A Comparative Study of GARCH (1,1) and Black-Scholes Option Prices*

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Abstract

This paper examines the behaviour of European option price (Duan (1995)) and the Black-Scholes model bias when stock returns follow a GARCH (1,1) process. The GARCH option price is not preference-neutral and depends on the unit risk premium (λ) as well as the two GARCH (1,1) process parameters (α_1 , β_1). In general, the GARCH option price does not seem overly sensitive to these parameters. Deep-out-of-the-money and short maturity options are an exception. The variance persistence parameter, $\gamma = \alpha_1 + \beta_1$, has a material bearing on the magnitude of the Black-Scholes model bias. The risk preference parameter, l, on the other hand, determines the so called "leverage effect" and can be important in determining the direction of the Black-Scholes model bias. Consequently, a time varying risk premium (l) may help explain a general underpricing or overpricing of traded options (Black (1975)).

Consistent with "volatility smile" and similar to the bias noted by Merton (1976), deep-out-of-the-money and deep-in-the-money (at-the-money) options with a very short time to expiration are found to be underpriced (overpriced) by the Black-Scholes model. The direction of striking price bias for longer maturities is mostly influenced by GARCH option valuation parameters, a result that could be useful in resolving conflicting striking price biases observed empirically.

This paper makes a novel attempt to decompose the Black-Scholes model bias into components related to three important features of GARCH option valuation: level of the unconditional variance of the locally risk-neutral return process, relative level of the initial conditional variance, and path dependence of the terminal stock price distribution. An analysis of their behaviour sheds light on the making of the overall systematic biases mentioned above as well as the time to maturity bias reversal phenomenon (Rubinstein (1985) and Sheikh (1991)).

One modification to the Black-Scholes model that corrects only the unconditional variance bias does not improve accuracy enough to justify the additional input requirement. Another modification corrects the unconditional variance bias and the conditional variance bias, but not the path dependence bias. This latter modification, which we call the Pseudo-GARCH or PGARCH formula, performs rather well when the impact of a given period's variance innovation is low (small α_1) but nearly permanent (γ close to 1.0). In these empirically relevant situations, the Black-Scholes type PGARCH formula offers practical approximations to the theoretically correct simulated GARCH prices.

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A key determinant of option value is the variance of the underlying asset. Variance also affects the sensitivity of option value with respect to the asset price, option's time to maturity, interest rate, and variance itself. Thus, in principle, an accurate specification of variance could have significant bearings on the valuation of options, and the calculation of option value sensitivities.

In deriving their seminal European option valuation formula, Black and Scholes (1973) assumed a normal diffusion process for the stock return with a constant instantaneous variance. If the instantaneous variance is a deterministic function of time, the average instantaneous variance over the life of the option can instead be used in the Black-Scholes formula (Merton (1973)). However, neither constancy nor a time-deterministic behaviour is supported by empirical studies.[1] **A type of variance behaviour which has gained widespread acceptance in the literature is Generalized Autoregressive Conditional Heteroskedasticity or GARCH (Engle (1982), Bollerslev (1986)).**

Using a discrete time equilibrium asset pricing framework (Rubinstein (1976), Brennan (1979)), Duan (1995) has recently developed a European stock option valuation model when the continuously compounded stock returns follow a GARCH process. Duan's model contains the Black-Scholes model as a special case of homoskedasticity. As shown by Duan (1994a,1994b), the basic GARCH option pricing framework can be extended or generalized to handle alternative specifications of conditional variance such as Nelson's (1991) EGARCH, stochastic interest rates, and the bivariate stochastic variance diffusion cases (Hull and White (1987), Wiggins (1987), Scott (1987), Stein and Stein (1991), and Heston (1993)) in the limit.

In this paper we use simulations to examine the comparative behaviour of the Black-Scholes (BS) and Duan's (1995) GARCH stock option prices. The recently proposed Empirical Martingale Simulation (EMS) method of Duan and Simonato (1995) is combined with standard simulation methods to generate the GARCH option prices. We also propose two modifications to the Black-Scholes formula and examine their accuracy in tracking the GARCH option price. One modification, which we shall refer to as the Modified Black-Scholes formula (MBS), uses the unconditional variance under the locally risk-neutral GARCH process of Duan (1995) in the Black-Scholes formula. Under the other modification, called the Pseudo-GARCH (PGARCH)

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e, for example, Fama (1965), Black (1976), Merton (1980), Christie (1982), Poterba and Summers (1986), French, Schwet, and Stambaugh (19 Schwert (1989).

formula, the expected average conditional variance under the same process is inserted into the Black-Scholes formula.

Our results show that the GARCH option valuation effect, i.e., the divergence between the BS and the GARCH option price, varies widely from a negligible magnitude to a substantial one. The same can be said about the accuracy of the proposed formulas (MBS, PGARCH). Further, the sampling error associated with the simulated GARCH option price varies with parameter combination. The results of this study should thus help a researcher or practitioner in assessing the importance of GARCH in a given option valuation situation and in selecting the BS, MBS, or PGARCH as an approximation if desired.

In some early works following the Black-Scholes model, heteroskedasticity was incorporated in the form of stock-price-dependent variance (Cox and Ross (1976)), jumps in the stock price which otherwise follows a lognormal diffusion process (Merton (1976)), and dependence of the stock return variance on leverage (Geske (1979) or asset structure (Rubinstein (1983)). More recently, heteroskedasticity of the optioned asset's returns and its effect on option valuation have been the primary focus in a number of papers where the asset return variance is modelled as a separate stochastic variable from the asset return. A majority of these works assume a bivariate diffusion model.[2] In a similar vein, Madan and Seneta (1990) assume a gamma distribution for asset return variance. All models are in continuous form.

Modelling heteroskedasticity has several notable implications for option valuation in theory as well as in practice. First, even in the limit (continuous time), Black-Scholes type preference-free option valuation no longer prevails; an equilibrium asset pricing relationship is needed to determine an unique option value.

Second, the unconditional distribution of asset return is no longer lognormal even when the conditional distribution is lognormal. In most cases, the unconditional asset return distribution is either quite intricate or analytically intractable. Thus a series approximation or the use of a Monte Carlo simulation method is usually required to calculate the option value. These methods are certainly computationally more involved than using a Black-Scholes type formula. Further, the simulated option price is subject to sampling error and the series approximation

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See Hull and White (1987, 1988a,1988b), Johnson and Shanno (1987), Wiggins (1987), Heston (1993), Melino and Turnbull (1990), Scott (1
sney and Scott (1989), Finucane (1989), and Stein and Stein (1991). A stochastic volatility framework has been closely examined by Lee, Lee,
(1991), Finucane (1994), and Hull and White (1993).

method also leads to approximation error even when the series is convergent.[3] Similar comments also apply to calculation of the hedge ratio and other option value sensitivities such as the theta or the gamma.

Third, for bivariate diffusion models, parameter estimation could be a challenging task as the asset return variance is not directly observable. While the GARCH parameters can be estimated using historical returns, an additional assumption needs to be made about parameter stability and sometimes estimation may not converge (Figlewski (1994)).

Fourth, it is not clear how a practitioner could imply volatility expectations by the market from the observed option prices. Even if it was possible to invert a theoretical option valuation model that incorporates heteroskedasticity (see Engle and Mustafa (1992) for an attempt), one could at best hope to estimate the implied parameter(s) of the assumed volatility process but not the implied volatility in the traditional sense. While some researchers (Day and Lewis (1992), Harvey and Whaley (1992), Stein (1989)) have examined the behaviour of the volatilities implied by the Black-Scholes model, the results are in general tenable only to the extent that the Black-Scholes model produces reasonable estimate of the theoretical option price incorporating heteroskedasticity.[4]

The above considerations lead to some interesting issues. First, how important is the preference-based nature of option pricing under heteroskedasticity? In other words, are option prices sensitive to the preference related parameter? We may also ask the same question about the heteroskedasticity related parameters. Answers to these questions may lead to simpler approximations to the theoretical valuation model or they may identify parameters which should be estimated with care by practitioners in implementing the model.

Second, relative to the Black-Scholes model, is there any systematic pattern in the option valuation effect of heteroskedasticity? Is it possible to decompose the effect into components that can be identified with specific aspects of the option valuation model? [5] If so, the

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all and Roma (1994, p.597) note potential instabilities when using a Hull and White (1987) type third-order series approximation.

See, for example, footnote 3 of Stein (1989), p.1012.

or example, in the stochastic variance option valuation model of Hull and White (1987), the unconditional distribution of the terminal stock p t lognormal. Further, when the stock return and the change in its variance rate are instantaneously correlated, the conditional (upon an ave ance rate) distribution of the terminal stock price is not lognormal either and its mean depends on the specific path followed by the variance s two specific aspects of the Hull and White (1987) model are the departure from lognormality and the path dependence of the stochastic vari on price.

decomposition may offer insights into the option valuation effect of heteroskedasticity and the observed empirical biases of the BS model.

Third, when is the Black-Scholes formula (with a constant variance) in gross error so that a more complex option valuation model incorporating heteroskedasticity may be worth pursuing despite the accompanying sampling or approximation errors? The 'when' can of course be defined according to the option types (call, put), option features such as moneyness and time to maturity, the option exercise rule (European, American), the parameters of the variance process, the preference or equilibrium asset pricing parameter(s), and the riskfree rate or parameters of an interest rate process.

Fourth, if practitioners continue to rely on the Black-Scholes formula (with a single variance measure) because of its convenience and intuitive appeal, is it worthwhile to use a single variance measure that attempts to capture the nature of assumed heteroskedasticity?

In this paper, we address the above issues, focusing exclusively on the European stock option valuation problem when the stock returns follow a GARCH (1,1) process. While Duan's (1995) GARCH option valuation model encompasses higher order GARCH processes, GARCH (1,1) seems by and large to be a popular choice in modelling volatility behaviour parsimoniously.[6]

Regarding the first and third issues, Duan presents simulation results for only one set of parameter values for the GARCH(1, 1) process. In this paper, we present a more comprehensive set of simulation results with respect to the GARCH parameters, namely, the unit risk premium and the two slope parameters of the GARCH (1,1) process. In general, we do not find the GARCH option price or the BS model bias to be overly sensitive to the preference parameter except for the deep-out-of-the-money options with a very short time to expiration. This latter group of options are also the ones where the BS model bias and the simulation sampling error of the GARCH price are the largest in percentage terms.

In most cases, the two GARCH (1,1) process slope parameters have similar effects on the GARCH price, the simulation error, and the bias of the Black-Scholes model, when one of the two parameters is held constant. At higher levels of either parameter, the simulation error is relatively higher and most notably the bias of the Black-Scholes model can be substantial. This

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'hat low-order GARCH models describe stock return volatility behaviour very well is shown by Akgiray (1989) and Pagan and Schwert (1
ng others.

suggests that in general the sum of the two parameters, viz., the variance persistence parameter, γ plays an important role and needs to be estimated carefully.

The issue of using a modified variance estimate in the Black-Scholes formula is motivated by the continued widespread use of the formula by practitioners despite the growing evidence of heteroskedasticity in stock returns. If a practitioner, aware of the possible temporal variations in the variance, wishes to use the Black-Scholes formula, more efforts are likely to be made to capture the temporal behaviour in the single variance estimate to be used in the formula.

If the variance behaviour is presumed to be of GARCH type, one such estimate would be the unconditional variance under Duan's locally risk-neutral GARCH process. This leads to our MBS (Modified Black-Scholes) formula. The BS formula in Duan's paper, on the other hand, uses the unconditional variance of the assumed GARCH process. These two variance measures differ in that the former is affected by the preference parameter while the latter is not.

Like the BS formula, the MBS formula ignores the conditional nature of the variance. A variance estimate which attempts to capture this conditional nature is the average of the expected conditional variances at various points during the life of the option. Noh, Engle, and Kane (1994) have recently found that using an average GARCH (1,1) variance forecast in the Black-Scholes formula returns greater profits from trading straddles on the S&P 500 Index than using an implied standard deviation (ISD) (Whaley (1982), Day and Lewis (1988)). However, they have not examined the average expected conditional variance under the locally risk-neutral pricing measure of Duan (1995). Our PGARCH (Psuedo-GARCH) formula uses this variance measure in the Black-Scholes formula. Previously, Heynen, Kemna, and Vorst (1994) found that the average expected volatility under Duan's locally risk-neutral measure is close to the implied standard deviation which equates the Black-Scholes price to Duan's GARCH option price for at-the-money and near-the-money options. But they do not report simulation results for deep-out-of-the-money and deep-in-the-money options. Their report is also limited to just one set of values for the preference and GARCH process parameters.

Our simulation results suggest that the MBS formula does not offer particular benefit over the BS formula given the additional input requirements (unit risk premium, GARCH process parameters). The PGARCH formula, however, improves substantially over the BS formula when the persistence in variance is high (γ close to 1.0, nearly integrated variance) and when

the options are at-the-money or out-of-the-money. In the nearly integrated variance situations the PGARCH error is under 5% (with the exception of very short maturity deep-out-of-the-money options) and thus offers a computationally attractive alternative to the more accurate simulated GARCH prices.

Additionally, an important benefit of the two new formulas considered in this paper, viz., MBS and PGARCH, is that they allow a rough breakdown of the GARCH option valuation effect (difference between BS and GARCH) into three components: the effect of change in the unconditional variance under Duan's locally risk-neutral measure (BS - MBS), the effect of the conditional nature of the variance process (MBS - PGARCH), and the nonlinear and path-dependent nature of GARCH option pricing (PGARCH - GARCH). An examination of these effects offers a number of useful insights into GARCH option valuation and the associated bias of the Black-Scholes model.

Our simulation results indicate that the three components of the BS model bias are not always of the same sign. Their relative importance (magnitude) also varies across different option valuation situations. The interaction of these factors leads to the determination of the direction of the BS model bias in a given option valuation situation. These include the "smile effect", the conflicting striking price biases (Black (1975), MacBeth and Merville (1979), and Rubinstein (1985)), and the general overpricing or underpricing bias (Black (1975)).

The rest of this paper is organized as follows. In section I, we first review Duan's (1995) GARCH option pricing model, then discuss conceptually the BS model bias and its three components, and finally describe the MBS and the PGARCH valuation formulas. Section II delineates the design of our simulation study. The simulation results are presented in section III. Lastly, concluding remarks are given in section IV.

I. GARCH Option Pricing

A. Duan's (1995) GARCH (1,1) Option Valuation Model

Let S_t be the stock price at time t. The one-period stock price relative is assumed to follow the following process:

$$S_t/S_{t-1} = \exp(r + l\sqrt{h_t} - 0.5\, h_t + e_t)$$

where e_t has a normal distribution with mean 0 and conditional variance h_t under probability measure P; r is the constant one-period continuously compounded risk-free rate; and l is the constant unit risk premium on the stock. It is further assumed that h_t, the conditional variance of e_t, follows a GARCH (1,1) process of Bollerslev (1986) under measure P:

$$e_t \,|f_{t-1} \sim \; N(0, \; h_t)$$

$$h_t = a_0 + \alpha_1 e^2_{t-1} + \beta_1 h_{t-1}$$

where f_{t-1} is the s-field generated by all information up to and including time t - 1; $a_0>0$, $\alpha_1>0$, $\beta_1> 0$. To ensure covariance stationarity of e_t, $\gamma\,(=\alpha_1+\beta_1)$ is assumed be less than one. The lognormal process for the stock price with a constant variance is a special case of GARCH (1,1) with $\alpha_1=0$, $\beta_1= 0$. The parameter, g, can be viewed as a measure of the persistence of shocks to the conditional variance. [7] A high value of g means a very slow rate of decay for the effect of any innovation in the conditional variance process on the future conditional variances. [8] This may cause the conditional variance to deviate from its long-term mean (stationary level) for a long time. The degree of persistence or accumulation of innovations in the conditional variance process as typified by the parameter, g, could thus have important implications for option valuation in a GARCH (1,1) environment.

