

Emulation in Teams and Families

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Abstract

We present a model where the probability distribution over the space of an agent's achievements depends not only on her ability and effort, but also on the goals set for her (aka the intensity of her training program). The agent chooses her effort according to her utility net of perceived cost of effort. The principal incurs costs in setting goals, i. e. coaching the agent, and subtracts these costs from the maximand. The agent's perceived unit cost of effort is inversely proportional to an 'emulation function' that depends on the training undertaken by rival agents. In a situation where two principals separately attempt to motivate two rival agents we find a unique Nash equilibrium if each agent's ability exceeds the difficulty of training, both from the point of view of the agent's difficulty and the principal's. If not there may be no Nash equilibrium, or a stable-unstable pair. In the second half of the paper we turn our attention, not to teams, but to families where parents have a more caring attitude than coaches. We show that in many circumstances the emulation process reverses our naive intuition. There are now two distinct ways of prodding children or athletes to success: direct coaching and emulation through sibling rivalry. It is not a pretty world!

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1 Introduction

Emulation has long been recognized as an major force that shapes behaviour. Veblen (1924) defined emulation as “the stimulus of an invidious comparison which prompts us to outdo those with whom we are in the habit of classing ourselves.” He claimed that “with the exception of the instinct for self-preservation, the propensity for emulation is probably the strongest and most alert and persistent of economic motives proper.” According to Veblen “the motive that lies at the root of ownership is emulation; and the same motive of emulation continues active in the further development of institution to which it has given rise and in the development of all those features of the social structure which this institution of ownership touches.” Emulation interacts with the instinct of workmanship: “Man is an agent seeking in every act an accomplishment of some concrete, objective, impersonal end. This aptitude or propensity may be called the instinct of workmanship. Whenever the circumstances of traditions of life lead to an habitual comparison of one person with another in point of efficiency, the instinct of workmanship works out in an emulative or invidious comparison of persons.”

Educators have debated whether emulation should be encouraged. Rousseau was well known for his rejection of emulation. On the other hand, the use of ranking is often believed to stimulate performance. This paper explores some possible consequences of emulation. As a metaphor, we model the setting of goals for athletes by their coaches or for children by their parents, who know that their charges may perform better with some suitably chosen degree of emulation.

Recently there has been some empirical evidence from economists (Mas and Moretti, 2006) that indicate that emulation does influence workers’ performance. They found that introduction of highly productive workers into a shift increases the productivity of the existing workers. A 10% increase in

average co-worker productivity is associated with 1.7% increase in a worker's effort.

We consider the case where the principals are interested in the actual performance (which may or may not exceed the set goals), rather than just achieving the goal, with success or failure as the only possible outcomes.

2 The Model

Goal setting has been known to be a crucial element in achieving success, be it in sports, education, academia, or in the business world. Psychologists, researchers in sports science and management science have emphasized the importance of appropriate goal setting. (See Locke and Latham (1900a, 1990b), Hardy et al. (1986), Bell (1983), Botterill (1983), Cury and Sarrazin (1993), among others.) It is generally recognized that, subject to goals being realistic, performance increases as the goals become more difficult. This is known as "the hard goal effect" . (For empirical evidence in sports, see Beggs (1990), Cury and Sarrazin (1993).

The prospect of success by an individual or a firm depends on their own efforts and abilities but might also depend on other factors, such as how they perceive their environment and others. Their probability of success, hence their performance is possibly influenced by it.. The salesperson does not operate in an isolated environment. There are other sales people in the same firm or perhaps in rival firms. They also have set targets which also influence their likelihood of success. The central premise of this paper is that the goals, or targets, for which an individual's rivals are striving have a real influence on the probability of success of that individual. Agents are aware of the targets set for other agents and this forms the basis of the process we refer to as 'emulation'.

Emulation, defined by the Webster's dictionary as "ambition or endeavor

to equal or excel others (as in achievement)" is thus the main focus of the paper. We use the word 'emulation' in its common meaning¹. This concept has been investigated in the social sciences from the time of Bentham (1811) and later Veblen (1924) as discussed above.

Consider an athlete or a child that faces a set goal S . We assume that, given the pair (E, S) of effort level and goal, the probability that her actual performance, A , exceeds a given number a is

$$\Pr(A \geq a; E, S) = \frac{\lambda E}{\lambda E + S^{-1}a^2} = \frac{1}{1 + \frac{1}{\lambda ES}a^2} \quad (1)$$

where $E \geq 0$ is her effort level, λ denotes her ability.

The probability of reaching or exceeding a result increases with the set goal S because (i) the athlete has a more positive attitude the higher is the goal, and (ii) the setting of goals is actually accompanied by a training program; the higher S , the better the program.

Define

$$\alpha \equiv \frac{k}{\lambda ES}$$

Then

$$\Pr(A \geq a; \alpha) = \frac{1}{1 + \alpha a^2}$$

and

$$\Pr(A \leq a) = \frac{\alpha a^2}{1 + \alpha a^2} \equiv G(a)$$

where $G(a)$ is the cumulative distribution. The corresponding density function is

$$g(a) = G'(a) = \frac{2\alpha a}{(1 + \alpha a^2)^2}$$

¹In developmental psychology, emulation has a more specific meaning. Thompson and Russell (2004) distinguish emulation from three other behaviors relevant to observational learning (mimicking, stimulus enhancement, and imitation). According to their definition, in emulation, the subject "replicates, as a means to an end, not the body movement of the model but the dynamic result of the model's action, having noted how the manipulated element caused the desired result" (p. 883).

The expected performance is

$$\begin{aligned} \mathcal{E}(A) &= \int_0^\infty \frac{2\alpha a}{(1 + \alpha a^2)^2} da \\ &= \left\{ \frac{-a}{1 + \alpha a^2} + \frac{1}{\sqrt{\alpha}} \arctan(\sqrt{\alpha}a) \right\}_0^\infty = \frac{\pi}{2\sqrt{\alpha}} = \frac{\pi}{2} \sqrt{\lambda ES} \end{aligned} \quad (2)$$

This is the expected value of the actual performance, A , given the agent's ability, effort and coaching programme (the set goal).

Note: Expected performance increases with both effort and set goal so that it seems that setting ever higher goals will provide an ever larger reward. Clearly this is not so, as we now show by taking into account the behaviour of the agent.

2.1 Utility of the Agent

The agent will experience more utility, the higher her actual performance. However she has factored in the set goal S and falling short of it will disappoint her, so utility is decreasing in S . We take

$$U(A, S) = \frac{\rho A}{T + S}$$

where $T > 0$ and $\rho > 0$ is an arbitrary scaling constant. Let b be the perceived cost of effort to the agent. The agent maximizes expected utility net of effort cost:

$$\frac{\pi\rho}{2} \frac{1}{T + S} \sqrt{\lambda ES} - bE$$

We normalize so that $\pi\rho/2 = 1$. The first-order condition yields the agent's chosen effort level

$$E^* = \frac{\lambda S}{4b^2(T + S)^2} \quad (3)$$

The function $E^*(S)$ is single-peaked: it reaches its maximum at $S = T$. Therefore setting ever higher goals eventually becomes counterproductive.

Note also that an agent with a higher T will, ceteris paribus, exert less effort. The maximum effort exerted by the agent is

$$\frac{\lambda}{16Tb^2}$$

It is inversely proportional to T . Therefore agents with a higher T require a higher set goal, or more coaching, to produce their maximum effort and that effort is lower than for an individual with a lower T . This leads naturally to the interpretation of T as a measure of the *difficulty to motivate an agent, be they an athlete or a child*.

