping curve

$$\psi = KX^{-\beta} - R'(0) - s^*$$

Along this curve, as X tends to zero, ψ tends to infinity, and as X tends to infinity, ψ tends the the negative number $-R'(0) - s^*$; at $\psi = 0$, $X = \bar{X}$ where

$$\bar{X} = \left[\frac{R'(0) + s^*}{K}\right]^{1/\beta}$$

In region B, we can write

$$q = q(X, \psi)$$

where

$$\frac{\partial q}{\partial X} = \frac{H_{qX}}{-H_{qq}} = -\phi'(Z)\beta K X^{-\beta-1} > 0$$

$$\frac{\partial q}{\partial \psi} = \frac{H_{q\psi}}{-H_{qq}} = \frac{1}{-R''} = -\phi'(Z) > 0$$

In fact, in region B, where $KX^{-\beta} - s^* - \psi < R'(0)$,

$$q(X, \psi) = R^{'-1}(KX^{-\beta} - s^* - \psi) \tag{35}$$

Define

$$\phi(.) = R^{'-1}(.)$$

and

$$Z \equiv KX^{-\beta} - s^* - \psi$$

We now show that there exists a unique steady state in region B. Denote the steady state pair by (X_∞, ψ_∞). Then, setting $\dot{\psi} = 0$, we get

$$\psi_\infty = \left(\frac{\beta}{r + \delta}\right) K X_\infty^{-\beta-1} q(X_\infty, \psi_\infty) \tag{36}$$

And setting $\dot{X} = 0$, we get

$$q(X_\infty, \psi_\infty) = \delta X_\infty \tag{37}$$

Substituting (37) into (36), we get

$$\psi_\infty = K X_\infty^{-\beta} \tag{38}$$

Using (35), we can write (37) as

$$KX_{\infty}^{-\beta} - s^* - \psi_{\infty} = R'(\delta X_{\infty}) \tag{39}$$

Substituting (38) into (39) we get

$$-s^* = R'(\delta X_{\infty}) \tag{40}$$

Comparing (10) with (40) we deduce that

$$\delta X_{\infty} = q^{so} \tag{41}$$

It follows that the unique steady state is

$$(X_{\infty}, \psi_{\infty}) = \left(\frac{q^{so}}{\delta}, \frac{K}{(q^{so}/\delta)^{\beta}} \right) \tag{42}$$

Now we show that the steady state has the saddlepoint property. Define

$$M(\psi, X) = (r+\delta)\psi - \beta K X^{-\beta-1} q(\psi, X)$$

and

$$N(\psi, X) = q(\psi, X) - \delta X$$

We must show that the matrix

$$\begin{bmatrix} M_{\psi} & M_X \\ N_{\psi} & N_X \end{bmatrix}$$

(evaluated at the steady state) has a negative determinant (.i.e., has one negative root and one positive root).

$$M_{\psi} = r + \delta + \beta K X^{-\beta-1} \phi'(Z)$$

$$M_X = \beta(1+\beta) K X^{-\beta-2} q + \beta^2 K^2 X^{-2\beta-2} \phi'(Z)$$

$$= \delta\beta(1+\beta) K X^{-\beta-1} + \beta^2 K^2 X^{-2\beta-2} \phi'(Z)$$

$$N_{\psi} = -\phi'(Z)$$

$$N_X = -\phi'(Z) \beta K X^{-\beta-1} - \delta$$

Thus

$$M_\psi N_X = -\left(r+\delta+\beta K X^{-\beta-1}\phi'(Z)\right)\left(\phi'(Z)\beta K X^{-\beta-1}+\delta\right)$$

$$= -r\left(\phi'(Z)\beta K X^{-\beta-1}+\delta\right)-\left(\delta+\beta K X^{-\beta-1}\phi'(Z)\right)^2$$

$$M_\psi N_X = -\left[\phi'(Z)\beta K X^{-\beta-1}+\delta\right]^2 - r\delta - r\phi'(Z)\beta K X^{-\beta-1}$$

$$= -\left[\phi'(Z)\beta K X^{-\beta-1}\right]^2-\delta^2-r\delta-\{2\delta+r\}\beta\phi'(Z)KX^{-\beta-1}$$

and

$$N_\psi M_X = (-\phi')\delta\beta(\beta+1)KX^{-\beta-1}-\left[\phi'(Z)\beta K X^{-\beta-1}\right]^2$$

Therefore,

$$M_\psi N_X - N_\psi M_X = -\left[\phi'(Z)\beta K X^{-\beta-1}\right]^2-\delta^2-r\delta$$

$$-\{2\delta+r\}\beta\phi'(Z)KX^{-\beta-1}+\phi'\delta\beta(\beta+1)KX^{-\beta-1}+\left[\phi'(Z)\beta K X^{-\beta-1}\right]^2$$

$$M_\psi N_X - N_\psi M_X = -\delta^2-r\delta-\{2\delta+r-\delta(\beta+1)\}\beta\phi'(Z)KX^{-\beta-1}$$

$$M_\psi N_X - N_\psi M_X = -\delta^2-r\delta-\{\delta+r-\delta\beta\}\beta\phi'(Z)KX^{-\beta-1}$$

recalling that $\beta\delta = r+\delta$,

$$M_\psi N_X - N_\psi M_X = -\delta^2 - r\delta < 0$$

We have thus shown that the steady state $(X_\infty, \psi_\infty) = \left(\frac{q^{so}}{\delta}, \frac{K}{(q^{so}/\delta)^\beta}\right)$ has the saddlepoint property ■

Appendix C:

Solving the differential equation (6) along the optimal production path and using

$$X\left(0\right) = X_0$$

yields

$$X\left(t\right) = \left(X_0 - \frac{q^{so}}{\delta}\right)e^{-\delta t} + \frac{q^{so}}{\delta}$$

Let s_0 and s_∞ denote the initial subsidy rate and the steady state subsidy rate, we have

$$s^* - s_\infty = KX_\infty^{-\left(\frac{r}{\delta}+1\right)} \text{ and } s^* - s_0 = KX_0^{-\left(\frac{r}{\delta}+1\right)}$$

combining these two equations yields

$$s^* - s_\infty = K X_\infty^{-\left(\frac{r}{\delta}+1\right)} = K X_0^{-\left(\frac{r}{\delta}+1\right)} \left(\frac{X_0}{X_\infty}\right)^{\left(\frac{r}{\delta}+1\right)}$$

that is

$$s^* - s_\infty = (s^* - s_0) \left(\frac{X_0}{X_\infty}\right)^{\left(\frac{r}{\delta}+1\right)}$$

Using (22) (i.e. $\frac{\bar{q}}{\delta} > X_0$) and the fact that $X_\infty = q^{so}/\delta$, the last equation gives along with (21) (i.e. $s_0 < \bar{s}$) and $\bar{s} < s^*$

$$s^* - s_\infty = (s^* - s_0) \left(\frac{X_0}{X_\infty}\right)^{\left(\frac{r}{\delta}+1\right)} < (s^* - s_0) \left(\frac{\bar{q}}{q^{so}}\right)^{\left(\frac{r}{\delta}+1\right)}$$

or

$$s_\infty > s^* - (s^* - s_0) \left(\frac{\bar{q}}{q^{so}}\right)^{\left(\frac{r}{\delta}+1\right)} \quad \blacksquare$$