The slope parameter, α_1, measures the marginal impact of the most recent innovation in the conditional variance. The slope parameter, β_1, on the other hand, captures the combined marginal impacts of the lagged innovations. Empirical studies of financial returns show that the β_1 estimates are markedly higher than α_1 estimates, i.e., variance persistence is often characterized by a low but prolonged effect of variance innovation in a given period [9]. This corresponds to a low but slowly decaying autocorrelation of squared returns (Taylor (1986)). Roughly speaking, α_1's primary impact is on the degree of autocorrelation while β_1's primary impact is on the decay of autocorrelation through the persistence parameter, γ. While the rate of decay underscores the importance of the conditional nature of the variance process, a higher α_1 increases the conditional kurtosis of multiperiod returns and can thus have important effect on option values (Engle and Bollerslev (1986), Engle and Mustafa

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See Engle and Mustafa (1992, p.292) and Bollerslev, Chou, and Kroner (1992) for discussions on the persistence of volatility shocks. Bollerslev (1 5) shows that the second and higher order autocorrelations of shocks to the conditional variance are increasing in the persistence parameter, γ. Empiric legree of persistence in the stock return variance seems to be related to the size of the firm with larger stocks exhibiting a greater degree of persistence ler stocks (Engle and Mustafa (1992), Engle and Gonzalez-Rivera (1991), Schwert and Seguin (1990)).

Nelson (1990, p.325) points out that the *persistence* in conditional variance in the sense of Engle and Bollerslev (1986) actually means a near perma t on the forecast moments of the conditional variances of future periods.

See, e.g., Taylor (1986), Akgiray (1989), Lamoureux and Lastrapes (1990), Ng (1991), Engle and Mustafa (1992), and Heynen and Kat (1994).

(1992)). In the GARCH model, there is just one source of randomness. Unlike the bivariate diffusion models, the return volatility over the next period of time is known with certainty in the GARCH model, given the information set f which includes the current and past realizations of the stock returns. This allows Duan (1995) to define an equilibrium price measure Q which is absolutely continuous with respect to the measure P, and under which one plus the conditionally expected stock return is exp(r) instead of exp(r+ l$\sqrt{h_t}$); the conditional variance, however, remains the same almost surely as under measure P. [10] Since the conditional mean of one plus the one-period stock return under Q is independent of any preference-related parameter and is equal to exp(r), measure Q is said to satisfy a locally risk-neutral valuation relationship (LRNVR). In the case of a constant variance, the LRNVR reduces to the conventional risk-neutral valuation relationship. It is important to note that measure Q does not lead to global risk neutralization.

Under pricing measure Q, the stock return process is as follows:

$$\ln (S_t / S_{t-1}) = r - 0.5\, h_t + z_t$$

$$z_t \,|f_{t-1} \sim N(0, h_t) \text{ and}$$

$$h_t = a_0 + \alpha_1 (z_{t-1} - l\sqrt{h_{t-1}})^2 + \beta_1 h_{t-1}$$

Note that the above conditional variance model is in fact the Nonlinear Asymmetric GARCH model of Engle and Ng (1993) and is a special case of the Generalized Asymmetric GARCH family of Hentschel (1995). In this model, for $\lambda>0$, conditional variance is negatively related to lagged return, i.e., the volatility impact of a negative news (return surprise) is greater than that of a positive news. This asymmetric volatility effect is sometimes referred to as the "leverage effect". Thus, Duan's results show that options on an asset following the traditional linear and symmetric GARCH model should be valued as if the asset follows the Nonlinear Asymmetric GARCH model instead with the risk premium parameter, λ, affecting the degree of departure.

By repeated substitution, h_t can be expressed as a function of the lagged values of a non-central chi-square variable z_t^2 with the unit risk premium, l, being the non-centrality parameter:

h_t~=~h_0`G_t~+~alpha_0`sum from {k`=`0} to {t`-`1}`G_k

where $z_t = (z_{t-1}/\sqrt{h_{t-1}}) - l$, and $G_k = G_{k-1} (\alpha_1 z_{t-k}^2 + \beta_1)$, and $G_0 = 1$.

The terminal stock price under measure Q can be expressed as:

Duan's (1995) GARCH option valuation model is for a general GARCH(p,q) process. In this paper, we limit our attention to the GARCH (1,1) pro

S_T~=~S_t`exp left [`(T~-~t`)`r~-~0.5`sum from {s`=`t} to T`h_s~+~sum from {s`=`t} to T` zeta_s``right]

The value of a European call option with strike price X is obtained by taking conditional expectation of the terminal payoff under measure Q and then discounting at the risk-free rate:

GARCH: $C_t^{GH} = \exp(-r\,(T-t))\, E^Q[\, \max(S_T - X, 0) \mid f_t\,]$

The put option value can be calculated using the European put-call parity relationship. In this paper, we only focus upon call options.

B. The Black-Scholes Model vs. Duan's GARCH (1,1) Option Valuation Model

The Black-Scholes formula for a European call option is a special case of GARCH option valuation where the variance rate is constant through time, i.e., $h_t = s_P^2$ for all t. In this situation, the call option formula assumes the familiar form:

BS: $C_t^{BS} = S_t\, N(d_1) - X \exp(-r(T-t))\, N(d_2)$

where $d_1 = [\ln(S_t/X) + r(T-t) + 0.5\, s_P^2(T-t)] / s_P\sqrt{(T-t)}$, $d_2 = d_1 - s_P\sqrt{(T-t)}$, and s_P^2 is the unconditional variance under measure P and is calculated as $s_P^2 = a_0/(1 - \alpha_1 - \beta_1)$.

If the variance follows a lognormal diffusion process as in Hull and White (1987) and others, the terminal stock price is lognormally distributed conditional on the path followed by the variance. When the stock price is instantaneously uncorrelated with the variance, this conditional lognormal distribution depends on the average variance only and is not affected by other attributes of the variance path. Thus, conditional upon an average variance, the option price is merely the Black-Scholes price; the unconditional or final option price is then the expected Black-Scholes price, with the expectation being taken over the distribution of the average variance.[11] However, the distribution of the average variance is not lognormal, and Hull and White propose a Taylor series approximation involving the moments of this distribution.

When the stock price and the variance are correlated, the mean of the conditional lognormal distribution of the stock price depends on the specific path followed by the variance and not just on the average variance along the path. The option price conditional upon the path followed by the variance is

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[11] A similar conclusion has been drawn by Amin and Ng (1993), Stein and Stein (1991), Madan and Seneta (1990).

no longer the Black-Scholes price (with the average variance inserted). In this case, Hull and White (1987) use simulation to calculate the option price.

In Duan's GARCH option pricing model, the stock return variance over the next time period is known with certainty conditional upon the current information set, and hence the conditional distribution of one-period ahead stock return is lognormal with a known conditional variance. Beyond the immediate period, the one-period conditional variances evolve stochastically according to the assumed GARCH process. While the distribution of a future-period conditional variance may be tractable according to H-function properties (Springer (1979)), it is certainly not lognormal. Hence, unlike the bivariate diffusion models, the joint distribution for the logarithmic stock price and the logarithmic (conditional) variance is not bivariate normal; accordingly, the distribution of the terminal stock price is unlikely to be lognormal given the path of the one-period conditional variance.

Further, local risk-neutralization in Duan's model increases the unconditional stock return variance; it also induces correlation between the conditional variance and the lagged stock return when the unit risk premium parameter, l, is non-zero.

To better understand the relationship between the BS option price and the GARCH (1,1) option price, assume, for a moment, the following option valuation conditions prevail:

(a) any correlation between stock return and conditional variance can be ignored (say, l is small);

(b) the time to maturity of the option is quite long ; and

(c) the conditional distribution of the terminal stock price under measure Q can be reasonably approximated by a lognormal distribution (say, the GARCH slope parameters are small);

Then, due to (a) and (c), following Hull and White's (1987, pp.285-286) logic, the GARCH option price is approximately the expected Black-Scholes price, with the expectation taken over the distribution of average conditional variance under measure Q. Using Hull and White's series approach, the GARCH option price can thus be calculated as a function of the moments of the average conditional variance. Because of (b), the expected average conditional variance would be fairly close to the unconditional variance under measure Q and the second- and higher-order moments of the average conditional variance would be negligible.[12] Given a small or negligible unit risk premium in (a), the unconditional variance under measure Q would be roughly the same as that under the original probability measure P. Consequently, the BS price would be fairly close to the GARCH price.

For an at-the-money option, the derivatives would be small too.

If condition (a) is not applicable, the BS price would deviate from the GARCH price for two reasons. First, as mentioned by Duan (1995), the BS price is based upon an incorrect unconditional variance, viz., the measure P unconditional variance. Since the measure Q unconditional variance is higher for a nonzero l, this reason by itself would cause the BS price to be lower than the GARCH price for any degree of moneyness and any initial conditional variance situation. Let us call this GARCH option valuation effect the u.v. (unconditional variance) bias. Second, the GARCH price would deviate from the expected BS price as a function of the average conditional variance. This second GARCH option valuation effect, which we shall refer to as the p.d. (path dependence) bias, would also result when condition (c) is violated. The direction of the p.d. bias is not immediately clear.

If condition (b) does not hold, i.e., the option in question has a short time to expiration, the initial conditional variance situation is likely to play an important role. If the initial conditional variance is lower than the measure Q unconditional variance, the conditional variances through maturity would tend to stay below the measure Q unconditional variance. One clear implication is that the expected average conditional variance would be lower than the measure Q unconditional variance. Similar results apply when the initial conditional variance is high.[13] Hence even if we were to use the measure Q (as opposed to measure P) unconditional variance in the BS formula, a pricing difference between the BS and GARCH option valuation models would arise due to a lack of recognition of the initial condition. This GARCH option valuation effect shall henceforth be referred to as the c.v. (conditional variance) bias.

Note that the three biases that we have identified are not necessarily of the same sign for a given option. For example, consider a situation where the initial conditional variance is below measure Q (as well as measure P) unconditional variance. The u.v. bias would always cause the BS price to be low. The c.v. bias, on the other hand, is likely to cause the BS price to be high. As we shall see later, the direction of the p.d. bias is ambiguous and depends upon other option valuation parameters or variables such as moneyness and time to maturity of the option. Thus the direction of the overall GARCH option valuation effect is not uniform. Our simulation study considers variation in a broader set of parameters and is expected to shed light on this situation-specific nature of the overall GARCH option valuation effect and its components. An additional area where we hope to gain insight is the relative importance of the three GARCH effects.

Higher-order moments of the average conditional variance would also likely depend on the initial condition.

C. Modified Black-Scholes (MBS) and Pseudo-GARCH (PGARCH) Models

Depending upon the specific option valuation situation, it is possible that either the BS model or the BS formula with some GARCH-based variance figure would yield option value not too far from Duan's GARCH option value. Duan's simulation results already bear some evidence in this regard. In 49 of the 63 cases that Duan reports, the GARCH option price is within 5% of the BS model (with measure P unconditional variance) value. Motivated by such possibilities, we explore in this paper two other variants of the BS model, viz., the Modified Black-Scholes (MBS) model and the Pseudo-GARCH (PGARCH) model. Both use the Black-Scholes functional form. These models differ from the conventional BS model in that measure Q-based variance figures are inserted in the Black-Scholes pricing function:

MBS: $C_t^{MBS} = S_t\, N(d_1) - X \exp(-r(T-t))\, N(d_2)$

where $d_1 = [\ln(S_t/X) + r(T-t) + 0.5\, s^2(T-t)] / s\sqrt{(T-t)}$, $d_2 = d_1 - s\sqrt{(T-t)}$, s^2 is the unconditional variance under measure Q and is calculated as $s^2 = a_0 / [1 - \alpha_1(1+l^2) - \beta_1]$; and

PGARCH: $C_t^{PGH} = S_t\, N(d_1) - X \exp(-r(T-t))\, N(d_2)$

where $d_1 = [\ln(S_t/X) + r(T-t) + 0.5\, v(T-t)] / \sqrt{v(T-t)}$, $d_2 = d_1 - \sqrt{v(T-t)}$, and v is the average expected conditional variance under measure Q. The average expected conditional variance over an n-day period is calculated as follows. Let h_0 be the known initial variance. Define $D = \alpha_1(1+l^2) + \beta_1$. Then, as shown in Duan (1995), the expected conditional variance for day *k* is given by

E`(h_k`)~=~h_0`DELTA^{k}~+~alpha_0`{1~-~DELTA^k} over {1~-~DELTA}.

Therefore the average annualized expected conditional variance over the n-day period is:

360 over n``sum from {k`=`1} to n`E`(h_k`)~=~{alpha_0`n} over {1~-~DELTA}~+~left [`h_0~-~alpha_0 over {1~-~DELTA}

right]`{DELTA``(1~-~DELTA^n`)} over {1~-~DELTA}.

The MBS model attempts to eliminate the u.v. bias of the BS model by using the unconditional variance under measure Q which is greater than that under measure P. The MBS price is thus always higher than the BS price. The PGARCH model attempts to eliminate both the u.v. bias and the c.v. bias. It does so by using the average expected conditional variance under measure Q, v, which is lower than, equal to, or greater than s^2 depending upon whether h_1 is lower than, equal to, or greater than s^2. Accordingly, the PGARCH price is lower than, equal to, or greater than the MBS price depending upon whether h_1 is lower than, equal to, or greater than s^2. Since the PGARCH model is subject only to the path dependence bias, it is expected to track the true GARCH price closely in option valuation situations where the path dependence (unconditional variance plus conditional variance) bias is a relatively small (large) part of the overall bias of the Black-Scholes formula.

The MBS and PGARCH prices allow us to estimate the three components of the BS model bias. This can be seen by expressing the BS model bias as follows:

BS - GARCH = (BS - MBS) + (MBS - PGARCH) + (PGARCH - GARCH)

The u.v. bias can be estimated as (BS - MBS), the c.v. bias can be estimated as (MBS - PGARCH), and the p.d. bias can be estimated as (PGARCH - GARCH).