Substitute E^* into (2); we obtain the expected performance as a function of S

$$\mathcal{E}(A, E^*(S), S) = \left(\frac{\pi}{2}\right) \frac{\lambda S}{2b(T + S)} \quad (4)$$

Again we see that, given S , a higher T reduces expected performance. Also, while if S is set at the individual's own T , the expected performance is independent of T , the higher T individual nonetheless needs a higher set goal (more coaching) to reach this result; she is indeed harder to motivate.

Note also that (3) indicates that optimal effort is inversely proportional to the square of the perceived unit cost of effort to the agent, b . This remark will turn out to be crucial in the interpretation of our results.

2.2 The Principal's Objective

In a situation where there is one coach and one agent, the coach chooses S to maximize the expected performance net of training cost ωS

$$\left(\frac{\pi}{2}\right) \frac{\lambda S}{2b(T + S)} - \omega S = \left(\frac{\pi}{2}\right) \frac{\lambda}{2b(1 + S^{-1}T)} - \omega S$$

Define

$$w = \frac{2\omega}{\pi}$$

The first-order condition for the coach's problem yields:

$$S^* = T \left[\sqrt{\frac{\lambda}{2bwT}} - 1 \right] = \sqrt{\frac{\lambda T}{2bw}} - T \quad (5)$$

2.3 Emulation

We have argued earlier, and it is the central premise of this paper, that emulation (from the goal setting of rivals or siblings) has a positive influence on an agent's behaviour. The simplest way to model this is to assume it lowers the agent's perceived cost of effort. In practice this can take many forms, such as a fellow PhD student's aim to finish their thesis by December fortifying another student's resolve to work on weekends, a boy's concentration's on his piano lessons being made somewhat more palatable because his little sister now has violin lessons three times a week. Here we simply assume that effort cost for agent 1 is a decreasing function of the rival agent's goal, S_2

$$\frac{1}{2b} = F(S_2) \quad (6)$$

where $F'(S_2) > 0$. This means that an increase in S_2 decreases b . Emulation lowers effort cost.

Consider the situation where there are two agents and two principals: two principals coaching two rival athletes.

Coach i takes S_j as given. The first-order condition with respect to S_i is

$$S_i = \sqrt{\frac{F(S_j)\lambda T}{w}} - T$$

Suppose the emulation function is linear:

$$F(S_j) = \theta + \sigma S_j$$

The first-order condition for coach 1 yields

$$S_1^* = \sqrt{\frac{\lambda_1 T_1 (\theta + \sigma S_2)}{w}} - T_1 = \sqrt{\frac{\lambda_1 T_1}{\frac{w}{(\theta + \sigma S_2)}}} - T_1 \quad (7)$$

or,

$$S_i^* = \max \left\{ 0, \sqrt{\frac{\lambda_i T_i (\theta + \sigma S_j)}{w}} - T_i \right\} \quad (8)$$

This yields a positively-sloped and concave reaction function $S_i^* = R_i(S_j)$ when $S_i^* > 0$. Note that when $S_j = 0$ then $S_i^* > 0$ if and only if

$$\sqrt{\frac{\lambda_i T_i \theta}{w}} > T_i, \text{ or } \lambda_i \theta > w T_i \quad (9)$$

Proposition 1: If condition (9) holds then **there exists a unique equilibrium** $(S_1^N, S_2^N) > (0, 0)$.

Proof: $R_i(0) > 0$ and $R_i' > 0$, $R_i'' < 0$. So, in a diagram with S_2 measured along the horizontal axis and S_1 along the vertical axis, the reaction curve $S_1 = R_1(S_2)$ is concave with $R_1(0) > 0$, while the other reaction curve has a convex shape, with $R_2(0) > 0$. Therefore there is a unique intersection in the positive orthant. This requires agent i 's ability to be large enough compared with the difficulty of training her, from hers and her coach's point of view, wT_i . Similarly $\lambda_j \theta > wT_j$ guarantees a positive intercept for $R_2(S_1)$. If neither of these conditions are met the outcome could be either no Nash equilibrium or two equilibria, one of them unstable.

3 Teamwork

Now consider the situation where there is only one coach but two athletes. The coach is interested in the overall performance of the team, e. g. how many medals they bring back. He therefore maximizes the sum of expected performances, net of his own efforts, that is

$$\frac{\lambda_1 S_1 F(S_2)}{T_1 + S_1} + \frac{\lambda_2 S_2 F(S_1)}{T_2 + S_2} - w(S_1 + S_2)$$

Again, assume

$$F(S_j) = \theta + \sigma S_j$$

The first-order condition with respect to S_1 is

$$\frac{\lambda_1 F(S_2)}{(T_1 + S_1)^2} = w - \frac{\lambda_2 S_2 \sigma}{(T_2 + S_2)}$$

Hence

$$\frac{\lambda_1}{(T_1 + S_1)^2} = \frac{w}{\theta + \sigma S_2} - \frac{\lambda_2 S_2 \sigma}{(T_2 + S_2)(\theta + \sigma S_2)}$$

and thus

$$S_1 = \sqrt{\frac{\lambda_1 T_1}{\frac{w}{\theta + \sigma S_2} - \frac{\lambda_2 S_2 \sigma}{(T_2 + S_2)(\theta + \sigma S_2)}}} - T_1 \equiv r_1(S_2) \quad (10)$$

The right-hand side of (10) is larger than the right-hand side of (8) for any S_2 value. So, in a diagram with S_2 measured along the horizontal axis and S_1 along the vertical axis, the curve $r_1(S_2)$ lies above the curve $R_1(S_2)$.

Similarly, the first-order condition with respect to S_2 yields

$$S_2 = \sqrt{\frac{\lambda_2 T_2}{\frac{w}{\theta + \sigma S_1} - \frac{\lambda_1 S_1 \sigma}{(T_1 + S_1)(\theta + \sigma S_1)}}} - T_2 \equiv r_2(S_1)$$

It follows that the intersection of the two curves $S_1 = r_1(S_2)$ and $S_2 = r_2(S_1)$ must be to the North-East of the intersection of two curves $S_1^* = R_1(S_2)$ and $S_2^* = R_2(S_1)$. This shows that:

Proposition 2: The coach's optimal values of S_1 and S_2 exceed the Nash equilibrium (S_1^N, S_2^N) . We have shown that, left to his own devices, a coach will set higher goals, hence devote more resources to training athletes if he is not competing against another coach, but managing a team of athletes. The inclusion of emulation has stood on its head the old adage that competition is beneficial for the performance of everyone.

4 Families

Suppose now that the situation is that of a family trying to decide on an education policy for their children. It is not just a matter of how much to spend on school fees and college tuition. This is to a large extent determined by their financial circumstances. Parents have a deeper responsibility. They must set goals and boundaries for their children. Their guidance will make a real difference to their children's achievements. It shapes their expectations of happiness and success. It thus affects how they feel. All children are different, even in the same family. (One could speculate that one of the main objectives of families is to make their children more alike, but this is not the object of our enquiry here.)

Therefore some of the pertinent questions may be:

How are children of different abilities treated?

How are children who may be easier or harder to motivate treated?

How do parents with different resources cope? Is there a pattern that shows parents of different means react differently to the characteristics of their children?

What is the influence of emulation in all that?

What consequences does the parents' attitude towards the children have on how they are treated and on how they perform?

We attempt to answer these questions in the framework of our model.

4.1 Families Are Different from Teams

Parents have preferences over their childrens' achievements. Their own utility depends on it. They also have constraints on how they deal with the demands of settings goals and boundaries for them. As every parent knows, it is not simply a matter of deciding how far Junior will go to school, it must also

be explained, talked about; resources must be used to make the point and it all takes a psychic toll on the parents. Most parents love their children and care for them, but it does have costs, and the costs we are focusing on here are that of setting goals for children. It does take many forms, financial, psychological and time constraints.

In the framework of this model parents maximize a combination of their children's expected performances, subject to the cost of enforcing these goals not exceeding an 'educational budget' Y .

This can be justified by assuming strong separability between children's education and more mundane matters in the parents' utility.

Parents also have preferences over their childrens' achievements. We use a simple Cobb-Douglas function to represent these. The reason why we have chosen this function is because, although parents may not treat their children in the same manner, in the end parents would not be satisfied if one child achieved a very low performance. When we modelled the performance of a team in the previous section, we used a simple additive form. What mattered was the performance of the team, no matter who achieved it –how many medals did the swimming team take home for instance. We argue that parents take a more individual approach and favour a more even distribution of performances, hence the use of a Cobb-Douglas utility function to reflect the fact that a very poor performance by one of the children would severely affect the parents' utility. Coaches consider the athletes' achievements to be perfect substitutes while parents have a more neutral elasticity of substitution equal to 1.

Parents choose S_1 and S_2 to maximize

$$V(S_1, S_2) = [E(A_1)]^{\varepsilon_1} \cdot [E(A_2)]^{\varepsilon_2} \quad , \quad \varepsilon_i > 0$$

subject to

$$w(S_1 + S_2) = Y$$

or

$$S_1 + S_2 = y, \quad \text{where } y \equiv Y/w$$

The maximand is

$$\epsilon_1 L n \frac{\lambda_1 S_1 F(S_2)}{T_1 + S_1} + \epsilon_2 L n \frac{\lambda_2 S_2 F(S_1)}{T_2 + S_2}$$

The first order condition is

$$\frac{\epsilon_1}{S_1} - \frac{\epsilon_1}{S_1 + T_1} + \frac{\epsilon_2 F'(S_1)}{F(S_1)} = \frac{\epsilon_2}{S_2} - \frac{\epsilon_2}{S_2 + T_2} + \frac{\epsilon_1 F'(S_2)}{F(S_2)}$$

We will examine several alternative preferences of the parents.

(a) If $\epsilon_1 = \epsilon_2$, parents are completely even-handed, no matter what the individual characteristics of the children are.

(b) If parents prefer one child over the other, we can represent that by letting ϵ_1 and ϵ_2 be different while λ_1 and λ_2 , as well as T_1 and T_2 are the same (twins).

(c) If parents prefer the more able child, we could represent that by letting ϵ_1 and ϵ_2 be proportional to λ_1 and λ_2 , respectively.

(d) If parents prefer the less able child, we could represent that by letting ϵ_1 and ϵ_2 be proportional to λ_2 and λ_1 , respectively.

(e) Parents may also have preferences over the relative difficulties of motivating their children and so may take ϵ_1 and ϵ_2 to be proportional to T_2 and T_1 , respectively.

(f) Conversely they may favour the child who is harder to motivate and take ϵ_1 and ϵ_2 to be proportional to T_1 and T_2 , respectively.

We assume that the emulation function takes the form

$$F(S_i) = (S_i)^\beta, \quad 0 < \beta < 1$$

A multiplicative constant in the above formulation would not measure the strength of the emulative process, as it drops out of the calculations. The size of β however is such an indicator. When β is very small, close to zero, the value of S_i has little influence on $F(S_i)$ as the function is almost constant for all values of S_i that are not too small. On the contrary, when β is close to 1, the emulation factor is almost proportional to S_i . Therefore we interpret β as the sensitivity of one child to the goal set for their sibling, the strength of the emulation process.

Under this formulation the first-order condition is

$$\frac{\epsilon_1 + \beta\epsilon_2}{S_1} - \frac{\epsilon_1}{S_1 + T_1} = \frac{\epsilon_2 + \beta\epsilon_1}{S_2} - \frac{\epsilon_2}{S_2 + T_2} \quad (11)$$

In the next section we carry out comparative statics experiments in an attempt to answer the questions raised above.

4.2 Analysis of Family Decisions

We use the following notation in the Analysis.

$$T \equiv \frac{T_1 + T_2}{2}, \quad x \equiv \frac{\lambda_2}{\lambda_1}, \quad y^c \equiv 2T \left(\frac{1 - \beta}{\beta} \right), \quad y \equiv ny^c, \quad t \equiv \frac{T_2}{T_1}, \quad T_1 = \frac{2T}{1 + t}, \quad T_2 = \frac{2tT}{1 + t}$$

REMARK. We must comment on the critical 'educational budget' $y^c \equiv 2T \left(\frac{1 - \beta}{\beta} \right)$. When emulation is negligible and β is close to 0, y^c is very large and it is likely that the actual budget is below the critical value. When emulation is strong and β is close to 1, y^c is very small and it is almost certain that the actual budget is above the critical value. Recall that a goal set at T , in the absence of emulation, induces the largest possible effort, so that the values of S , y and T are on similar scales. The critical income value y^c is the budget above which emulation takes on a stronger, possibly

determinant role; we shall see that it yields results counterintuitive to a 'naive' understanding that doesn't factor in the consequences of emulation.

The optimum values are denoted by S_1^* and S_2^* . The first-order condition is $V_1 - V_2 = 0$, where

$$V_1 \equiv \frac{\epsilon_1 + \beta\epsilon_2}{S_1} - \frac{\epsilon_1}{S_1 + T_1} \quad \text{and} \quad V_2 \equiv \frac{\epsilon_2 + \beta\epsilon_1}{S_2} - \frac{\epsilon_2}{S_2 + T_2}$$

The derivatives of $V_1 - V_2$ with respect to S_1 and S_2 are negative and positive, respectively, because of the second-order condition.

We use the following method to ascertain which goal is larger at the optimum.

We evaluate V_1 and V_2 at (S, S) (Note that $S = y/2$ then). $V_i(S, S) > V_j(S, S)$ implies $S_i^* > S_j^*$ because $V_1 = V_2$ at the optimum and $S_1 = S_2$ along the constraint.

(a) Parents have no preference for either child. $\epsilon_1 = \epsilon_2$. (11) becomes

$$\frac{1}{S_1} - \frac{1}{S_1 + T_1} + \frac{1F'(S_1)}{F(S_1)} = \frac{1}{S_2} - \frac{1}{S_2 + T_2} + \frac{1F'(S_2)}{F(S_2)}$$

Suppose $T_2 > T_1$ then $V_2(S, S) > V_1(S, S)$ therefore $S_2^* > S_1^*$.

$d(V_1 - V_2)/dT_1 > 0$ and $d(V_1 - V_2)/dT_2 < 0$, hence $dS_1/dT_2 < 0$ and $dS_2/dT_2 > 0$.

If both children are willing to work equally hard, parents will set identical goals for them, irrespective of their individual abilities. Otherwise they will set a higher goal for the more difficult child and consequently coach their sibling less.

(b) If favouritism is arbitrary we let $\epsilon_1 = 1$, without loss of generality, and $\epsilon_2 > 1, T_1 = T_2 = T$ and $\lambda_1 = \lambda_2$. This is the case of a preferred twin.