II. Simulation Design

In this paper, we simulate a total of 525 option valuation situations. An option valuation situation is a specific combination of the values of the initial conditional variance (h_1), the unit risk premium (l) , the GARCH process slope parameters (α_1, β_1), the moneyness (S/X) and the time to maturity (T) of the option. In all 525 cases, we assume r = 0 and s = 0.25. (σ is the unconditional volatility under measure Q.) For a given combination of the GARCH process slope parameters (α_1, β_1) and the unit risk premium (λ), the a_0 value is adjusted to maintain a constant s of 0.25. We do so to focus upon the option valuation effect of the process for the variance as opposed to its level. The specific values that we consider for the different variables and parameters are as follows:

$\sqrt{h_1}$: 0.75 s (low initial conditional variance), σ (equal to unconditional volatility), 1.25 σ (high initial conditional variance);

λ : 0.01 (low), 0.10 (medium), 0.20 (high);

α_1 : 0.05 (low), 0.175 (medium), 0.30 (high);

β_1 : 0.50 (low), 0.65 (medium), 0.80 (high);

S/X : 0.8 (deep-out-of-the-money), 0.9 (near-out-of-the-money), 1.0 (at-the-money), 1.1 (near- in-the-money), and 1.2 (deep-in-the-money); and

T : 30 days, 90 days, 180 days, 360 days, and 720 days.

Whenever an option valuation situation involves the low or the high value for one of the three parameters (α_1, β_1, l), the medium value is assumed for the other two parameters. For example, when $\alpha_1 = 0.05$ or $\alpha_1 = 0.30$, we use $\beta_1 = 0.65$ and l = 0.10. Thus the variance persistence parameter, g (=α_1+ β_1), varies between 0.675 ($\alpha_1 = 0.175$, $\beta_1 = 0.50$) and 0.975 ($a_1 = 0.175$, $\beta_1 = 0.80$). Given the empirical evidence of heavy persistence (g close to 1.0) driven by a high β_1 value in many situations, we also separately examine in a later section of this paper an additional set of cases with g=0.99 ($a_1 = 0.05$, $\beta_1 = 0.94$).

For each option valuation situation (h_1, l, a_1, β_1, S/X, T), we calculate four option prices. The BS, MBS, and PGARCH option prices are calculated using the formulas in sections I.B and I.C. For the GARCH option price, we use stratified simulations with 1000 strata and 50 runs. Therefore, the GARCH option price in each option valuation situation is based on 50,000 runs. To ensure that the simulated GARCH option prices do not violate the rational option pricing bounds, the underlying stock prices in these runs are generated using the Empirical Martingale Simulation (EMS) method recently proposed by Duan and Simonato (1995). The EMS method has the added benefit of a reduced standard error for the simulated option price. To further reduce simulation errors, we use a combination of the antithetic variable technique and the control variate technique. In the latter, the control variate is the BS price based on the unconditional variance with daily innovations.

III. Simulation Results

We organize this section in six sub-sections. In III.A, we study how various GARCH parameters and option contract parameters such as moneyness and time to maturity affect the GARCH option price and the associated sampling error. Section III.B studies the bias behaviour when we approximate the GARCH option price using the BS price, the volatility input of the latter being the unconditional volatility under measure P. In section III.C, we breakdown the overall pricing bias of the BS model into three separate components, and examine each component's individual contribution to the direction and magnitude of the overall bias. Next, in section III.D, we shall attempt to resolve a practical question: among the approximation models (BS, MBS, PGARCH), is there a model which is consistently more accurate in approximating the GARCH option price? Further, under what circumstances the BS model is more precise than the simulated GARCH option price despite the fact that the BS model is biased? In section III.E, we discuss the results for the nearly integrated variance situations that are often reported for financial time series. Finally, in section III.F, we discuss the implications of the findings for other valuation situations such as deposit insurance valuation.

A. Behaviour of the GARCH Price and the Simulation Sampling Error

As is customary, we shall refer to the *estimated* GARCH price of an option from simulations as the *GARCH price.* As such the GARCH price is subject to sampling error and the standard error of the GARCH price (GARCHSR) is estimated from the simulations.[14] For comparative purposes, the standard error is reported as a percentage of the GARCH price.

Table 1 presents the GARCH price and the standard error as a percentage of the GARCH price (GARCHPSR) averaged over all cases of an initial conditional variance situation (low, equal, or high) and also under an initial variance condition averaged over all cases of a given level of moneyness, time to maturity, unit risk premium, or a GARCH process parameter.[15]

Table 1 here

As expected, the GARCH option price increases with the level of the initial conditional variance, the level of moneyness, and the time to maturity of the option. [16] However, in general, the GARCH option price does not seem overly sensitive to the level of the initial conditional variance. Variation in

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The standard error reported in this paper for a given option valuation situation is the standard deviation of the simulated option prices divided by re root of the number of runs. The standard error calculated in this usual way, call it the *naive standard error*, assumes that the simulated option price pendent. However, this is not a valid assumption due to the martingale correction of the simulated stock price paths under the Empirical Marti ılation (EMS) method. As shown by Duan and Simonato (1995), the martingale correction reduces simulation errors. Hence the *naive standard e* ted in this paper overestimate the true standard errors of the simulated GARCH prices. We also ran simulations without the martingale correction ılated standard errors (which are valid estimates) that are slightly greater than the *naive standard errors*. The magnitude of these differences in the stan s are not material enough to affect the major findings of this paper.

With some exceptions, the comparisons based upon the averages in Table 1 and the other tables to follow fairly reflect the full set of 525 cases (avai request from the authors).

While we do not report the results in the table, we have also examined whether the GARCH price of an in-the-money Europea on falls below its intrinsic value and found that it does not. This is not surprising since we employed the Equivalent Martingale ulations technique. Even when this technique is not used, the use of a 0% riskfree interest rate in the BS formula would yield th e of a pure option (option on futures with futures-style margining) when the spot asset price follows a lognormal diffusion with stant variance rate (Lieu (1990)). It has been shown (Lie (1990), Chen and Scott (1993), Chaudhury and Wei (1994)) that the e of a pure European option never dips below its intrinsic value. This results also prevails in a stochastic interest rate regime (C Scott (1993)).

the GARCH option price across the different levels of moneyness and time to maturity is considerably more than that across the three situations of the initial conditional variance. For example, the average GARCH price moves from \$9.878 to \$10.021 as the initial conditional volatility (square root of the initial conditional variance) changes from its low level to its high level, i.e., a 1.45 percent change in average price for a 67% increase in initial conditional volatility.

A distinct feature of Duan's GARCH option valuation model is its preference-based nature. The unit risk premium, l, increases the unconditional variance under the locally risk-neutral pricing measure Q; it is the noncentrality parameter for the innovations driving the conditional variance process under measure Q; and it induces correlation between the conditional variance and the lagged asset return under measure Q. A priori, the net impact of the unit risk premium is unclear.

Results in Table 1 suggest that a change in the unit risk premium does not always move the GARCH price in the same direction. Further inspection of the full set of cases reveals that the GARCH option price is decreasing (increasing) in the unit risk premium for out-of-the-money and at-the-money (in-the-money) options. The economic significance of this intriguing result, however, becomes questionable once we consider the magnitude of price change for the different levels of l. As l increases from 0.01 to 0.20, the GARCH price change is less than 5 percent for all options other than the deep-out-of-the-money ones. For this latter group of options, the same change in l induces a large percentage drop (as high as 50 percent) in the GARCH price.

The two slope parameters (a_1, β_1) of the GARCH (1,1) conditional variance process are at the heart of the GARCH option valuation problem. If they were close to zero, a constant variance assumption would be reasonable and the BS price should provide a close approximation to the appropriate theoretical price. If, on the other hand, the two slope parameters and their sum are not negligible in value, the conditional and stochastic nature of the variance process becomes quite relevant.

Results in Table 1 confirm the above expectations. The two slope parameters have similar effects on the GARCH price and the simulation error when one of the two parameters is held constant. Since holding one of the slope parameters (a_1, β_1) constant while increasing the other leads to a higher γ or heavier persistence in variance, we also observe similar behaviour for the GARCH price and its standard error as γ varies. At higher levels of either parameter or their sum, the simulation error is relatively higher. This suggests that in general the sum of the two parameters, viz., the variance persistence parameter, γ, plays an important role and needs to be estimated carefully. However, no uniform directional pattern for the GARCH price can be discerned as the parameters vary.

The GARCHPSR (simulation standard error as a percentage of the GARCH price) average figures in Table 1 reveal that on average GARCHPSR is about 1 percent or less except in the case of deep-out-of-the-money and shortest maturity options. Case by case analysis indicates that GARCHPSR is in excess of 7 percent when the option is deep-out-of-the-money (S/X=0.8) and has a very short maturity (T = 30 days). While the volume of such exchange-traded options is typically low, they could still be relevant to practitioners and researchers dealing with custom-made derivative products and potential application of Duan's (1995) model to other option-like valuation situations.

GARCHPSR consistently goes down as the level of moneyness and time to maturity increases. The preference parameter λ has no material impact on the simulation precision level. In general, although not always, simulation precision is somewhat lower for the low initial conditional variance situations.

B. BS Model Bias

Table 2 presents summary statistics for the bias (BS -GARCH), the absolute bias (|BS - GARCH|), and the absolute percentage bias (100*|BS - GARCH|/GARCH) of the BS model when the appropriate theoretical price is the simulated GARCH option price.

Table 2 here

Averaging over all 175 cases of an initial conditional variance situation (Panel A), the BS bias is within a dime and is about 4 to 6 percent of the GARCH price. The initial conditional variance situation appears to have a bearing on the direction of the bias although not necessarily on the percentage bias. Given that the standard deviations are rather large compared to the means and also in the light of our discussion in the previous section, we shall now examine the bias figures classified by the moneyness (Panels B-F) and the time to maturity (Panels G-K) of options, the unit risk premium (Panels L-N), and the GARCH process parameters (Panels P-X).

The absolute percentage biases are more comparable across the various levels of a variable or a parameter since they are adjusted for the price level. Once again it appears from Panels B-K that the bias of the BS model is significant for the shortest maturity and deep-out-of-the-money options. Depending upon the initial conditional variance situation, the average absolute percentage bias of the

BS model ranges from 14 to 21 percent (15 to 22 percent) for deep-out-of-the-money (shortest maturity) options. In contrast, most other option categories have a mean bias of less than 5 percent.[17] We also note that the absolute percentage bias of the BS model decreases as the moneyness and the time to maturity of the option increase.

Notice that the standard deviation of the absolute percentage bias is rather large compared to the means for all categories of moneyness and time to maturity. Our examination of the full set of cases shows that this is due to the presence of deep-out-of-the-money and/or the shortest maturity options in each of the Panels B-K. The BS model bias in percentage terms is the highest for the deep-out-of-the-money shortest maturity options.

Regarding the direction of the BS model bias, it seems that the deep-in-the-money (S/X = 1.2) and the longest maturity (T = 720 days) options tend to be underpriced by the BS model, especially in equal and high conditional variance situations. An inspection of the individual cases also show that the deep-out-of-the-money and deep-in-the-money shortest maturity options are always underpriced by the BS model. At-the-money options are overpriced in low conditional variance situations, otherwise they are underpriced. This striking price bias is qualitatively similar to that noted by Merton (1976) when the stock returns follow a jump diffusion process and a low conditional variance situation prevails. However, unlike the jump diffusion context, the sign of the bias for at-the-money (S/X = 1.0) options reverses when the conditional variance is near to or higher than the stationary level of variance. Also, for near-the-money (S/X = 0.9, 1.1) options, the bias is not uniform and depends on other option valuation parameters. An inspection of the individual cases also reveal that the sign of the bias for deep-out-of-the-money and deep-in-the-money options varies for maturities longer than 30 days. For example, consider in Table 3 the BS model bias (BS-GARCH) for the option valuations situations where time to maturity is 180 days (0.5 year), $a_1 = 0.175$, $\beta_1 = 0.65$.

Table 3 here

When the initial conditional variance is low and the unit risk premium is at its low level (0.01), all options including the deep-out-of-the-money and deep-in-the-money ones are overpriced by the BS model. Under the same variance situation but now at a high unit risk premium level (0.20), all deep-

As can be seen in Panel C of Table 2, the average absolute percentage bias is 7 to 9 percent for the near-out-of-the-money ons in low and high initial conditional variance situations.

out-of-the-money options are overpriced while near-out-of-the-money, at-the-money and in-the-money options are underpriced. Moving to the high initial conditional variance situation, all options above are underpriced by the BS model.

One implication of the above bias behaviour for the same maturity is that the striking price bias may take different forms depending upon the variance situation and the level of the unit risk premium. Thus the GARCH option valuation model seems general enough to accommodate the various striking bias patterns and "reversals" that have been reported in empirical studies (e.g., Black (1975), MacBeth and Merville (1979), and Rusinstein (1985)).

It is also worth noting in Table 2 that the BS model bias tends to move from a general overpricing in low initial conditional variance situation to a general underpricing in high initial conditional variance situation.[18] This is in conformity with Duan's (1995) simulation results.[19] Duan (1995, p.23) mentions that the GARCH conditional variance process is known to generate more low-variance states more frequently. Accordingly, we should expect the BS model to overprice options more often. We, however, find that a high unit risk premium on the stock (l = 0.20) often leads to more options being underpriced by the BS model even in the low initial conditional variance situation. This can also be seen from Panels L-N of Table 2. For both the low and equal initial conditional variance situations, a positive average BS model bias at low unit risk premium changes to a negative average bias at high unit risk premium. An examination of the individual option valuation situations under the low and equal variance conditions also reveal a high frequency of overpricing (underpricing) by the BS model when the unit risk premium is at its low (high) level. Since the low and equal variance conditions are more likely than the high variance condition, a time varying risk premium may explain why the BS model with a constant variance overprices most traded options sometimes while underpricing them at other times (Black (1975), p.41).

Panels P-X as well as our examination of the full set of cases show that the alternate values for the GARCH process slope parameters (a_1 , β_1) do not have any noticeable impact on the direction of the BS model bias, given an initial conditional variance situation. However, as they increase (i.e. as the variance persistence increases), the absolute percentage bias of the BS model increases significantly. Panel X shows that when γ>0.90, the average absolute percentage bias is more than 8 percent regardless of the variance situation.

The zero bias point is, however, not necessarily at the exact at-the-money (S/X = 1.0) position.

Similar to our general finding regarding the behaviour of GARCH option prices, panels L-N indicate that the absolute percentage bias is not greatly affected by the level of the unit risk premium, l. The mean absolute percentage bias ranges from 4 to 7 percent for all three levels of l. But, as noted above, l could have an important bearing on the direction of the BS model bias.

C. The Three Components of the BS Model Bias

In the previous section, we have looked into the magnitude and the direction of the BS model bias when Duan's (1995) GARCH option valuation model is the appropriate theoretical model. Earlier in this paper, we have identified three sources of bias for the BS model, viz., the u.v. (unconditional variance) bias, the c.v. (conditional variance) bias, and the p.d. (path dependence) bias. It is useful to know how the three sources of bias interact to determine the overall or net BS model bias in a given option valuation situation. It is also useful to find out the principal source(s) of the BS model bias in a given option valuation situation, or how the relative importance of the three sources of bias vary (if at all) across different option valuation situations. It would be equally interesting to determine if the p.d. bias is always in one direction.