The first-order condition (11) is

$$\frac{1 + \beta\epsilon_2}{S_1} - \frac{1}{S_1 + T} - \left[\frac{\epsilon_2 + \beta}{S_2} - \frac{\epsilon_2}{S_2 + T} \right] = 0$$

It is straightforward to show, using the method outlined above, that $V_2 \geq V_1$ requires $y \leq y^c$ and implies $S_2^* \geq S_1^*$. Conversely $y \geq y^c$ implies $S_2^* \leq S_1^*$. Therefore when the budget is below the critical value we observe the parents lavishing more care, or coaching, on the preferred twin. However when the educational budget is high, the situation is reversed. The explanation is this. Reversal is more likely to occur the stronger the influence of emulation is because y^c is small then. In this situation the favouritism shown by parents for child number 2 manifests itself by a higher goal set for child number 1, so as to stimulate the preferred twin by lowering the cost of child 2's effort. See equations(4) and (6). It is worth noting that when emulation is moderate, say $\beta = 0.5$, the critical income is $2T$, just enough to stimulate both children to maximum effort in the absence of emulation.

A low income (hence a likely weak emulation process) implies the favourite child is set a higher goal, but at a high income (strong emulation likely) their less favoured sibling is set a higher goal in order to lower the cost of effort to the favourite. The reason why this apparent reversal of interest occurs is that at higher income parents have exhausted the effectiveness of direct coaching on the 'chosen one' and resort to indirect influence through emulation by additional expense on the other child. This reversal (because of high income) is more likely to occur when emulation forces are strong.

It is also possible to find bounds on the differences between S_2^* and S_1^* .

Evaluating V_1 and V_2 at $S_2 = \varepsilon_2 S_1$, we find

$$V_1 = \frac{1 + \beta \varepsilon_2}{S_1} - \frac{1}{S_1 + T}, V_2 = \frac{\varepsilon_2 + \beta}{\varepsilon_2 S_1} - \frac{\varepsilon_2}{\varepsilon_2 S_1 + T} \text{ hence } V_2 < V_1 \text{ and } S_2^* < \varepsilon_2 S_1^*.$$

Conversely, evaluating V_1 and V_2 at $S_1 = \varepsilon_2 S_2$, we find in a similar fashion that $S_1^* < \varepsilon_2 S_2^*$. Neither goal can exceed the other by a factor of ε_2 or more generally by the ratio $\frac{\varepsilon_2}{\varepsilon_1}$. The simple ratio of preferences bounds the discrepancy between set goals.

Consider now the influence of the strength of preferences of parents.

The first-order condition (11) can be written as

$$\varepsilon_2 \left[\frac{1}{S_2 + T} - \frac{1}{S_2} + \frac{\beta}{S_1} \right] - \left[\frac{1}{S_1 + T} - \frac{1}{S_1} + \frac{\beta}{S_2} \right] = 0$$

The partial derivative of the first-order condition with respect to ε_2 is the first bracket, which has the same sign as the second bracket. These signs are those of

$$[\beta S_2(S_2 + T) - TS_1] \text{ and } [\beta S_1(S_1 + T) - TS_1] \text{ respectively}$$

hence it also the sign of their sum

$$[\beta(S_1^2 + S_2^2) + Ty(\beta - 1)] \text{ or } [\beta((S_1 + S_2)^2 - 2S_1S_2) - Ty(\beta - 1)].$$

This last expression has the sign of $[y^2 - 2S_1S_2 - tT(1 - \beta)/\beta]$, or

$$\left[y - T\left(\frac{1 - \beta}{\beta}\right) \right]^2 - T^2(1 - \beta)^2/4\beta^2 - 2S_1S_2 \text{ and finally}$$

$$\left[T\left(\frac{1 - \beta}{2\beta}\right) - y \right]^2 - \left[T\left(\frac{1 - \beta}{2\beta}\right) \right]^2 - 2S_1S_2 < 0.$$

Therefore

$$\frac{dS_2^*}{d\varepsilon_2} > 0$$

The more favoured one twin is, the higher the goal set for that child, everything else equal. However, if income is high (emulation is strong), that goal is still lower than that of the other twin.

The influence of T is not determinate. The partial derivative of the first-order condition with respect to T has the sign of $[(S_2 + T) - \sqrt{\varepsilon_2}(S_1 + T)]$.

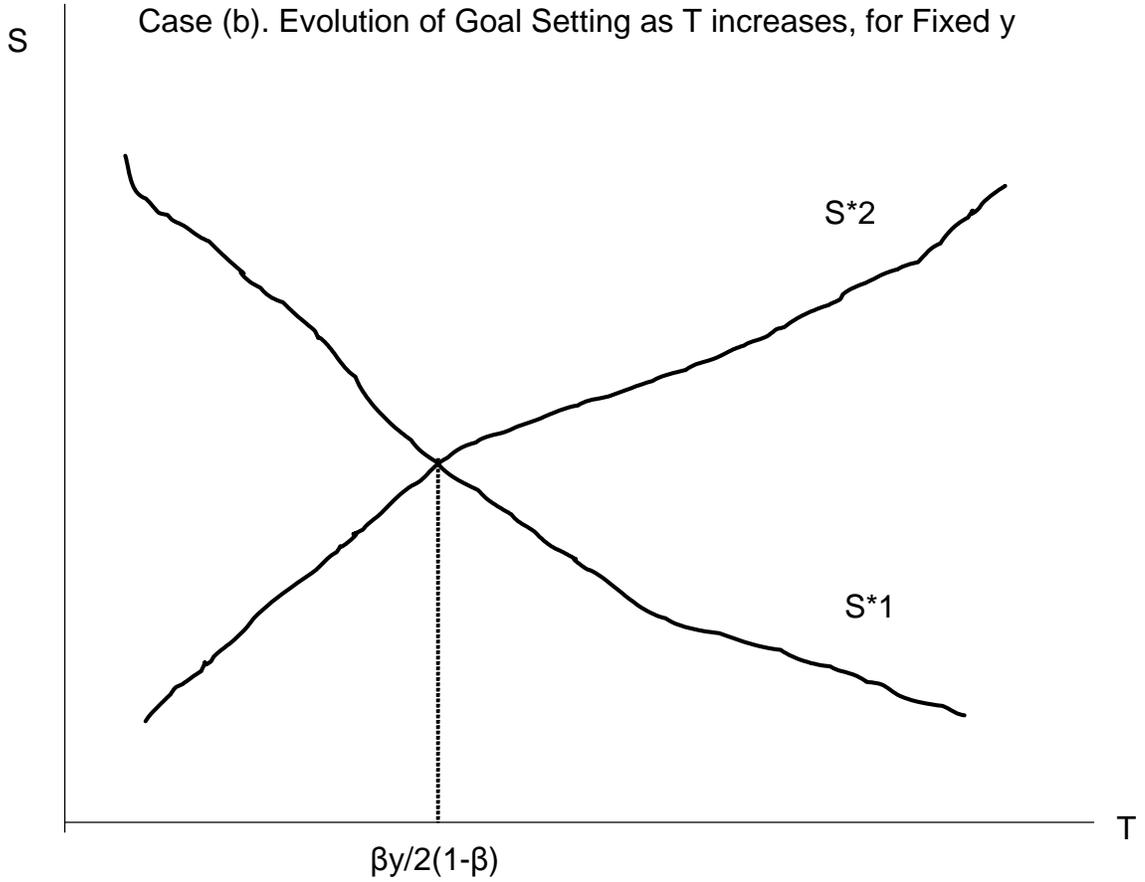


Figure 1

If $S_1 = S_2 = S$, that sign is negative (recall $\varepsilon_2 > 1$ here.) If $S_1 = \sqrt{\varepsilon_2}S_2$ or if $S_2 = \sqrt{\varepsilon_2}S_1$, the sign is also negative. Only if S_2 is very large compared to S_1 and T (and recall that $S_2^* < \varepsilon_2 S_1^*$) can the derivative be positive. In all our configurations it is negative and we take this to be the 'normal' result.

$$\text{Therefore 'normally' } \frac{dS_2^*}{dT} > 0 \text{ and } \frac{dS_1^*}{dT} < 0.$$

In this context, for fixed y , the most favoured child is set a higher goal

when the twins are harder to motivate. The reverse is true when the twins are keen to excel. It seems that the difficulty of educating these children exacerbates their different treatment; in this case unambiguously towards the favoured twin. The turning point, in Figure ?, corresponds to the critical value associated with reversal and is higher, the larger β is, or the stronger emulation works.

(c) If ϵ_1 and ϵ_2 are proportional to λ_1 and λ_2 , respectively, the first-order condition (11) becomes

$$\frac{\lambda_1 + \beta\lambda_2}{S_1} - \frac{\lambda_1}{S_1 + T_1} = \frac{\lambda_2 + \beta\lambda_1}{S_2} - \frac{\lambda_2}{S_2 + T_2}$$

First consider the case where $T_1 = T_2 = T$ and $x > 1$.

At (S, S) , $V_2 \leq V_1$ can be written as $\frac{1+\beta x}{S} - \frac{1}{S+T} \geq \frac{x+\beta}{S} - \frac{x}{S+T}$ which is true if and only if $y \geq y^c$ – Recall $S = y/2$.

Therefore when $t = 1$ and $x > 1$, $S_2^* < S_1^*$ if and only if $y > y^c$. By continuity this also true for t near 1.

In order to trace the boundary between the regions where $S_2^* < S_1^*$ and $S_2^* > S_1^*$, in the (x, t) space we use the first-order condition evaluated at (S, S) , $\frac{1+\beta x}{S} - \frac{1}{S+T_1} = \frac{x+\beta}{S} - \frac{x}{S+T_2}$. We obtain for the boundary

$$x = \frac{A^{-1} - \beta}{B^{-1} - \beta}, \text{ where } A \equiv 1+n \left(\frac{1-\beta}{\beta} \right) (1+t), \quad B \equiv 1+n \left(\frac{1-\beta}{\beta} \right) \left(\frac{1+t}{t} \right)$$

This boundary can be plotted for selected values of β and n . It is possible that the numerator and the denominator are of different signs, in which case one of the goals is always above the other. It is easy to show that, independently of the value of β ,

The numerator has the sign of $n - \frac{2}{1+t}$ and the denominator that of $n - \frac{2t}{1+t}$

From this result we can draw the following diagram that indicates the relative values of T_1 and T_2 and y (relative to T) where one of the goals

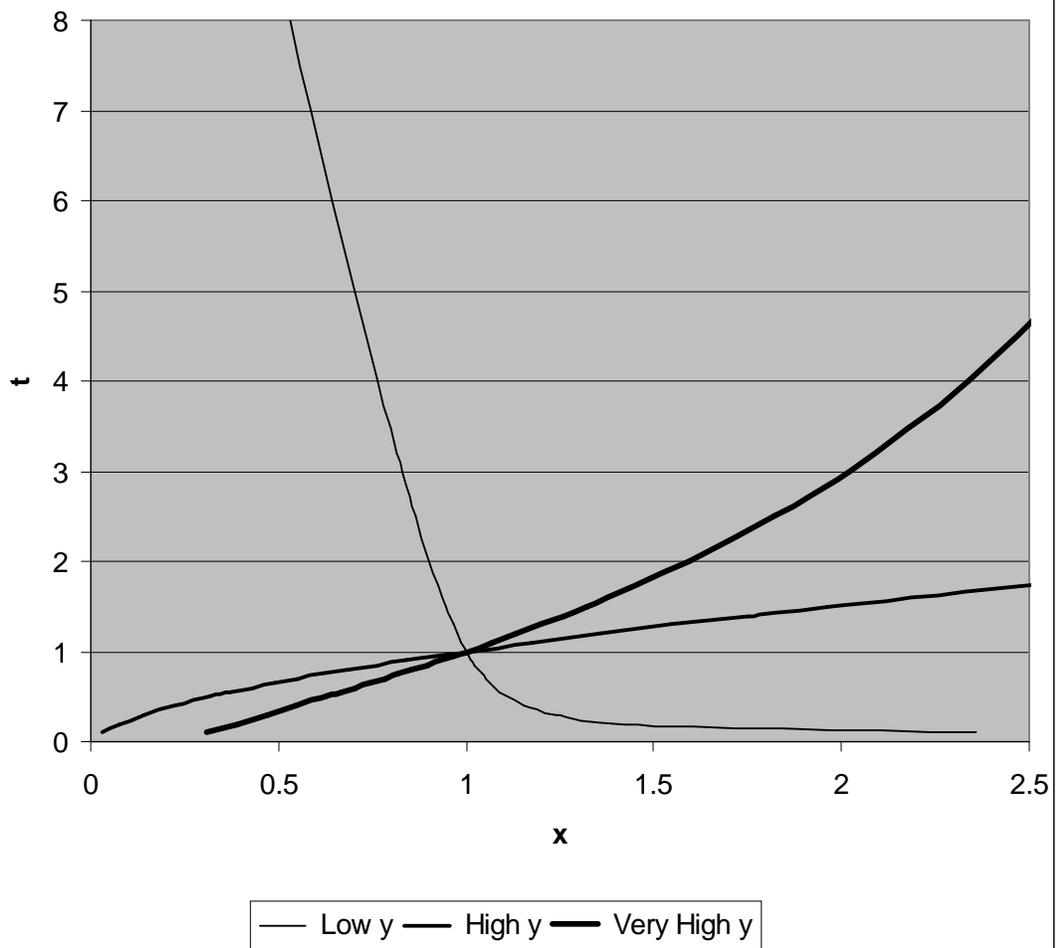
always dominates. This happens when y is not too far from y^c and T_1 and T_2 not too close together.