Table 4 contains some simulation results that should help address these issues. In this table, we report the mean figures for the u.v. bias (BS - MBS), the c.v. bias (MBS - PGARCH), the p.d. bias (PGARCH - GARCH), and the respective percentage bias proportions (PROP). For each option valuation situation (combination of S/X, t, a_1, β_1, l, initial conditional variance), the bias proportions (PROP) are calculated as follows:

UVPROP = 100* |BS - MBS| / ABSUM

CVPROP = 100* |MBS - PGARCH| / ABSUM

PDPROP = 100* |PGARCH - GARCH| / ABSUM

where ABSUM = |BS - MBS| + |MBS - PGARCH| + |PGARCH - GARCH|. PROP for a bias is an indicator of how important its magnitude is in determining the overall or net bias of the BS model. A large (small) PROP figure for a bias means that it is relatively more (less) important.

Strictly speaking, the difference between PGARCH and GARCH has two components: the p.d. bias (expected Black-Scholes price as a function of the average conditional variance - GARCH), and the bias (PGARCH - expected Black-Scholes price as a function of the average conditional variance)

Duan (1995) reports bias figures as: GARCH - BS. In this paper, we use: BS - GARCH.

of the BS formula due to its nonlinearity in the average conditional variance rate. Following Hull and White's (1987) series approach, we attempted to estimate the expected BS price using the first and second central moments of the average conditional variance (based upon Duan's (1995) results) in a second-order Taylor series. This, however, resulted in considerable instability in some option valuation situations. Hence we decided not to pursue the decomposition of the difference between PGARCH and GARCH. Consequently, while we refer to this difference as the p.d. bias, strictly speaking, it is also inclusive of the nonlinearity bias.[20]

Some comments about the biases can be made on an a priori basis. First, the u.v. bias is always negative since the measure Q unconditional variance is greater than the measure P unconditional variance as long as the unit risk premium, l , is nonzero. Second, the u.v. bias should increase (decrease) as the magnitude of l increases (decreases). Third, the c.v. bias is positive, zero, or negative depending upon whether the initial conditional variance is less than, equal to, or greater than the measure Q unconditional variance.

If we consider the possibility of either a positive or negative p.d. bias in conjunction with the above comments, it seems that the overall or net GARCH option valuation effect would very likely depend on the option valuation situation at hand. To see this, we now turn to Table 4.

Table 4 here

Panel A indicates that averaging across all 175 cases under an initial conditional variance situation, the p.d. bias is positive for all three (low, equal, high) initial conditional variance situations. In the low initial conditional variance situation, the positive p.d. bias (0.085) combined with a positive c.v. bias (0.064) outweighs the negative u.v. bias (-0.083). This results in an average net GARCH option valuation effect of 0.067 (Panel A, Table 2) or general overpricing by the BS model. In the equal initial conditional variance situation, the c.v. bias is zero and the positive p.d. bias (0.085) barely exceeds the negative u.v. bias (-0.083). This leads to a small overpricing by the BS model, the average magnitude of the net GARCH valuation effect being the lowest (0.003, Panel A, Table 2) of the three variance situations. A negative net GARCH Option valuation effect (-0.077) or a general underpricing by the

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For an at-the-money option, the BS formula is approximately linear in the variance. The nonlinearity bias is thus roughly zero, and the difference betv
RCH and GARCH solely reflects the p.d. bias.

BS model occurs in the high initial conditional variance situation; the negative u.v. bias (-0.083) and the negative c.v. bias (-0.081) together outweigh the positive p.d. bias (0.087).

Thus we have a general overpricing by the BS model turning into a general underpricing as the initial conditional variance moves from a low to a high state, a pattern that we have previously noted. This pattern is also supported by a redistribution of the relative importance (measured by mean PROP figures, Panel A, Table 4) away from the p.d. bias to the c.v. bias when the initial conditional variance is high rather than low.[21]

The nature of the p.d. bias (which also includes the nonlinearity bias in this paper) is an interesting issue since this is the component that mainly captures the effect of both path dependence and departure from lognormality of the terminal stock price. Since under the GARCH model the (log) stock return is leptokurtic (under both P and Q), both out-of-the-money and in-the-money options should be more valuable (Duan (1995), p.20). Accordingly, we would expect the p.d. bias to be negative (underpricing by the BS model).

Considering the average p.d. bias figures for the different levels of moneyness and time to maturities in Panels B-K of Table 4, the high variance, shortest maturity options (Panel G) are the only group where the average p.d. bias is negative. Case by case analysis confirms the preponderance of negative p.d. bias in high variance situations. Across all three variance situations, deep-in-the-money options maturing in 180 days or earlier have mostly negative p.d. bias figures. A negative p.d. bias is not as prevalent for the deep-out-of-the-money options. For these options, a negative p.d. bias is more common when the option maturity is short and the conditional variance equals or exceeds the stationary level. For maturities longer than 30 days, the p.d. bias is largely positive for deep-out-of-the-money options. Further, while the average p.d. bias is positive for at-the-money, near-out-of-the-money, near-in-the-money, and deep-in-the-money options under all three variance situations (Panels C-F, Table 4), studying the full set of cases we find that the p.d. bias is most consistently positive for the at-the-money options.

An evaluation of the average PROP figures in Panels B-K of Table 4 indicates that: (a) the average p.d. bias PROP reaches its highest values in the case of the deep-out-of-the-money (45.04%, 64.09%, 40.27%) and the shortest maturity (45.23%, 77.00%, 45.65%) options; (b) the p.d. bias is consistently the most dominant source of bias for deep-out-of-the-money, deep-in-the-money, and the shortest

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This redistribution pattern is largely supported by a case-by-case analysis except for the shortest maturity (T = 30 days) options.

maturity options only; and (c) as the option's maturity gets longer, the relative importance of the p.d. bias tends to diminish although not monotonically in the high variance situation. Further analysis of the full set of cases reveals that the individual (not average) p.d. bias PROP reaches its highest values in the case of the shortest maturity deep-out-of-the-money and deep-in-the-money options, and the dominance of the p.d. bias is also the greatest in these cases.

There are several insights that we can draw from the above simulation results. First, the p.d. bias (inclusive of the nonlinearity bias in this paper) is not uniformly negative for an out-of-the-money option, that is, it does not always lead to underpricing of an out-of-the-money option by the BS model. While leptokurtosis in (log) stock returns caused by the GARCH process creates underpricing by the BS model, for longer maturities this effect tends to be more than offset by other features of the distribution of the terminal stock price.

Second, the effect of these other features is not as important in the case of a deep-in-the-money option, thus resulting in a negative p.d. bias (underpricing) in a large number of cases.

Third, the overall underpricing of the shortest maturity deep-out-of-the-money and deep-in-the-money options by the BS model noted earlier is due to the dominance of the negative p.d. bias in these cases augmented by the universally negative u.v. bias.

Previously, we found at-the-money options to be overpriced (underpriced) by the BS model in low and equal (high) initial conditional variance situations (Panel D, Table 2). It appears (from Panel D, Table 4) that the positive p.d. bias of an at-the-money option either by itself (equal variance situation) or pairing with the positive c.v. bias (low variance situation) outweighs the universally negative u.v. bias to cause this underpricing. In the high initial conditional variance situation, the negative c.v. bias coupled with the negative u.v. bias outweighs the positive p.d. bias (Panel D, Table 4) and leads to an underpricing of an at-the-money option (Panel D, Table 2). Notice that the main source of this variance bias of at-the-money options is the change in the signed value of the c.v. bias and its relative importance across the three variance situations.

Rubinstein (1985) and Sheikh (1991) noted a time to maturity bias reversal phenomenon over different time periods for at-the-money options when the BS model is inverted to imply volatility from the observed option prices. Duan (1995) shows that under the GARCH model, when the initial conditional variance is low, the BS model ISD increases with maturity.[22] The ISD pattern reverses

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This means that the BS bias (BS-G) becomes more negative, i.e., underpricing by the BS model increases with maturity.

when the initial conditional variance is high.[23] To facilitate a better understanding of this phenomenon, we report in Table 5 the three components of the BS model bias and their relative magnitude figures (PROP) for at-the-money options, under the low and high initial conditional variance situations and the middle values of the GARCH valuation parameters. Under both variance situations, the absolute as well as the relative magnitude of the c.v. bias decreases as the maturity gets longer. Given that the c.v. bias is positive under the low variance situation, overall overpricing is more (underpricing is less) for shorter maturity options relative to longer maturity options. This of course translates a relatively lower ISD for a shorter maturity option in Duan's (1995) Table 4.1. Thus it appears that the c.v. bias or the initial conditional variance situation under the GARCH (1,1) model could be at the heart of the time to maturity bias reversal phenomenon.

Table 5 here

Previously we noted a general tendency of overpricing by the BS model turning into underpricing as the unit risk premium, l, increases. The unit risk premium, l, is of course the key determinant of the u.v. bias. Panels L-N of Table 4 show that average u.v. bias PROP increases from about 1% to over 50% as l changes from its low state (0.01) to its high state (0.20). When l is low, the positive p.d. bias either by itself (equal variance) or coupled with the positive c.v. bias (low variance) produces overpricing . However, when l is high, the dominance of the negative u.v. bias produces underpricing in all three variance situations.

By examining Panels P-U of Table 4, we find that the GARCH process slope parameter a_1 affects the relative importance of the three types of bias much more than the parameter β_1. With a higher a_1, the average p.d. bias and the average p.d. bias PROP go up significantly. For example, under the low variance condition, the average p.d. bias (p.d. bias PROP) goes up from 0.006 (30%) when $a_1 = 0.05$ to 0.291 (49%) when $a_1 = 0.3$. To a lesser extent, this effect also shows up in Panels V-X where the bias results are reported for alternative levels of the variance persistence parameter γ.

While both a_1 and l lead to significant shifts in the relative importance of the three types of bias of the BS model, the simulation results from Tables 1-4 suggest that the unit risk premium, l, is perhaps more important in determining the direction of the overall bias or GARCH option valuation effect. The

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This means that the BS bias (BS-G) becomes less negative, i.e., underpricing by the BS model decreases with maturity. While we do not report the ts here, we have confirmed Duan's explanation for various combinations of the unit risk premium and the GARCH process parameters.

GARCH process persistence parameter γ, on the other hand, is more important when it comes to the size of the overall bias while the slope parameter a_1 has significant influence on the composition of the bias.

D. Comparison of BS, MBS, PGARCH, and GARCH Prices

From the simulation results discussed so far, we gather that the BS price is a biased estimate of the true option price when the stock returns follow a GARCH (1,1) process. The direction and the magnitude of the bias vary across different option valuation situations and depend on the interaction of the three components of the bias (u.v. bias, c.v. bias, p.d. bias). While the BS price is biased, it has no variance as an estimate of the true option price when the unconditional stock return variance is known. The simulation-based GARCH option price, on the other hand, has no bias, but it has variance as an estimate of the true option price caused by simulation sampling error. As noted earlier, this sampling error varies across different option valuation situations too. Thus, it is possible, at least in theory, that the BS model absolute percentage bias in a given option valuation situation is less than the GARCHPSR (simulation standard error as a percentage of the simulation-based GARCH price).

Econometricians often use a mean square error (MSE) criterion to choose between biased and unbiased estimates of population parameters. The MSE of an estimate is equal to its variance plus bias squared. The MSE of the BS price is its absolute bias squared. For the simulation-based GARCH option price, the MSE is the GARCHSR squared. Thus a comparison of the absolute percentage bias of the BS model and the GARCHPSR is similar to a comparison of their root mean square error (RMSE). With estimation risk, an estimate with a lower RMSE is preferred under a squared error loss criterion.

This opens up the possibility that a practitioner may be better off using the BS model instead of the GARCH option pricing model in some option valuation situations. In other situations where the GARCH option pricing model is to be preferred on a RMSE basis, it is possible that the gain in RMSE is not perhaps large enough to a practitioner facing the significantly higher computational involvement of the GARCH model.

Another interesting issue is whether removing one or more but not all of the three components of the BS model bias leads to an improvement (in RMSE sense) over either the BS price or the simulation-based GARCH price. In other words, does MBS (removes u.v. bias) or PGARCH (removes u.v. bias and c.v. bias) outperform the BS price or the simulation-based GARCH price in a

RMSE sense? The answer to this question is not clear a priori since the three biases are not always of the same sign.

Table 6 presents the absolute percentage bias for each of BS, MBS, PGARCH, and the simulation standard error as a percentage of the GARCH price (GARCHPSR), averaged over all option valuation situations under a given (low, equal, high) initial conditional variance state and also by monyeness, time to maturity, the unit risk premium, and the GARCH process slope parameters. The absolute percentage bias for each of BS, MBS and PGARCH is in fact its RMSE as a percentage of the GARCH price. Similarly, the GARCHPSR is the RMSE of the simulated GARCH price expressed as a percentage.[24] We report the percentage figures as they are more comparable across different option valuation situations. However, for the sake of brevity, we shall refer to these percentage figures as simply RMSE in the discussion to follow.

Table 6 here

Panel A of Table 6 shows that on average the BS model's RMSE is about 4 to 6 percent higher than that of the simulated GARCH model. Removing either the u.v. bias (using MBS) or both the u.v. bias and the c.v bias (using PGARCH) does not offer any significant advantage over the BS model on average. However, as noted earlier, the BS model bias, its components, and GARCHPSR vary widely across different option valuation situations. Hence the average across all option valuation situations could be potentially misleading. One indication of this are the large standard deviations reported beside the mean RMSE figures in Table 6.

Panels B-K indicate that on average for any level of moneyness and time to maturity, the GARCH model's RMSE is lower than either of BS, MBS, or PGARCH. This advantage is, however, marginal for at-the-money and in-the-money (near and deep) options and those maturing in 6 months (180 days) or later. The RMSE advantage of the GARCH model is about 4 (2) percent or less for at-the-money (in-the-money) and 180 (360, 720) days options. In these and other cases, on average the MBS and PGARCH formulas do not offer any particular benefit over the BS model. In fact, it appears that sometimes trying to remove the u.v. and/or the c.v. bias leads to a higher RMSE. But once again we should not ignore the rather large standard deviations of the RMSE.

Strictly speaking, the true absolute percentage bias for BS, MBS and PGARCH and the true standard error of the simulated GARCH price as a percer e GARCH price (GARCHPSR) are not known. This is because the true GARCH price is not known. Since the simulated GARCH price is an estimate i eported absolute percentage bias and GARCHPSR figures are actually estimates of the corresponding true figures.

The RMSE results in Panel C (near-out-of-the-money options), Panel H (90 day options), and Panels L-N (different levels of the unit risk premium, λ) of Table 6 are fairly similar. With the exception of near-out-of-the-money options under a low variance condition, the MSE gain of the GARCH model over the BS model is on average in the range of 3 to 7 percent and the alternative formulas (MBS,PGARCH) do not seem to offer any notable improvement over the BS model. Largely similar comments also apply to the RMSE results in Panels S and T for low to medium β_1 values and the RMSE results in Panels P-Q for low and medium a_1 values.[25] The RMSE results in Panels Rand U for the high a_1 and β levels are different in that the GARCH model's RMSE gain over the BS model is on average 8 percent or more for these cases. Panels V, W, and X show that a higher γ produces similar RMSE behaviour as the high a_1 and β levels.