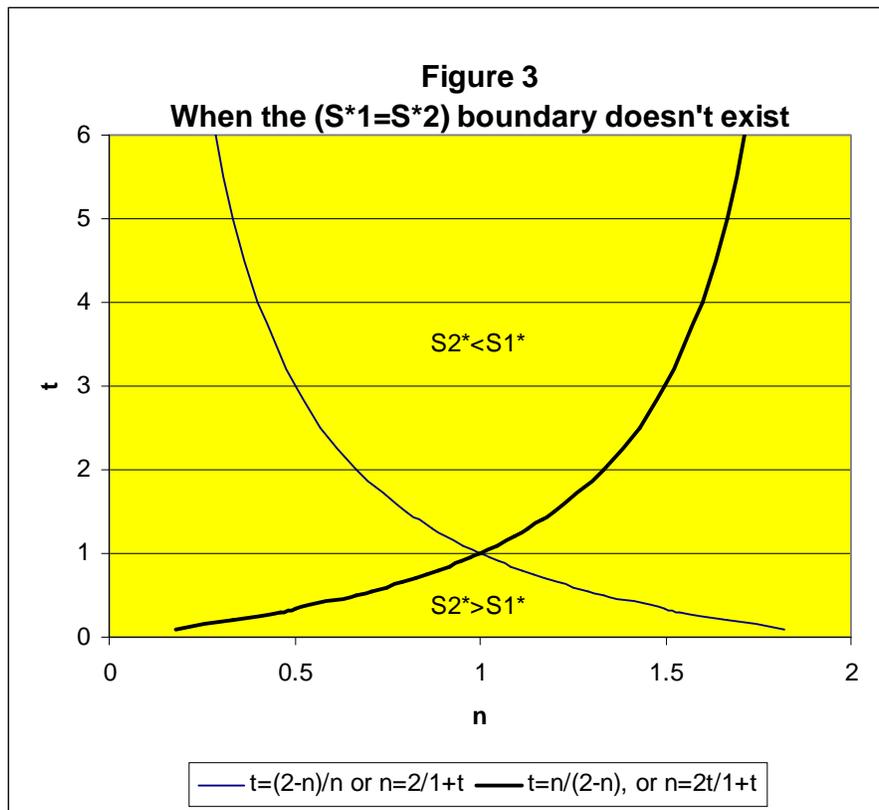
In the (x, t) space we illustrate how the $S_1^* = S_2^*$ boundary varies with income y .

The boundary rotates anti-clockwise as income increases. When y is close to y^c the boundary is almost a straight line at $t = 1$ and valid for a very small interval of t values, as expected from Figure 3.

In order to understand the partitionning of the (x, t) space let us rethink our description of 'high income'. It means $y > y^c$. This is more likely to occur when emulation is strong because in that case y^c is small. So let us now refer to situations of 'high income' as situations of 'strong emulation'. We have depicted the case of weak emulation (low income) in Fig. 4. Six regions are identified. The notations S_m^* and S_l^* refer to more gifted child and less gifted, respectively, as do T_m and T_l . In region I child 2 is more gifted and $S_m^* > S_l^*$; also $T_2 > T_1$ hence $T_m > T_l$. In region I" child 1 is more gifted and $S_m^* > S_l^*$ still; also $T_1 > T_2$ hence $T_m > T_l$ as well. In region II for instance $\lambda_1 > \lambda_2$ and $T_2 > T_1$ as well as $S_2^* > S_1^*$, so we have $S_m^* < S_l^*$ and $T_m < T_l$. Similar relations are deduced from Figure 2 for all six regions of Fig. 5. First note that regions I and I", II and II" etc... are homeomorphic in the terminology of 'more gifted' and 'less gifted'; their definitions are identical, as can be seen on the diagram. The terminology of child 1 and child 2, while useful in the calculations, must now give way to the more relevant distinction of ability. The interpretation of the results is as follows. When emulation is weak a child who is both more gifted and more difficult will attract more coaching (region I); this is expected when parents' preferences follow ability. This will hold true even if the more gifted child is a little easier to motivate. Only if one child is only slightly more gifted and much more difficult to motivate, will parents set him or her a lower goal (region II). They are then using the other child to lower the cost of effort to the more gifted and preferred one

FIGURE 2
S*1=S*2 Boundaries. Case(c)