The RMSE results that are noticeably different from the rest in Table 6 are the ones in Panel B (deep-out-of-the-money options) and Panel G (30 day options). In these cases, the RMSE of the BS model is on average 14 percent or more while the GARCH model's RMSE is on average about 2 percent. Thus for these options, there is an average gain of 12 percent or more in accuracy (RMSE) in computing the GARCH simulation-based price rather than using the conventional BS model. While the alternative formulas (MBS, PGARCH) attempt to remove some biases of the BS model, this does not result in any clear improvement in accuracy (RMSE) over the BS model.

On the basis of the average RMSE results in Table 6, it seems that GARCH option pricing is most important for deep-out-of-the-money options (S/X = 0.8), very short maturity (T = 30 days) options, and options on stocks with high γ or variance persistence.

While the above results compare the average RMSE figures for the GARCH model, the BS model, and the alternative models (MBS, PGARCH) by one option variable (S/X, T) or parameter (a_1, β_1, l) at a time, it would be useful to know under what option valuation situations the BS model fares better (lower RMSE) or worse than the GARCH model.

The RMSE of the BS model is less than that of the GARCH model in 18 of the 175 low variance cases, in 32 of the 175 equal variance cases, and in 9 of the 175 high variance cases. The corresponding numbers for the MBS (PGARCH) formula are 7 (14), 17 (7), and 18 (23). In a large majority of these cases, the RMSE gain over the GARCH model is, however, quite modest (typically less than 1 percent). While the MBS and PGARCH formulas sometimes offer a more accurate price estimate

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For the highest b_1 level, the RMSE averages are a bit higher.

(lower RMSE) than the BS model and the simulated GARCH price, the gain in RMSE is not significant. Further, compared to the BS model, implementation of the MBS or PGARCH model leads to additional data requirements (the unit risk premium and the GARCH process parameters).

Some of the distinguishing features of the cases where the BS model has a RMSE advantage over the GARCH model are: (a) the highest a_1 value (0.30) occurs in none of the low variance, 1 of 32 equal variance, and 3 of 9 high variance cases; (b) the highest β value (0.80) occurs in 2 of 18 low variance, 3 equal variance, and 1 high variance cases; (c) the highest λ value (0.20) occurs in only 1 low variance, none of the equal and high variance cases; in contrast, the lowest λ value (0.01) occurs in 4 equal variance and 5 high variance cases;[26] and (d) the time to maturity is 6 months or longer in 17 of 18 low variance, 25 of 32 equal variance, and all 9 high variance cases.[27] None of the cases involve the shortest maturity (T=30 day) options.

As for moneyness, the cases seem well spread over all levels including the deep-out-of-the-money level. As the maturity gets longer (6 months or more), the BS model RMSE drops off significantly even when the option is deep-out-of-the-money. It is only when the option maturity is short (90 days) and γ is not low, or when the option maturity is very short (30 days), that the BS model RMSE is quite high for out-of-the-money, especially deep-out-of-the-money options.

As shown in Panel A of Table 7, for out-of-the-money options (S/X = 0.8, 0.9) which are maturing in 90 days or sooner, the average RMSE of the BS model is 24 percent, 22 percent, and 32 percent respectively for low, equal, and high variance conditions. In contrast, the average RMSE for all other options is 2.17 percent, 1.06 percent, and 1.48 percent respectively for low, equal, and high variance conditions.

Table 7 here

Panel B of Table 7 shows that for the shortest maturity deep-out-of-the-money options (T=30 days, S/X = 0.8), the average RMSE of the BS model is 54 percent, 65 percent, and 75 percent respectively for low, equal, and high variance conditions. When the shortest maturity deep-out-of-the-money options are excluded, the average RMSE for the remaining 168 cases is 3.60 percent, 1.85 percent, and 3.56 percent respectively for low, equal, and high variance conditions. Examining the bias proportions for the shortest maturity deep-out-of-the-money options vs. all other options, we find that

The middle value of λ is found in 17 of 18 low variance, 28 of 32 equal variance, and 4 of 9 high variance cases.

All 9 high variance cases involve the longest maturity (T=720 day) options.

the p.d. bias is at the source of the worst errors of the BS model. For the 7 (168) shortest maturity deep-out-of-the-money (all other) options, the p.d. bias PROP is 70 (36) percent, 95 (57), and 75 (32) percent respectively for low, equal, and high initial variance conditions.

Unfortunately, neither MBS nor PGARCH provides any significant improvement in RMSE over the BS model for the shortest maturity deep-out-of-the-money options. We further examine the cases (14 low variance, 9 equal variance, and 15 high variance) where the BS model RMSE is greater than 20 percent. Panel C of Table 7 shows that the average RMSE of the BS model in these cases is 44 percent for low variance condition, 59 percent for equal variance condition, and 55 percent for high variance condition.[28] The corresponding average RMSE figures for the MBS (PGARCH) formula are 46 (41) percent, 52 (52) percent, and 50 (32) percent respectively.

E. Nearly Integrated Cases

In dealing with asset returns, empirical studies often find a very high level of variance persistence or a situation of nearly integrated variance process as indicated by estimated γ values exceeding 0.90. Ng (1991) reports γ values in the range of 0.90 to 0.93 for monthly excess returns on both small and large size portfolios of U.S. stocks during 1931-1987. Akgiray (1989) finds γ values exceeding 0.96 for the CRSP value-weighted and equally-weighted daily returns during the various subperiods of 1963-1986. Engle and Mustafa's (1992) γ values for daily returns (July 1962 to December 1985) on the S&P 500 and 5 large U.S. stocks are all around 1.0. In Heynen and Kat's (1994) study of the 1980-1987 daily returns, 4 of the 7 stock indices and 3 of the 5 currencies show γ values in excess of 0.97. Lamoureux and Lastrapes (1990) examine the daily returns of 20 actively traded stocks with CBOE options during the 1980-1984 period. They report γ values in excess of 0.97 for 5 of the 20 stocks in their sample. In all these and other cases, γ values are dominated by the β_1 parameter for which a value greater than 0.90 is not uncommon. Thus the empirical evidence on returns of many financial assets seems to indicate a relatively small immediate impact of a variance innovation which however is nearly permanent.

Given the empirical relevance of these nearly integrated situations and the fact that our simulation results so far also indicate larger biases for the BS model and larger simulation errors for the GARCH price, we report in Appendix A the case by case simulation results for some nearly integrated (α_1=0.05,

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Panel C of Table 7 indicates that when the cases where the RMSE of the BS model is more than 20 percent are compared with the cases where the RI e BS model is less than 20 percent, the former cases are characterized by a significantly higher and dominant P.D. bias proportion. Thus the p.d. ars to be responsible for the worst errors of the BS model.

β_1=0.94, γ=0.99) cases. For the sake of brevity, we only consider the middle value (0.10) of λ, and ignore the equal variance cases.

Some key features noted earlier show up in the nearly integrated cases as expected albeit with a greater intensity. GARCH simulation errors and the BS model bias in percentage terms decrease with moneyness and time to maturity. Compared to the results reported earlier, the importance of the GARCH valuation effect is not limited to the deep-out-of-the-money shortest maturity (30 days) options only. Even at-the-money options maturing in 6 months can have BS error in excess of 10 percent.

A very important feature of the nearly integrated cases is that the GARCH valuation effect (BS model bias) is dominated by the conditional variance bias and the p.d.bias is relatively unimportant. As a result, the BS model consistently overprices (underprices) options in low (high) variance conditions. With near integration (with α_1 approaching zero and β_1 approaching 1.0), the conditional variance innovations are nearly perfectly correlated and their effect decays extremely slowly; if conditional variance starts at a high level, it is expected to remain high for a long time. Since the future conditional variances are highly predictable in this situation, not much variance is expected for them and the average expected variance is like an average over a deterministic path of variance (the complete stochastic path of the variance becomes less important).

This suggests that our PGARCH formula should approximate the GARCH price fairly well in the nearly integrated situations (where β_1 is the major component) since it corrects the c.v. bias as well as the u.v. bias and the p.d. bias is relatively a small component. The results in Appendix A lend support to this contention. Except for the shortest maturity deep-out-of-the-money options, the percentage error of the PGARCH formula is under 6%. For the widely traded at-the-money options, the PGARCH bias is about 2% or less. Thus, in commonly found empirical situations, the PGARCH formula may offer an attractive practical alternative to the simulated GARCH price which is more accurate but computationally more involved.

F. Some Implications

In what follows, we discuss some key implications of our simulation results for option valuation, the empirical biases of the BS model, and option-like economic situations.

F.1 Option valuation

A number of option valuation implications emerge from our simulation results. First, among the GARCH process parameters, the level of variance persistence, γ, is relatively more consequential for the GARCH option price. This is especially so for out-of-the-money options maturing in 90 days or earlier. For these options, the standard error of the simulated GARCH price is also relatively high. Researchers and practitioners should thus strive for accurate estimates of γ in implementing Duan's GARCH option valuation model for the out-of-the-money short maturity options.

Second, the magnitude of the Black-Scholes (BS) model bias is the largest in percentage terms (often exceeding 14 percent) for the deep-out-of-the-money options maturing in 30 days or earlier. Hence, implementation of the GARCH option valuation model is recommended (strongly) for out-of-the-money options (deep-out-of-the-money) maturing in 90 (30) days or earlier. These options should, of course, be avoided if the BS model is inverted to imply the GARCH process parameters from the observed market prices.

Third, for options other than the deep-out-of-the-money shortest maturity ones, the absolute percentage bias of the BS model is under 4 percent on average. In fact, the BS model absolute bias is less than the standard error of the simulated GARCH price in about 11 percent of the cases that we have considered. These are typically options maturing in 180 days or later.

With the advent of the new breed of long-dated options (e.g., LEAPS and FLEX) and the increasing body of evidence indicating that volatility is not constant over an extended period of time, it is somewhat reassuring to know that the Black-Scholes model with a constant (unconditional or stationary) volatility provides a good approximation to the GARCH(1,1) theoretical option price for longer maturities. For some major asset classes, Figlewski (1994) finds that over horizons extending up to 10 years the historical volatility estimate produces a better forecast of the future volatility than the GARCH (1,1) model. Our results suggest that there is hope in implying the unconditional or stationary volatility from the Black-Scholes model using the observed long-dated option prices. We should however caution that we have not explored the accuracy of the Black-Scholes model with a constant volatility in approximating the hedge ratio of long-dated theoretical options under the GARCH (1,1) model.

Fourth, if the variance process is nearly integrated, the BS model bias can be significant even for options other than the deep-out-of-the-money very short maturity ones. Given that various empirical studies report nearly integrated situations with a high β_1 value, caution should be exercised in using the BS model in these situations for valuation, ISD estimation, and other purposes. Duan's (1995) GARCH option valuation model is appropriate under these circumstances.

However, for options other than the deep-out-of-the-money very short maturity ones, the Pseudo-GARCH formula presented in this paper offers an attractive alternative to the more accurate simulated GARCH price of Duan. This is specially so for at-the-money options where the Pseudo-GARCH error is 2% or less. At-the-money options are , of course, the most actively traded contracts on the organized exchanges.

F.2 Empirical Biases of the BS Model

To finance researchers, the direction of the BS model bias is of equal, if not more, interest as the magnitude of the bias. As suggested by Duan (1995), the GARCH option pricing model helps explain some of the well-known empirical biases of the BS model. In this paper, we decompose the bias into three components. Simulation results on the three components of the BS model bias or the GARCH valuation effect sheds further light on this issue.

Consistent with the popular "smile effect" in implied volatility and similar to the Black-Scholes bias under Merton's (1976) jump diffusion model, deep-in-the-money and deep-out-of-the-money (at-the-money) options with a very short time to expiration are underpriced (overpriced) by the BS model. This striking price bias of the BS model is caused by the direction and the relative importance of the bias component related to the nonlinear and path-dependent nature of GARCH option pricing. However, it should be mentioned that the striking price bias can take different forms depending upon the initial conditional variance situation and the unit risk premium level. Thus the GARCH option pricing model seems general enough to accommodate the conflicting striking price biases reported in a number of empirical studies (e.g., Black (1975), MacBeth and Merville (1979), and Rubinstein (1985)).

Averaging across all maturities, the at-the-money options are overpriced (underpriced) by the BS model when the initial conditional variance is lower than or equal to (higher than) the unconditional variance. This pattern is driven by a change in the direction of the bias component related to the conditional nature of the variance process. The behaviour of this conditional variance bias also helps explain the time to maturity bias reversal phenomenon (Rubinstein (1985), Sheikh (1991)). On the other hand, the relative importance of the bias component related to a change in the unconditional variance under Duan's locally risk-neutral pricing measure is responsible for consistent underpricing (by the BS model) of options with a very long time to maturity.

Two decades ago Black (1975, p.41) observed that there are times when most traded options seem underpriced and times when most traded options seem overpriced relative to the BS model price. One of the two possible explanations that Black provided was that ".. may be that the market is expecting volatilities to be generally lower or generally higher than the estimates used in the formula, ..", alluding in his discussion to a mean reverting conditional variance process. Consistent with this, our simulation results show that in general the BS model overprices (underprices) options when the initial conditional variance is lower than (higher than or equal to) the unconditional variance. A key factor here is the opposing influences of the change in the unconditional variance and the nonlinear and path-dependent nature of GARCH option pricing.

The second explanation advanced by Black was that ".. it may be that factors unrelated to option values are affecting the option prices." From our simulation results, it seems that one such factor could be a time varying risk premium. As the unit risk premium increases from its low level to its high level, a general pattern of overpricing by the BS model turns into a pattern of underpricing under the low and equal initial conditional variance situations. The low and equal initial conditional variance situations are of course more common place than the high initial conditional variance situation.

We should, however, note that while the unit risk premium affects the direction of the BS model bias, the GARCH (1,1) process persistence level is more important in determining the size of the bias. The immediate impact parameter, α, on the other hand plays an important role in determining the composition of the bias in most cases.

F.3 Option-like situations

Option valuation models are often used to gain insights into other economic situations that are option-like. Our simulation results have some important implications in this regard.

First, equity in a levered firm can be viewed as a call option on the assets of the firm (Black and Scholes (1973)). For solvent firms, which is the typical situation, value of the assets exceeds the debt obligations and as such the equity interest would be an in-the-money call option. Our results suggest that if the asset value is heteroskedastic and follows a GARCH(1,1) process, the constant volatility Black-Scholes model can still be relied upon to estimate the option-theoretic value of equity in a levered firm.

Second, for firms which are in a near bankruptcy situation, i.e., the market value of the assets is not nearly enough to cover the debt obligations and the debt payment date is close, the equity can be viewed as a deep-out-of-the-money option with a very short time to expiration. Since situations like this are often characterized by a sharp and sustained increase in volatility of asset value (e.g., real estate and resource-based companies, financial institutions lending to real estate and resource-based companies, firms with significant business interests in locations experiencing political instability, etc.), volatility models such as the GARCH (1,1) specification may be appropriate. Consequently, as indicated by our results, the use of a constant volatility Black-Scholes model may lead to serious errors (underpricing) in estimating the equity value. In the absence of taxes and other market imperfections, this would also mean errors (overpricing) in estimating the debt value. Practitioners should thus exercise caution in using the Black-Scholes model with a constant volatility to estimate the value of risky debt or to assess the implied political risk in the case of sovereign debt.

Third, deposit insurance obtained by a financial institution can be viewed as a put option (Merton (1977,1978)) on its assets. Obviously, on the basis of our results and the put-call parity relationship, the insurance will be mispriced if the insurer uses the Black-Scholes model with a constant volatility to determine the premium when the value of assets follow a GARCH (1,1) process. For solvent financial institutions, the put option would be deep-out-of-the-money and the mispricing would be modest. However, when the financial institution is near bankruptcy, the put option is deep-in-the-money with a short maturity (the next audit date is close) and use of the Black-Scholes model with a constant volatility will lead to a significant mispricing (undervaluation) of the deposit insurance.

IV. Summary

In this paper, we study the behaviour of European stock option prices when the stock returns follow a GARCH (1,1) process. The appropriate theoretical price of an option in this case is provided by Duan's (1995) GARCH option pricing model. Since the terminal stock price distribution does not conform to known functional forms, the GARCH option price is calculated using simulations and is thus subject to sampling error.

The GARCH option price is a function of the initial conditional variance, the unit risk premium on the stock, and the GARCH process parameters. Our simulation results for a variety of option valuation situations suggest that the GARCH option price is not, in general, very sensitive to the level of initial conditional variance. The level of variance persistence appears more consequential for the GARCH option price. This is especially so for out-of-the-money options maturing in 90 days or earlier. For these options, the standard error of the simulated GARCH price is also relatively high. Researchers and practitioners should thus strive for accurate estimates of γ in implementing Duan's GARCH option valuation model for the out-of-the-money short maturity options.

The magnitude of the Black-Scholes (BS) model bias is the largest in percentage terms (often exceeding 14 percent) for the deep-out-of-the-money options maturing in 30 days or earlier. For other options, the absolute percentage bias is under 4 percent on average. In fact, the BS model absolute bias is less than the standard error of the simulated GARCH price in about 11 percent of the cases that we have considered. These cases are typically the ones with low to moderate α_1 and λ values and options maturing in 180 days or later.

We have also tried two modifications to the BS model. The first one, modified BS, inserts the unconditional stock return variance under Duan's locally risk-neutral price measure into the BS formula. The second one, Pseudo-GARCH, uses the average expected conditional variance (under Duan's measure Q) in the BS formula. Our simulation results suggest that the two modified formulas do not in general result in any material improvement over the BS model. However, in the commonly found nearly integrated variance situations, the Pseudo-GARCH formula offers significant improvement over the BS model. At-the-money options with maturity more than a month are the most actively traded options. For these options, the Pseudo-GARCH formula’s error is about 2 percent or less for stocks with nearly integrated variance process.

Duan (1995) suggested that the GARCH option valuation model helps explain some of the well-known empirical biases of the Black-Scholes model. An important benefit of the two new formulas considered in this paper (MBS and PGARCH) is that they allow a rough breakdown of the GARCH option valuation effect (difference between BS and GARCH) into three components: the effect of change in the unconditional variance under Duan's locally risk-neutral measure (BS - MBS) or the u.v. bias, the effect of the conditional nature of the variance process (MBS - PGARCH) or the c.v. bias, and the nonlinear and path-dependent nature of GARCH option pricing (PGARCH - GARCH) or the p.d. bias. Our simulation results indicate that the three components of the BS model bias are not always of the same sign. Their relative importance (magnitude) also varies across different option valuation situations. The interaction of these factors leads to the determination of the direction of the BS model bias in a given option valuation situation. These include the “smile effect”, the conflicting striking price biases (Black (1975), MacBeth and Merville (1979), and Rubinstein (1985)), and the general overpricing or underpricing bias (Black (1975)).

Additionally, we discuss implications of GARCH effect in some option-like situations, e.g., equity of a levered firm, claims on firms nearing bankruptcy, and deposit insurance.

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Table 1
The Behaviour of GARCH European Call Option Prices (GARCH) and the Associated Simulation Standard Error as a Percentage of the Price (GARCHPSR)[a]

(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns):

		0.75		1.00		1.25	
	N	Mean GARCH	Mean GARCHPSR	Mean GARCH	Mean GARCHPSR	Mean GARCH	Mean GARCHPSR
All	175	9.878	0.636	9.942	0.569	10.021	0.54
S/X=0.8	35	1.560	2.364	1.587	2.090	1.624	1.949
S/X=0.9	35	3.528	0.508	3.591	0.444	3.670	0.431
S/X=1.0	35	7.512	0.167	7.617	0.165	7.743	0.171
S/X=1.1	35	14.230	0.087	14.306	0.087	14.402	0.091
S/X=1.2	35	22.559	0.056	22.607	0.057	22.668	0.059
T= 30 Days	35	6.622	1.931	6.689	1.649	6.772	1.529
T= 90 Days	35	7.611	0.492	7.688	0.451	7.785	0.430
T=180 Days	35	8.977	0.300	9.048	0.290	9.137	0.287
T=360 Days	35	11.290	0.238	11.349	0.236	11.422	0.236
T=720 Days	35	14.889	0.218	14.935	0.218	14.991	0.218
l =0.01	25	9.983	0.538	10.017	0.510	10.061	0.484
l =0.10	125	9.837	0.668	9.913	0.583	10.007	0.553
l =0.20	25	9.974	0.572	10.009	0.555	10.055	0.534
a_1=0.050	25	10.007	0.206	10.027	0.169	10.053	0.234
a_1=0.175	125	9.904	0.635	9.968	0.572	10.048	0.535
a_1=0.300	25	9.620	1.072	9.727	0.953	9.859	0.871
b_1=0.50	25	10.002	0.427	10.020	0.423	10.044	0.428
b_1=0.65	125	9.913	0.587	9.959	0.542	10.017	0.524
b_1=0.80	25	9.578	1.089	9.776	0.850	10.020	0.731
a_1+b_1:							
0.675,0.700	50	10.000	0.316	10.020	0.296	10.050	0.331
0.825	75	9.979	0.553	10.014	0.529	10.058	0.506
0.950,0.975	50	9.600	1.080	9.750	0.901	9.940	0.801

(a) When comparing prices for the alternative levels of a given GARCH process or preference parameter (say, l), the middle values for the other such parameters (a_1=0.175, b_1=0.65) are assumed. Thus the number of cases (N) corresponding to the middle value of any of these parameters is high (125).

Table 2
The Black-Scholes Prices (BS) Compared to GARCH Prices (G) for European Call Options[a]

		(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns): 0.75		1.00		1.25	
	N	Mean	Stdev	Mean	Stdev	Mean	Stdev
A. All cases							
BS-G	175	0.067	0.154	0.003	0.107	-0.077	0.114
|BS-G|	175	0.096	0.137	0.065	0.085	0.095	0.099
100*|BS-G|/G	175	5.628	13.556	4.398	14.236	6.426	16.887
B. Deep-out-of-the-money (S/X=0.8)							
BS-G	35	0.034	0.085	0.007	0.065	-0.030	0.063
|BS-G|	35	0.046	0.079	0.032	0.057	0.044	0.054
100*|BS-G|/G	35	14.380	23.908	16.507	28.509	21.256	31.318
C. Near-out-of-the-money (S/X=0.9)							
BS-G	35	0.101	0.155	0.038	0.110	-0.041	0.094
|BS-G|	35	0.113	0.146	0.064	0.097	0.074	0.071
100*|BS-G|/G	35	8.996	13.617	3.257	5.105	7.364	12.570
D. At-the-money (S/X=1.0)							
BS-G	35	0.160	0.219	0.055	0.133	-0.071	0.118
|BS-G|	35	0.181	0.201	0.091	0.110	0.105	0.089
100*|BS-G|/G	35	3.919	5.784	1.456	1.916	1.973	2.384
E. Near-in-the-money (S/X=1.1)							
BS-G	35	0.056	0.132	-0.021	0.093	-0.117	0.122
|BS-G|	35	0.092	0.109	0.064	0.069	0.128	0.110
100*|BS-G|/G	35	0.637	0.738	0.450	0.441	0.971	0.922
F. Deep-in-the-money (S/X=1.2)							
BS-G	35	-0.017	0.075	-0.064	0.081	-0.126	0.129
|BS-G|	35	0.048	0.059	0.072	0.074	0.127	0.128
100*|BS-G|/G	35	0.206	0.234	0.322	0.325	0.566	0.575
G. T = 30 Days							
BS-G	35	0.052	0.125	-0.014	0.055	-0.097	0.100
|BS-G|	35	0.063	0.120	0.035	0.044	0.097	0.100
100*|BS-G|/G	35	17.462	25.504	15.126	28.309	21.840	31.055
H. T = 90 Days							
BS-G	35	0.067	0.151	-0.011	0.090	-0.107	0.116
|BS-G|	35	0.089	0.139	0.054	0.073	0.109	0.114
100*|BS-G|/G	35	4.618	8.043	3.495	8.545	6.252	12.436
I. T = 180 Days							
BS-G	35	0.076	0.161	0.005	0.109	-0.084	0.111
|BS-G|	35	0.097	0.149	0.064	0.087	0.098	0.098
100*|BS-G|/G	35	2.911	4.974	1.272	2.208	2.253	3.532
J. T = 360 Days							
BS-G	35	0.072	0.164	0.013	0.122	-0.061	0.109

|BS-G|	35	0.110	0.141	0.079	0.094	0.089	0.086
100*|BS-G|/G	35	1.919	3.223	1.187	1.891	1.080	1.106
K. T = 720 Days							
BS-G	35	0.066	0.171	0.021	0.141	-0.036	0.123
|BS-G|	35	0.122	0.135	0.093	0.106	0.083	0.096
100*|BS-G|/G	35	1.227	1.822	0.911	1.347	0.706	0.977

Table 2 (Continued)
The Black-Scholes Prices (BS) Compared to GARCH Prices (G) for European Call Options

		(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns): 0.75		1.00		1.25	
	N	Mean	Stdev	Mean	Stdev	Mean	Stdev
L. l=0.01							
BS-G	25	0.043	0.036	0.009	0.019	-0.034	0.028
|BS-G|	25	0.044	0.034	0.018	0.012	0.036	0.026
100*|BS-G|/G	25	3.967	12.852	3.855	14.750	5.609	16.849
M. l=0.10							
BS-G	125	0.099	0.162	0.023	0.109	-0.071	0.123
|BS-G|	125	0.108	0.156	0.066	0.089	0.096	0.104
100*|BS-G|/G	125	6.314	14.408	4.667	14.634	6.638	17.395
N. l=0.20							
BS-G	25	-0.070	0.092	-0.106	0.085	-0.151	0.085
|BS-G|	25	0.088	0.075	0.106	0.085	0.151	0.085
100*|BS-G|/G	25	3.858	9.278	3.596	11.996	6.183	14.805
P. a_1*=0.050*							
BS-G	25	0.015	0.016	-0.005	0.007	-0.031	0.018
|BS-G|	25	0.015	0.016	0.006	0.006	0.031	0.018
100*|BS-G|/G	25	1.257	2.522	0.790	3.624	2.763	7.745
Q. a_1*=0.175*							
BS-G	125	0.046	0.140	-0.018	0.086	-0.098	0.105
|BS-G|	125	0.083	0.122	0.055	0.068	0.099	0.104
100*|BS-G|/G	125	5.330	13.287	4.153	14.130	6.500	16.960
R. a_1*=0.300*							
BS-G	25	0.221	0.198	0.115	0.172	-0.017	0.167
|BS-G|	25	0.241	0.172	0.172	0.111	0.143	0.084
100*|BS-G|/G	25	11.488	18.914	9.231	19.657	9.718	22.273
S. b_1*=0.50*							
BS-G	25	0.009	0.022	-0.010	0.014	-0.033	0.019
|BS-G|	25	0.016	0.017	0.013	0.012	0.033	0.019
100*|BS-G|/G	25	2.501	7.902	2.145	9.657	3.554	11.918
T. b_1*=0.65*							
BS-G	125	0.045	0.138	-0.001	0.111	-0.059	0.098
|BS-G|	125	0.084	0.119	0.065	0.090	0.084	0.077
100*|BS-G|/G	125	4.878	12.437	4.127	13.749	5.909	16.057
U. b_1*=0.80*							
BS-G	25	0.232	0.193	0.034	0.134	-0.210	0.147
|BS-G|	25	0.238	0.185	0.116	0.071	0.212	0.145
100*|BS-G|/G	25	12.500	20.208	8.007	19.497	11.884	23.501

Table 2 (Continued)
The Black-Scholes Prices (BS) Compared to GARCH Prices (G) for European Call Options

		(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns):					
		0.75		1.00		1.25	
	N	Mean	Stdev	Mean	Stdev	Mean	Stdev
V. a_1+b_1*=0.675,0.700*							
BS-G	50	0.012	0.019	-0.007	0.011	-0.032	0.018
|BS-G|	50	0.016	0.016	0.009	0.010	0.032	0.018
100*|BS-G|/G	50	1.879	5.839	1.470	7.250	3.160	9.960
W. a_1+b_1*=0.825*							
BS-G	75	-0.004	0.078	-0.038	0.072	-0.082	0.074
|BS-G|	75	0.054	0.056	0.049	0.064	0.083	0.073
100*|BS-G|/G	75	3.880	11.000	3.540	13.200	5.690	15.600
X. a_1+b_1*=0.950,0.975*							
BS-G	50	0.227	0.194	0.074	0.158	-0.114	0.184
|BS-G|	50	0.240	0.177	0.144	0.096	0.177	0.122
100*|BS-G|/G	50	11.990	19.380	8.620	19.390	10.080	22.690

(a) When comparing prices for the alternative levels of a given GARCH process or preference parameter (say, l), the middle values for the other such parameters (a_1=0.175, b_1=0.65) are assumed. Thus the number of cases (N) corresponding to the middle value of any of these parameters is high (125).