Goal Setting for (c)-Low Budget, or (d)-High Budget

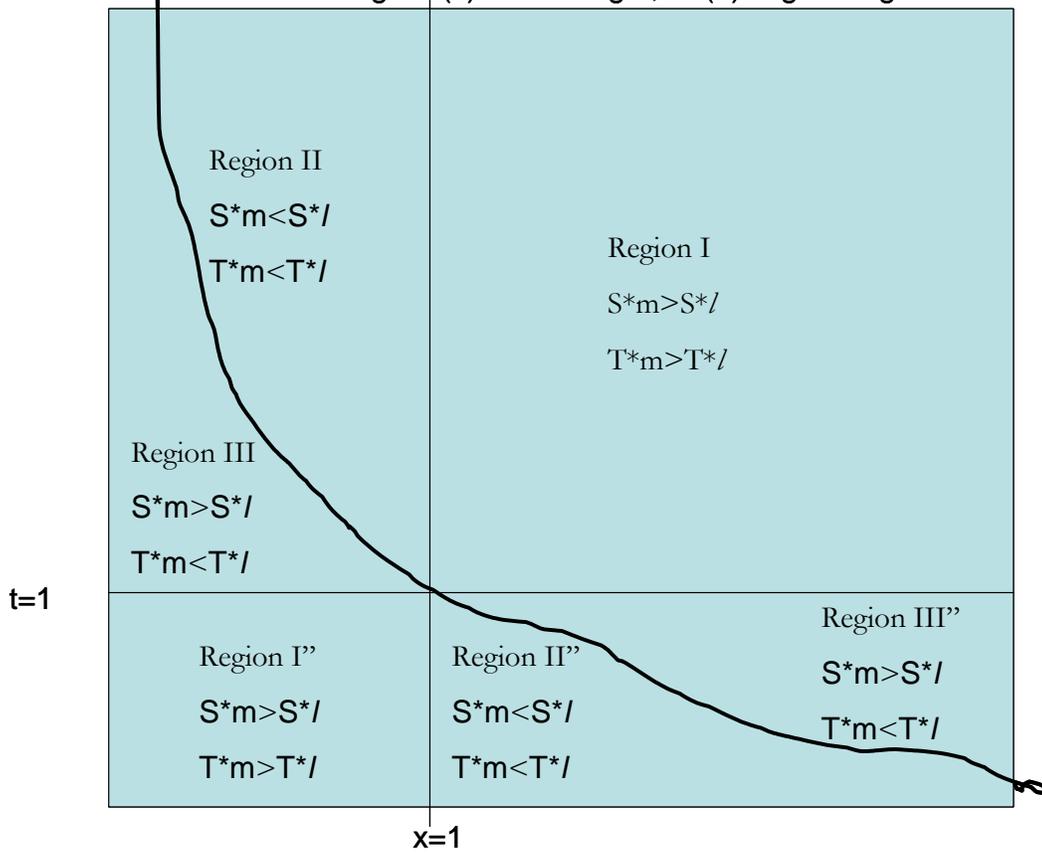


Figure 4

Goal Setting for (c)-High Budget, or (d)-Low Budget

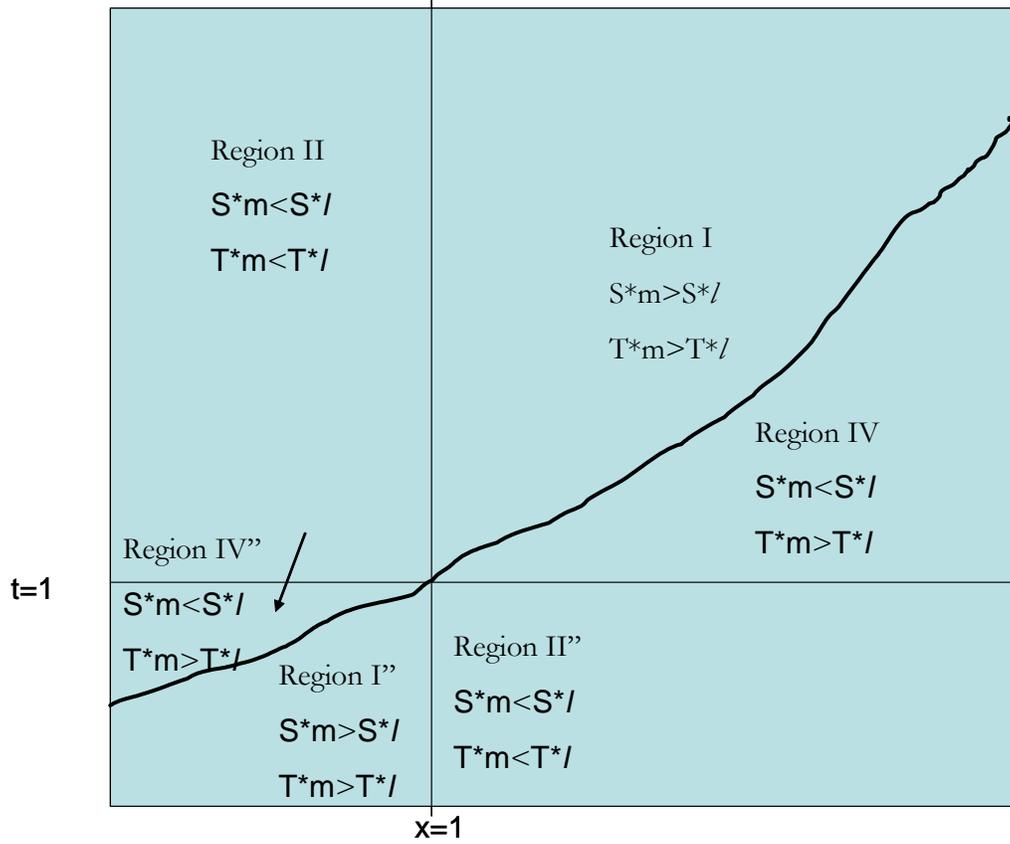


Figure 5

through emulation. This, in the (x, t) space seems to happen less frequently.

We now turn to the high budget (strong emulation) case; see Fig. 2. The boundary has rotated counter-clock wise, from the low budget situation and regions III and III" have disappeared when the boundary moved into the $t = 1$ line. Regions IV and IV" have emerged. They, and regions II and II", now dominate the diagram. When the emulation process is much stronger (high budget), the situation is reversed: parents use their considerable budget to help the favoured child indirectly, by lowering that child's effort cost through emulation and coaxing him or her into higher effort by coaching the less favoured child more. As remarked earlier, we could also view the situation as one where parents have exhausted their direct influence on the more gifted child through additional coaching and now revert to indirect coaxing by lowering the child's effort cost through emulation.

(d) Conversely we suppose that parents favour the less gifted child, ϵ_1 and ϵ_2 are proportional to λ_2 and λ_1 , respectively,

The first-order condition (11) is

$$\frac{\lambda_2 + \beta\lambda_1}{S_1} - \frac{\lambda_2}{S_1 + T_1} = \frac{\lambda_1 + \beta\lambda_2}{S_2} - \frac{\lambda_1}{S_2 + T_2}$$

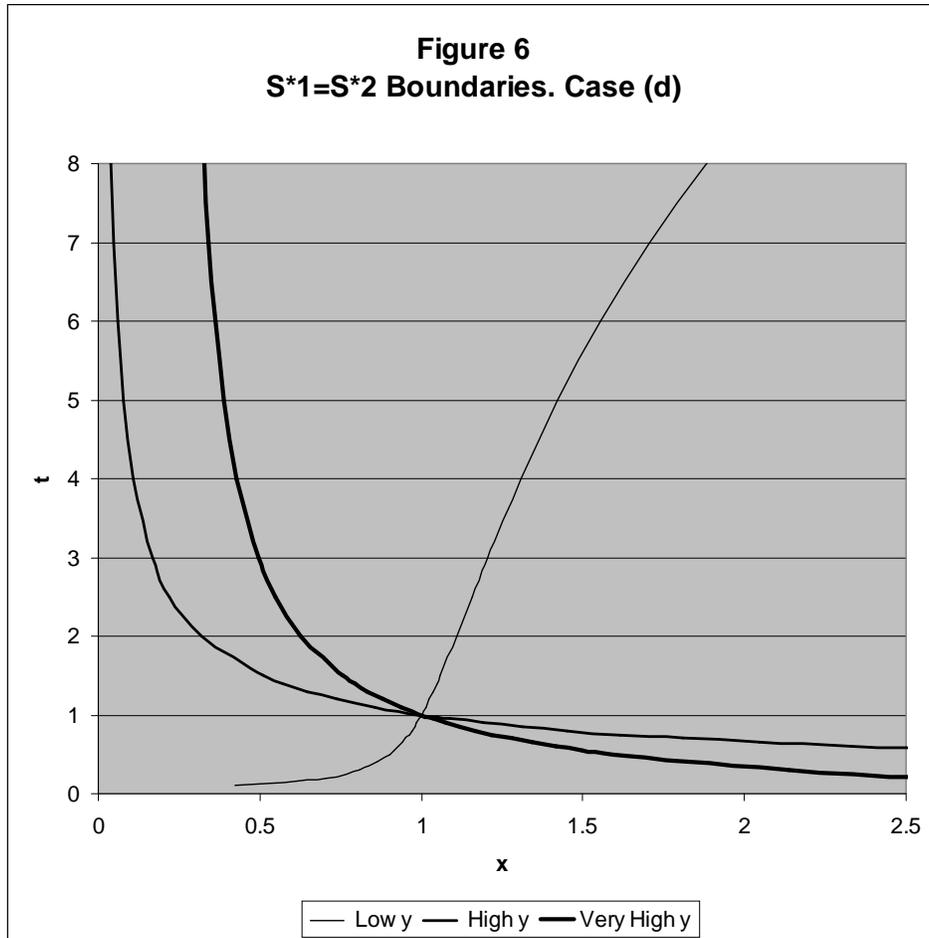
We proceed as in case (c).

The expression for the boundary $S_2^* = S_1^*$ is

$$x = \frac{(1 - \beta) - (1 + (B - 1)^{-1})^{-1}}{(1 - \beta) - (1 + (A - 1)^{-1})^{-1}} \text{ where A and B are as above.}$$

It is easy to show that, independently of the value of β , the numerator and the denominator are of the signs of

$$\frac{2t}{1+t} - n \text{ and of } \frac{2}{1+t} - n, \text{ respectively.}$$



Therefore the zones in (x, t) space where $S_2^* < S_1^*$ and $S_2^* > S_1^*$ are inverted compared to case (c).

The boundary rotates clockwise as income increases. When y is close to y^c the boundary is almost a straight line at $t = 1$ and valid for a very small interval of t values, as expected from Figure 3.

Now parents favour the child with the lower ability. An analysis of the diagram along the lines of what was done in case (c) reveals a startling fact. When emulation is weak (low income) in case (d) the broad shape of the

boundary and the exact definition of the regions match the results of case (c) when emulation is strong. Conversely, strong emulation with preference for the less able child match the results of weak emulation with preference for the more able child. This observation brings out a feature of the model. It is not the abilities of the child that drive the results: it is the parents' preferences, for whatever reason. The results in (c) and (d) are mirror images of one another.

(e) Parents may also have preferences over the relative difficulty of motivating their children. If they favour the child who is harder to motivate we can take

$$\text{Here } \epsilon_1 = \frac{T_1}{T_1 + T_2} \quad \epsilon_2 = \frac{T_2}{T_1 + T_2}, \text{ with, } \frac{T_2}{T_1} \equiv t > 1$$

Hence $T_1 = \frac{2T}{1+t}$ and $T_2 = \frac{2tT}{1+t}$. We also use the notation $y = zT = 2S$ in this case.