Table 3

The Bias of the Black-Scholes Model (BS - GARCH) Under Low, Equal and High Initial Conditional Variance for Options Maturing in 180 Days and the Middle Values of the GARCH Process Slope Parameters [Measure *Q* Unconditional Volatility (σ)=0.25; Initial Conditional Voaltility ($\sqrt{h_1}$)=0.75σ (low), σ (equal), 1.25σ (high)]

Initial Conditional Variance: Low	Equal	High	α_1	β_1	λ	T (Days)	S/X
0.011	-0.007	-0.031	0.175	0.65	0.01	180	0.8
0.052	0.013	-0.036	0.175	0.65	0.01	180	0.9
0.073	0.024	-0.038	0.175	0.65	0.01	180	1.0
0.053	0.010	-0.047	0.175	0.65	0.01	180	1.1
0.020	-0.011	-0.051	0.175	0.65	0.01	180	1.2
0.022	0.004	-0.020	0.175	0.65	0.10	180	0.8
0.050	0.011	-0.038	0.175	0.65	0.10	180	0.9
0.043	-0.006	-0.069	0.175	0.65	0.10	180	1.0
0.005	-0.039	-0.096	0.175	0.65	0.10	180	1.1
-0.029	-0.060	-0.101	0.175	0.65	0.10	180	1.2
0.009	-0.009	-0.033	0.175	0.65	0.20	180	0.8
-0.001	-0.041	-0.092	0.175	0.65	0.20	180	0.9
-0.053	-0.103	-0.168	0.175	0.65	0.20	180	1.0
-0.107	-0.153	-0.211	0.175	0.65	0.20	180	1.1
-0.127	-0.159	-0.201	0.175	0.65	0.20	180	1.2

Table 4
The Three components of the GARCH Option Valuation Effect: the Unconditional Variance (U.V.) Bias (BS-MBS), the Conditional Variance (C.V.) Bias (MBS-PGARCH), and the Path Dependence (P.D.) Bias (PGARCH-GARCH)[(a)]

(Initial Standard Deviation of Logarithmic Stock Returns)/
(Unconditional Standard Deviation of Logarithmic Stock Returns):
0.75 1.00 1.25

	N	Mean	Mean Bias	Mean PROP[(a)]	Mean Bias	Mean PROP	Mean Bias	PROP
A. All Cases								
U.V. Bias	175	-0.083	26.881	-0.083	41.172	-0.083	25.311	
C.V. Bias	175	0.064	35.945	0.000	0.000	-0.081	40.735	
P.D. Bias	175	0.085	37.174	0.085	58.828	0.087	33.955	
B. Deep-out-of-the-money (S/X=0.8)								
U.V. Bias	35	-0.053	24.075	-0.053	35.907	-0.053	23.145	
C.V. Bias	35	0.028	30.888	0.000	0.000	-0.037	36.586	
P.D. Bias	35	0.059	45.037	0.060	64.093	0.061	40.269	
C. Near-out-of-the-money (S/X=0.9)								
U.V. Bias	35	-0.084	24.770	-0.084	44.436	-0.084	24.628	
C.V. Bias	35	0.065	37.644	0.000	0.000	-0.084	45.280	
P.D. Bias	35	0.120	37.586	0.122	55.564	0.126	30.092	
D. At-the-money (S/X=1.0)								
U.V. Bias	35	-0.105	26.536	-0.105	42.532	-0.105	27.030	
C.V. Bias	35	0.106	35.992	0.000	0.000	-0.128	45.753	
P.D. Bias	35	0.159	37.472	0.160	57.468	0.162	27.217	
E. Near-in-the-money (S/X=1.1)								
U.V. Bias	35	-0.095	31.501	-0.095	46.365	-0.095	27.172	
C.V. Bias	35	0.076	43.588	0.000	0.000	-0.098	43.666	
P.D. Bias	35	0.074	24.911	0.074	53.635	0.076	29.162	
F. Deep-in-the-money (S/X=1.2)								
U.V. Bias	35	-0.076	27.521	-0.076	36.621	-0.076	24.578	
C.V. Bias	35	0.044	31.613	0.000	0.000	-0.059	32.388	
P.D. Bias	35	0.014	40.866	0.012	63.379	0.009	43.034	
G. T = 30 Days								
U.V. Bias	35	-0.014	8.591	-0.014	23.000	-0.014	6.632	
C.V. Bias	35	0.061	46.174	0.000	0.000	-0.078	47.717	
P.D. Bias	35	0.006	45.234	0.000	77.000	-0.005	45.650	
H. T = 90 Days								
U.V. Bias	35	-0.040	19.465	-0.040	37.476	-0.040	17.288	
C.V. Bias	35	0.079	47.384	0.000	0.000	-0.101	52.790	
P.D. Bias	35	0.027	33.150	0.029	62.524	0.034	29.922	
I. T = 180 Days								
U.V. Bias	35	-0.071	28.624	-0.071	45.814	-0.071	26.034	
C.V. Bias	35	0.074	39.999	0.000	0.000	-0.094	45.401	
P.D. Bias	35	0.073	31.377	0.076	54.186	0.081	28.565	

J. T = 360 Days							
U.V. Bias	35	-0.115	36.684	-0.115	50.349	-0.115	34.461
C.V. Bias	35	0.060	29.352	0.000	0.000	-0.076	34.974
P.D. Bias	35	0.127	33.965	0.128	49.651	0.130	30.565
K. T = 720 Days							
U.V. Bias	35	-0.173	41.039	-0.173	49.220	-0.173	42.139
C.V. Bias	35	0.045	16.814	0.000	0.000	-0.058	22.790
P.D. Bias	35	0.194	42.147	0.194	50.780	0.195	35.072

Table 4 (Continued)
The Three components of the GARCH Option Valuation Effect: the Unconditional Variance (U.V.) Bias (BS-MBS), the Conditional Variance (C.V.) Bias (MBS-PGARCH), and the Path Dependence (P.D.) Bias (PGARCH-GARCH)[(a)]

		(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns): 0.75		1.00		1.25	
	N	Mean Bias	Mean PROP[(a)]	Mean Bias	Mean PROP	Mean Bias	Mean PROP
L. l=0.01							
U.V. Bias	25	0.000	0.635	0.000	2.909	0.000	0.621
C.V. Bias	25	0.029	56.046	0.000	0.000	-0.037	65.177
P.D. Bias	25	0.015	43.319	0.010	97.091	0.003	34.202
M. l=0.10							
U.V. Bias	125	-0.091	26.963	-0.091	44.416	-0.091	25.161
C.V. Bias	125	0.078	35.436	0.000	0.000	-0.099	39.577
P.D. Bias	125	0.112	37.602	0.114	55.584	0.119	35.262
N. l=0.20							
U.V. Bias	25	-0.123	52.718	-0.123	63.217	-0.123	50.750
C.V. Bias	25	0.030	18.389	0.000	0.000	-0.039	22.080
P.D. Bias	25	0.023	28.893	0.018	36.783	0.011	27.170
P. a_1=0.050							
U.V. Bias	25	-0.005	17.993	-0.005	45.064	-0.005	17.178
C.V. Bias	25	0.014	52.379	0.000	0.000	-0.018	55.317
P.D. Bias	25	0.006	29.629	0.000	54.936	-0.007	27.505
Q. a_1=0.175							
U.V. Bias	125	-0.077	28.668	-0.077	41.391	-0.077	27.164
C.V. Bias	125	0.063	34.980	0.000	0.000	-0.080	40.430
P.D. Bias	125	0.060	36.353	0.060	58.609	0.060	32.406
R. a_1=0.300							
U.V. Bias	25	-0.185	26.834	-0.185	36.186	-0.185	24.177
C.V. Bias	25	0.116	24.336	0.000	0.000	-0.148	27.677
P.D. Bias	25	0.291	48.830	0.300	63.814	0.316	48.146
S. b_1=0.50							
U.V. Bias	25	-0.016	31.347	-0.016	48.196	-0.016	31.542
C.V. Bias	25	0.013	28.961	0.000	0.000	-0.016	35.970
P.D. Bias	25	0.013	39.692	0.007	51.804	0.000	32.488
T. b_1=0.65							
U.V. Bias	125	-0.069	25.878	-0.069	39.505	-0.069	24.453
C.V. Bias	125	0.044	37.163	0.000	0.000	-0.056	41.931
P.D. Bias	125	0.071	36.960	0.068	60.495	0.066	33.616
U. b_1=0.80							
U.V. Bias	25	-0.217	27.430	-0.217	42.484	-0.217	23.370
C.V. Bias	25	0.217	36.839	0.000	0.000	-0.274	39.516
P.D. Bias	25	0.232	35.731	0.251	57.516	0.280	37.114

Table 4 (Continued)
The Three components of the GARCH Option Valuation Effect: the Unconditional Variance (U.V.) Bias (BS-MBS), the Conditional Variance (C.V.) Bias (MBS-PGARCH), and the Path Dependence (P.D.) Bias (PGARCH-GARCH)[(a)]

		(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns): 0.75		1.00		1.25	
	N	Mean Bias	Mean PROP[(a)]	Mean Bias	Mean PROP	Mean Bias	Mean PROP
V. a_1+b_1=0.0.675,0.700							
U.V. Bias	50	-0.011	24.67	-0.011	46.63	-0.011	24.36
C.V. Bias	50	0.014	40.67	0.000	0.00	-0.017	45.64
P.D. Bias	50	0.009	34.66	0.004	53.37	-0.004	30.00
W. a_1+b_1=0.825							
U.V. Bias	75	-0.051	28.19	-0.051	38.76	-0.051	26.97
C.V. Bias	75	0.029	36.37	0.000	0.00	-0.038	42.22
P.D. Bias	75	0.019	35.45	0.013	61.24	0.007	30.81
X. a_1+b_1=0.950,0.975							
U.V. Bias	50	-0.201	27.13	-0.201	39.34	-0.201	23.77
C.V. Bias	50	0.166	30.59	0.000	0.00	-0.211	33.60
P.D. Bias	50	0.261	42.28	0.276	60.66	0.298	42.63

(a) For a given option valuation situation, we first take the absolute value of each of the three biases, sum these absolute values, and then express each absolute value as a percentage of the sum to arrive at the PROP figure. PROP for a bias is thus an estimate of the relative importance of the magnitude of that bias in determining the overall or net GARCH option valuation effect, BS-GARCH.

Table 5

The Black-Scholes Model Bias (BS-GARCH), U.V. Bias (BS-MBS), C.V. Bias (MBS-PGARCH), P.D. Bias (PGARCH-GARCH), and Respective Percentage Proportion (PROP) for At-the-Money (S/X=1.0) Options Under Low and High Initial Conditional Variance [Measure Q Unconditional Volatility (σ)=0.25; Initial Conditional Volatility ($\sqrt{h_1}$)=0.75σ (low), 1.25σ (high)]

U.V. Bias	C.V. Bias	P.D. Bias	BS-GARCH	α_1	β_1	λ	T (Days)	U.V. Bias PROP[a]	C.V. Bias PROP	P.D. Bias PROP
A. Low Variance Situation										
-0.014	0.102	0.067	0.154	0.175	0.65	0.10	30	7.89	55.64	36.47
-0.025	0.058	0.045	0.078	0.175	0.65	0.10	90	19.51	45.45	35.03
-0.035	0.041	0.038	0.043	0.175	0.65	0.10	180	30.93	35.85	33.22
-0.050	0.029	0.035	0.014	0.175	0.65	0.10	360	43.94	25.46	30.59
-0.070	0.020	0.049	-0.001	0.175	0.65	0.10	720	50.18	14.49	35.33
B. High Variance Situation										
-0.014	-0.126	0.030	-0.111	0.175	0.65	0.10	30	8.50	74.10	17.40
-0.025	-0.074	0.021	-0.078	0.175	0.65	0.10	90	20.85	61.73	17.42
-0.035	-0.052	0.019	-0.069	0.175	0.65	0.10	180	33.15	49.15	17.70
-0.050	-0.037	0.022	-0.064	0.175	0.65	0.10	360	45.72	33.86	20.42
-0.070	-0.026	0.041	-0.054	0.175	0.65	0.10	720	50.91	18.95	30.14

(a) For a given option valuation situation, we first take the absolute value of each of the three biases, sum these absolute values, and then express each absolute value as a percentage of the sum to arrive at the PROP figure. PROP for a bias is thus an estimate of the relative importance of the magnitude of that bias in determining the overall or net GARCH option valuation effect, BS-GARCH.

Table 6
Absolute Percentage Bias of Black-Scholes (BS), Pseudo-GARCH (PGARCH), and Modified Black-Scholes (MBS) Models, and Simulation Standard Error (as a Percentage of the Price) of GARCH Option pricing Model

		(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns): 0.75		1.00		1.25	
	N	Mean	Stdev	Mean	Stdev	Mean	Stdev
A. All Cases							
BS	175	5.628	13.556	4.398	14.236	6.426	16.887
MBS	175	7.148	14.740	5.040	13.551	5.931	16.023
PGARCH	175	5.533	14.595	5.040	13.551	4.792	12.547
GARCH	175	0.636	1.729	0.569	1.477	0.540	1.342
B. Deep-out-of-the-money (S/X=0.8)							
BS	35	14.380	23.908	16.507	28.509	21.256	31.318
MBS	35	16.158	21.615	17.244	26.265	19.933	30.244
PGARCH	35	18.628	28.239	17.244	26.265	16.021	24.145
GARCH	35	2.364	3.315	2.090	2.809	1.949	2.531
C. Near-out-of-the-money (S/X=0.9)							
BS	35	8.996	13.617	3.257	5.105	7.364	12.570
MBS	35	13.031	19.543	4.533	5.854	6.704	10.747
PGARCH	35	5.575	6.527	4.533	5.854	4.456	5.960
GARCH	35	0.508	0.559	0.444	0.424	0.431	0.375
D. At-the-money (S/X=1.0)							
BS	35	3.919	5.784	1.456	1.916	1.973	2.384
MBS	35	5.215	7.108	2.486	3.334	1.793	2.098
PGARCH	35	2.593	3.156	2.486	3.334	2.443	3.552
GARCH	35	0.167	0.113	0.165	0.116	0.171	0.117
E. Near-in-the-money (S/X=1.1)							
BS	35	0.637	0.738	0.450	0.441	0.971	0.922
MBS	35	1.023	1.468	0.647	0.994	0.809	0.845
PGARCH	35	0.597	0.977	0.647	0.994	0.719	1.030
GARCH	35	0.087	0.078	0.087	0.079	0.091	0.080
F. Deep-in-the-money (S/X=1.2)							
BS	35	0.206	0.234	0.322	0.325	0.566	0.575
MBS	35	0.315	0.577	0.292	0.433	0.418	0.454
PGARCH	35	0.271	0.425	0.292	0.433	0.321	0.446
GARCH	35	0.056	0.059	0.057	0.060	0.059	0.061
G. T = 30 Days							
BS	35	17.462	25.504	15.126	28.309	21.840	31.055
MBS	35	18.468	26.419	14.130	27.050	20.418	30.176
PGARCH	35	15.953	28.751	14.130	27.050	13.296	24.935
GARCH	35	1.931	3.453	1.649	2.945	1.529	2.665
H. T = 90 Days							
BS	35	4.618	8.043	3.495	8.545	6.252	12.436
MBS	35	6.351	10.748	3.887	6.806	4.723	10.237
PGARCH	35	4.519	8.281	3.887	6.806	3.435	5.752
GARCH	35	0.492	0.908	0.451	0.805	0.430	0.728
I. T = 180 Days							
BS	35	2.911	4.974	1.272	2.208	2.253	3.532
MBS	35	5.071	8.568	2.768	4.756	1.540	2.361
PGARCH	35	2.730	4.555	2.768	4.756	2.837	5.032
GARCH	35	0.300	0.467	0.290	0.441	0.287	0.419
J. T = 360 Days							
BS	*35*	*1.919*	*3.223*	*1.187*	*1.891*	*1.080*	*1.106*
MBS	*35*	*3.434*	5.956	2.444	4.437	1.519	2.790
PGARCH	35	2.470	4.426	2.444	4.437	2.437	4.474
GARCH	35	0.238	0.286	0.236	0.280	0.236	0.275

Table 6 (Continued)
Absolute Percentage Bias of Black-Scholes (BS), Pseudo-GARCH (PGARCH), and Modified Black-Scholes (MBS) Models, and Simulation Standard Error (as a Percentage of the Price) of Duan's (1995) GARCH Option pricing Model