The first-order condition (11) is

$$V_1 - V_2 = 0, \text{ with } V_1 \equiv \frac{1 + \beta t}{S_1} - \frac{1}{S_1 + T_1} \text{ and } V_2 \equiv \frac{t + \beta}{S_2} - \frac{t}{S_2 + T_2}$$

At (S, S) let us suppose that $V_1(S, S) \geq V_2(S, S)$, which would imply $S_1^* \geq S_2^*$. This means:

$$\frac{1 + \beta t}{S} - \frac{1}{S + T_1} \geq \frac{t + \beta}{S} - \frac{t}{S + T_2} \text{ or } (1 - \beta) \leq \frac{S^2}{(S + T_1)(S + T_2)} \text{ or}$$

$$(1 - \beta) \leq \frac{z^2}{(z + 4/(1+t))(z + 4t/(1+t))}$$

Developing and simplifying, $z^2(\beta/(1 - \beta)) - 4z - 16t/(1 + t)^2 \geq 0$ or

$$y/T \equiv z \geq 2((1 - \beta)/\beta) \{1 + \sqrt{1 + 4(\beta/(1 - \beta))(t/(1 + t)^2)}\}$$

or $S_1^* \geq S_2^*$, if and only if

$$y \geq y^c \{1 + \sqrt{1 + 4(\beta/(1 - \beta))(t/(1 + t)^2)}\} \equiv \phi y^c > y^c$$

When y is larger than the fraction ϕ of y^c , defined above, the less favoured child, who is also the easier one to motivate, is set a higher goal. That fraction ϕ of y^c is larger, the larger β is and the smaller t is; that is the stronger emulation is and the less difference there is between the siblings. When y is smaller than ϕy^c , the ordering of goals is reversed.

The results here are not entirely straightforward. On the one hand, weak emulation makes it more likely that $y \leq y^c$, thus also $y \leq \phi y^c$ and therefore $S_1^* \leq S_2^*$. However weak emulation also makes ϕ smaller, working in the opposite direction. This is due to the fact that T has two roles here, one determining parents preferences for the difficult child, and the other directly in the children's utilities, hampering their performances. Nonetheless, on balance, it seems that weak emulation leads to the favoured child being trained more intensively (as in previous cases) and the reversal occurring when emulation is strong. Note finally that strong differences in T values increase ϕ and therefore make it more likely that the less favoured child will be set a higher goal to stimulate their sibling.

(f) If willingness to work hard is foremost in the parents' minds and they favour the more willing child we take

$$\epsilon_1 = \frac{T_2}{T_1 + T_2}, \quad \epsilon_2 = \frac{T_1}{T_1 + T_2} \quad \text{with} \quad \frac{T_2}{T_1} \equiv t > 1 \quad \text{and} \quad y = zT$$

Then (11) becomes

$$\frac{T_2 + \beta T_1}{S_1} - \frac{T_2}{S_1 + T_1} = \frac{T_1 + \beta T_2}{S_2} - \frac{T_1}{S_2 + T_2}$$

$$\text{Now } V_1 \equiv \frac{t + \beta}{S_1} - \frac{t}{S_1 + T_1} \quad \text{and} \quad V_2 \equiv \frac{1 + \beta t}{S_2} - \frac{1}{S_2 + T_2}$$

We follow a procedure similar to case (e) to show that

$$S_1^* \geq S_2^* \quad \text{if and only if} \quad z^2 + 4z - 16((1 - \beta)/\beta)(t/(1 + t)^2) \leq 0 \text{ or}$$

$$y/T \equiv z \leq 2\{-1 + \sqrt{1 + 4((1 - \beta)/\beta)(t/(1 + t)^2)}\} \equiv \psi$$

When y is smaller than the fraction ψ of T defined above the most favoured child, is set a higher goal. That fraction ψ of T is larger, the smaller β and t are; that is the weaker emulation is and the less difference there is between the siblings.

Here the results are more clearcut than in case (e). Recall that child 1 is now the favourite. Since ψ is decreasing in β and t , weak emulation will make it more likely that $y \leq \psi T$, consequently $S_1^* \geq S_2^*$. Weak emulation implies a stronger training program for the favoured child who is also easier to motivate. This will also be helped if the differences between the children's attitudes to work are slight. Now T 's distinct roles work in the same direction. The consequence of strong emulation is reversal of this state of affairs.

5 Conclusion

We have presented a model where the probability distribution over the space of an agent's achievements depends not only on her ability and effort, but also on the goals set for her (aka the intensity of her training program). The agent chooses her effort according to her utility net of perceived cost of effort. Chosen effort is a single-peaked function of the set goal and is inversely proportional to the perceived unit cost of effort. One of the parameters in her utility function is the difficulty the principal has to motivate her. Unsurprisingly the more difficult she is to motivate, the more training she requires.

The principal incurs costs in setting goals, i. e. coaching the agent, and subtracts these costs from the maximand. The agent's perceived unit cost of effort is inversely proportional to an 'emulation function' that depends on the training undertaken by rival agents. This is one of the essential mechanisms of the model.

In a situation where two principals separately attempt to motivate two rival agents we find a unique Nash equilibrium if each agent's ability exceeds the difficulty of training, both from the point of view of the agent's difficulty and the principal's. If not there may be no Nash equilibrium, or a stable-unstable pair.

In the second half of the paper we turn our attention, not to teams, but to families where parents have a more caring attitude than coaches. For the latter, the performances of athletes are perfect substitutes; for parents the elasticity of substitution is 1. A crucial parameter in our analysis is the 'critical value' of the educational budget of parents. It is large if emulation is weak and small if emulation is strong. The size of this critical value is inversely related to the strength of emulation.

We envisage several versions of parents' preferences.

■ If parents care equally for both children they will allocate a larger share of the budget to the more difficult child.

■ If the children are twins but the parents nonetheless prefer one of them, they will devote more resources to the favourite if emulation is weak. However if emulation is strong the situation is reversed: parents devote more resources to the less favourite child so as to decrease the favourite's perceived cost, hence boost that child's effort. We also see that this reversal occurs at higher income – parents have done all they could for their preferred child; they now work through the twin.

For a fixed educational budget, an increasing difficulty to motivate the twins always turn to the advantage of the favourite.

■ If parents base their their preferences on the respective abilities of the children, we find that it doesn't matter whether they prefer the more able or the less able. The results are the same once put in terms of "more liked" and "less liked". When emulation is weak a child who is both preferred and more difficult will attract more coaching. only if a child is only slightly preferred and much more difficult to motivate will parents set him or her a lower goal. They are then using the other child to stimulate the preferred one; all other avenues having been exhausted.

When emulation is very strong (this means a relatively high educational budget) the situation is reversed. The preferred child is more often set a lower goal, the parents concentrating on stimulating him or her through emulation from the other child. This is only reversed when the preferred child is much more difficult to motivate.

■ When parents' preferences are dictated by the ease of motivating a child, weak emulation implies a stronger training program for the easier child. Strong emulation and strong differences in attitudes to work will reverse this.

Therefore we have shown that in many circumstances the emulation process reverses our naive intuition. There are now two distinct ways of prodding children or athletes to success: direct coaching and emulation through sibling rivalry. It is not a pretty world!

In a different situation where one principal coaches both agents and is keen on the sum of their performances (medal tally), we find that the principal tries harder and that (the sum of the) performances are better.

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