		(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns): 0.75		1.00		1.25	
	N	Mean	Stdev	Mean	Stdev	Mean	Stdev
K. T = 720 Days							
BS	35	1.227	1.822	0.911	1.347	0.706	0.977
MBS	35	2.419	3.910	1.973	3.330	1.456	2.656
PGARCH	35	1.993	3.334	1.973	3.330	1.954	3.341
GARCH	35	0.218	0.201	0.218	0.201	0.218	0.200
L. l = 0.01							
BS	25	3.967	12.852	3.855	14.750	5.609	16.849
MBS	25	3.972	12.851	3.858	14.750	5.605	16.850
PGARCH	25	4.028	15.609	3.858	14.750	3.797	14.217
GARCH	25	0.538	1.639	0.510	1.542	0.484	1.411
M. l = 0.10							
BS	125	6.314	14.408	4.667	14.634	6.638	17.395
MBS	125	8.229	15.946	5.515	13.979	6.396	16.411
PGARCH	125	6.067	15.041	5.515	13.979	5.275	12.849
GARCH	125	0.668	1.746	0.583	1.415	0.553	1.267
N. l = 0.20							
BS	25	3.858	9.278	3.596	11.996	6.183	14.805
MBS	25	4.924	8.635	3.849	10.000	3.933	13.433
PGARCH	25	4.368	11.254	3.849	10.000	3.375	9.045
GARCH	25	0.572	1.792	0.555	1.760	0.534	1.660
P. a_1 *= 0.050*							
BS	25	1.257	2.522	0.790	3.624	2.763	7.745
MBS	25	1.370	2.598	0.841	3.615	2.654	7.744
PGARCH	25	1.094	3.527	0.841	3.615	1.111	4.131
GARCH	25	0.206	0.742	0.169	0.613	0.234	0.887
Q. a_1 *= 0.175*							
BS	125	5.330	13.287	4.153	14.130	6.500	16.960
MBS	125	6.758	14.502	4.616	13.300	5.629	16.115
PGARCH	125	5.104	14.496	4.616	13.300	4.338	12.262
GARCH	125	0.635	1.803	0.572	1.562	0.535	1.413
R. a_1 *= 0.300*							
BS	25	11.488	18.914	9.231	19.657	9.718	22.273
MBS	25	14.879	19.594	11.360	18.531	10.720	20.591
PGARCH	25	12.117	19.484	11.360	18.531	10.742	17.184
GARCH	25	1.072	1.981	0.953	1.590	0.871	1.316
S. b_1 *= 0.50*							
BS	25	2.501	7.902	2.145	9.657	3.554	11.918
MBS	25	2.711	7.231	2.183	9.023	3.180	11.459
PGARCH	25	2.487	9.190	2.183	9.023	2.252	9.473
GARCH	25	0.427	1.370	0.423	1.376	0.428	1.397
T. b_1 *= 0.65*							
BS	125	4.878	12.437	4.127	13.749	5.909	16.057
MBS	125	5.906	12.866	4.624	13.228	5.521	15.411
PGARCH	125	5.041	14.107	4.624	13.228	4.430	12.465
GARCH	125	0.587	1.625	0.542	1.479	0.524	1.372
U. b_1 *= 0.80*							
BS	25	12.500	20.208	8.007	19.497	11.884	23.501
MBS	25	17.797	22.823	9.980	17.635	10.743	21.682

PGARCH	25	11.039	19.729	9.980	17.635	9.141	14.889
GARCH	25	1.089	2.424	0.850	1.589	0.731	1.146

Table 6 (Continued)
Absolute Percentage Bias of Black-Scholes (BS), Pseudo-GARCH (PGARCH), and Modified Black-Scholes (MBS) Models, and Simulation Standard Error (as a Percentage of the Price) of Duan's (1995) GARCH Option pricing Model

		(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns): 0.75		1.00		1.25	
	N	Mean	Stdev	Mean	Stdev	Mean	Stdev
V. a_1+b_1=0.675,0.700							
BS	50	1.879	5.839	1.470	7.250	3.160	9.960
MBS	50	2.041	5.420	1.512	6.837	2.920	9.680
PGARCH	50	1.790	6.925	1.512	6.837	1.680	7.260
GARCH	50	0.316	1.096	0.296	1.062	0.331	1.162
W. a_1+b_1=0.825							
BS	75	3.880	11.000	3.540	13.200	5.690	15.600
MBS	75	4.430	10.690	3.640	12.510	4.740	15.120
PGARCH	75	4.000	13.530	3.640	12.510	3.430	11.870
GARCH	75	0.553	1.687	0.529	1.618	0.506	1.507
X. a_1+b_1=0.950,0.975							
BS	50	11.990	19.380	8.620	19.390	10.800	22.690
MBS	50	16.340	21.100	10.670	17.920	10.730	20.930
PGARCH	50	11.580	19.410	10.670	17.920	9.940	15.930
GARCH	50	1.080	2.191	0.901	1.574	0.801	1.223

Table 7

Bias Proportion(PROP) and Percentage Error for BS, MBS, and PGARCH, and GARCH Standard Error as Percentage of GARCH Price (GARCHPSR) when (A) the Option is an Out-of-the-money (S/X=0.8,0.9), Short Maturity (T=30,90 Days) one Vs. Other Options,(B) the Option is a Deep-out-of-the-money (S/X=0.8), Shortest Maturity (T=30 Days) one Vs. Other Options, and (C) the BS Price is More Than Vs. Less Than 20 Percent Away from the GARCH Price[a]

(Initial Standard Deviation of Logarithmic Stock Returns)/ (Unconditional Standard Deviation of Logarithmic Stock Returns):	0.75		1.0		1.25	
A. Out-of-the-money(S/X=0.8,0.9) and Short Maturity(T=30,90 Days),OS, vs. Other Options (Other)						
	OS	Other	OS	Other	OS	Other
	N=28	N=147	N=28	N=147	N=28	N=147
U.V. Bias (BS-MBS) PROP[a]	14.085	29.318	36.697	42.024	12.414	27.767
C.V. Bias (MBS-PGARCH) PROP	45.843	34.059	0.000	0.000	51.314	38.719
P.D. Bias (PGARCH-GARCH)PROP	40.072	36.623	63.303	57.976	36.272	33.513
100*|BS-GARCH|/GARCH	23.770	2.172	21.949	1.055	32.390	1.481
100*|MBS-GARCH|/GARCH	26.334	3.494	20.634	2.070	29.317	1.477
100*|PGARCH-GARCH|/GARCH	23.653	2.081	20.634	2.070	19.001	2.086
GARCHPSR	2.925	0.200	2.521	0.197	2.335	0.198
B. Deep-out-of-the-money(S/X=0.8) and Shortest Maturity(T=30 Days),DVS, vs. Other Options (Other)						
	DVS	Other	DVS	Other	DVS	Other
	N=7	N=168	N=7	N=168	N=7	N=168
U.V. Bias (BS-MBS) PROP	5.363	27.777	4.805	42.687	2.594	26.257
C.V. Bias (MBS-PGARCH) PROP	24.202	36.434	0.000	0.000	21.883	41.520
P.D. Bias (PGARCH-GARCH)PROP	70.435	35.789	95.195	57.313	75.522	32.223
100*|BS-GARCH|/GARCH	54.190	3.604	65.461	1.854	75.288	3.557
100*|MBS-GARCH|/GARCH	48.561	5.423	62.315	2.654	73.343	3.123
100*|PGARCH-GARCH|/GARCH	66.419	2.996	62.315	2.654	58.523	2.553
GARCHPSR	8.310	0.316	7.175	0.293	6.609	0.287
C. BS is more than (MORE) vs. less than (LESS) 20 percent away from GARCH						
	MORE	*LESS*	*MORE*	*LESS*	*MORE*	*LESS*

	N=14	*N=161*	*N= 9*	*N=166*	*N=15*	*N=160*
U.V. Bias (BS-MBS) PROP	13.666	28.030	19.469	42.349	9.388	26.803
C.V. Bias (MBS-PGARCH) PROP	38.320	35.738	0.000	0.000	44.620	40.370
P.D. Bias (PGARCH-GARCH)PROP	48.014	36.232	80.531	57.651	45.991	32.826
100*|BS-GARCH|/GARCH	44.132	2.279	59.135	1.431	54.611	1.909
100*|MBS-GARCH|/GARCH	46.703	3.709	52.227	2.482	50.009	1.799
100*|PGARCH-GARCH|/GARCH	41.302	2.422	52.227	2.482	31.520	2.286
GARCHPSR	4.761	0.277	6.138	0.267	3.937	0.222

(a) For a given option valuation situation, we first take the absolute value of each of the three biases, sum these absolute values, and then express each absolute value as a percentage of the sum to arrive at the PROP figure. PROP for a bias is thus an estimate of the relative importance of the magnitude of that bias in determining the overall or net GARCH option valuation effect.

Appendix A

Analyses for the Nearly Integrated Variance Case

The table reports the GARCH price (GARCH), simulation standard error as a percentage of the GARCH price (GARCHPSR), Black-Scholes model mias (BS-GARCH), absolute percentage biases of the Black-Scholes model (BS), the Modified Black-Scholes formula (MBS), the Pseudo-GARCH formula (PGARCH), the relative importance of the unconditional variance (U.V.) bias (= BS-MBS), the conditional variance (C.V.) bias (= MBS-PGARCH), and the path dependence (P.D.) bias (= PGARCH-GARCH) as measured by the respective bias proportions[(a)].

GARCH Option Pricing Parameters are a_1 = 0.05, b_1 = 0.94, s = 0.25, and l = 0.10.

U.V. Bias PROP	C.V. Bias PROP	P.D. Bias PROP	T	S/X	GARCH Price	BS-GARCH	Absolute Percentage Bias: BS	MBS	PGARCH	GARCHPSR
A. Low Initial Conditional Variance (Öh_1/s =0.75)										
18.52	62.96	18.52	30	0.8	0.001	0.001	116.67	200.00	83.33	20.747
12.93	84.78	2.29	30	0.9	0.072	0.125	174.30	206.70	5.73	2.593
10.16	84.90	4.95	30	1.0	2.234	0.572	25.60	28.87	1.59	0.120
11.69	80.07	8.24	30	1.1	10.138	0.161	1.58	1.89	0.22	0.028
14.29	64.29	21.43	30	1.2	20.006	0.006	0.03	0.04	0.02	0.007
15.93	78.35	5.72	90	0.8	0.065	0.078	120.28	154.07	12.14	3.436
13.46	79.74	6.80	90	0.9	0.786	0.451	57.35	67.91	5.33	0.577
12.72	79.77	7.50	90	1.0	4.119	0.738	17.92	20.98	1.80	0.134
13.91	83.18	2.92	90	1.1	11.115	0.468	4.21	5.09	0.19	0.056
13.44	71.49	15.07	90	1.2	20.246	0.148	0.73	0.96	0.26	0.026
18.92	76.18	4.90	180	0.8	0.476	0.231	48.51	63.27	3.82	1.059
16.66	71.15	12.19	180	0.9	2.137	0.563	26.36	32.94	4.82	0.337
16.68	72.49	10.84	180	1.0	6.154	0.711	11.55	14.44	1.88	0.145
18.70	80.14	1.16	180	1.1	12.742	0.538	4.22	5.48	0.08	0.076
17.15	70.83	12.03	180	1.2	21.128	0.274	1.30	1.83	0.37	0.045
24.35	59.90	15.75	360	0.8	1.803	0.314	17.42	25.69	5.35	0.523
23.40	58.30	18.30	360	0.9	4.561	0.494	10.82	15.58	3.72	0.264
24.06	60.18	15.75	360	1.0	9.157	0.540	5.90	8.64	1.79	0.159
26.31	65.62	8.07	360	1.1	15.507	0.439	2.83	4.41	0.48	0.102
27.34	67.64	5.02	360	1.2	23.227	0.270	1.16	2.06	0.17	0.071
32.62	41.38	26.00	720	0.8	4.496	0.273	6.08	11.79	4.55	0.345
33.25	42.23	24.51	720	0.9	8.271	0.321	3.88	7.74	2.84	0.231
34.81	44.23	20.96	720	1.0	13.373	0.307	2.29	4.92	1.58	0.169
37.08	47.09	15.83	720	1.1	19.656	0.248	1.26	3.07	0.77	1.86
0.27	0.099									
B. High Initial Conditional Variance (Öh_1/s =1.25)										
2.56	63.08	34.36	30	0.8	0.021	-0.020	93.75	91.35	32.21	5.225
8.22	89.01	2.76	30	0.9	0.463	-0.267	57.57	52.56	1.69	0.683
9.68	83.78	6.55	30	1.0	3.460	-0.655	18.92	16.81	1.42	0.106
8.07	84.04	7.89	30	1.1	10.685	-0.386	3.62	3.33	0.29	0.038
3.33	58.49	38.18	30	1.2	20.102	-0.090	0.45	0.43	0.17	0.014
9.81	88.89	1.29	90	0.8	0.362	-0.218	60.36	54.28	0.80	1.250
10.70	77.75	11.55	90	0.9	1.833	-0.596	32.54	28.01	4.89	0.353
11.44	78.09	10.48	90	1.0	5.728	-0.871	15.20	13.00	2.01	0.133
12.10	86.90	1.00	90	1.1	12.377	-0.794	6.42	5.62	0.07	0.066
9.07	74.80	16.12	90	1.2	20.904	-0.510	2.44	2.22	0.39	0.039
13.31	73.76	12.93	180	0.8	1.099	-0.392	35.64	29.24	6.22	0.695
13.73	69.80	16.48	180	0.9	3.388	-0.687	20.29	16.13	4.99	0.292
14.35	71.25	14.40	180	1.0	7.748	-0.882	11.39	9.09	2.30	0.152
15.48	78.43	6.09	180	1.1	14.192	-0.912	6.42	5.29	0.45	0.088
14.42	77.09	8.49	180	1.2	22.186	-0.783	3.53	3.02	0.30	0.058
19.43	60.50	20.08	360	0.8	2.576	-0.459	17.82	12.04	5.98	0.464
19.76	59.72	20.52	360	0.9	5.703	-0.648	11.36	7.55	3.95	0.260
20.56	61.61	17.83	360	1.0	10.481	-0.784	7.48	5.09	2.07	0.168
22.07	66.65	11.28	360	1.1	16.802	-0.856	5.09	3.64	0.74	0.114
24.51	75.36	0.13	360	1.2	24.348	-0.851	3.49	2.64	0.00	0.082
28.49	45.41	26.09	720	0.8	5.200	-0.430	8.28	3.34	4.52	0.336

29.23	46.14	24.62	720	0.9	9.146	-0.553	6.05	2.57	2.94	0.233
30.54	48.07	21.39	720	1.0	14.339	-0.659	4.60	2.14	1.72	0.175
32.28	50.92	16.80	720	1.1	20.635	-0.731	3.54	1.82	0.90	0.136
34.51	54.77	10.73	720	1.2	27.844	-0.765	2.75	1.54	0.38	0.107

(a) For a given option valuation situation, we first take the absolute value of each of the three biases, sum these absolute values, and then express each absolute value as a percentage of the sum to arrive at the PROP figure. PROP for a bias is thus an estimate of the relative importance of the magnitude of that bias in determining the overall or net GARCH option valuation effect.