On the Effect of Resource Exploitation on Growth: Domestic Innovation vs. Foreign Direct Investment*

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Abstract

We introduce a renewable natural resource sector into an endogenous growth model with an expanding variety of productive inputs. We first study an economy that carries out domestic innovation. This hypothesis matches with the historical experience of industrialized countries, but it is less consistent with the reality of developing economies, whose technological progress can be weak or even non-existent. A second model takes this fact into account and relies on trade and technological foreign direct investment to solve the problem of sustainability. Technology diffuses from a technological leading country to the country endowed with the natural resource. The existence, uniqueness and stability of a sustainable growth path are proved for both models. The growth rates and welfares under both scenarios, domestic innovation and foreign direct investment, are compared.

Keywords: Trade, renewable natural resources, foreign direct investment, endogenous growth, sustainability, technological progress.

JEL Codes: C61, C62, Q20, F18.

1 Introduction

Developing economies typically linked to the extraction and transformation of natural resources face two important challenges: the management of their environmental richness should generate economic wealth and its sustainability must be guaranteed along the years.

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Despite the potentially beneficial impact of natural resource wealth on economic growth, many countries suffer from what has been called the "resource curse".. There is a large body of empirical work that tries to establish a negative relationship between resource abundance and poor economic performance. However, the literature also assets that some countries managed to take advantage of their environmental endowments and receive a "blessing". There is no a single explanation in the economic literature of what creates a "blessing" rather than a "curse" (Sachs and Warner (1997, 1999), Rodriguez and Sachs (1999), Stevens (2003), Papyrakis and Gerlagh (2004)).One of the purposes of this paper is to throw light on this question.

Regarding the problem of the sustainability of the economic growth, since the 90s, economists have relied on technological change as the solution. Authors like Grossman & Helpman (1991), Smulders (1995), Bovenberg & Smulders (1995) and Elbasa & Roe (1996) among others, suggest that chances of achieving sustainable growth critically depend on maintaining a steady flow of technological innovations. A conclusion which is roughly consistent with historical experience in industrialized countries. However, many developing economies rely on underdeveloped, or even inexistent, R&D sectors.

This paper develops two models of endogenous growth for a country owning a renewable natural resource. The first model follows the literature on endogenous growth and the environment developed during the 90s. It describes an economy endowed with a renewable natural resource that, at the same time, invests on R&D. It allows us to concentrate on the impact of natural resource wealth on the long-run growth rate, although it may not properly explain the reality of developing countries. As Todo (2000) observes, developing countries tend to rely on foreign direct investment (FDI), rather than domestic R&D, as the major source of technological development. This fact is taken into account in the second model in this paper, where technological improvements diffuse from a technological leader country to the developing one [1].

Technology diffusion from a technological leading country to a follower by FDI requires a certain degree of openness in both countries. However, most of the endogenous growth models which tackle environmental problems study an isolated country and do not take into account trade relationships. At the same time international trade poses a particular problem for developing countries trying to manage their environment. Since the exploitation of natural resources remains a large sector in their economies, needs for foreign exchange encourage many developing countries to overexploit their natural resources. Trade in tropical timber, for example, is one factor underlying tropical deforestation. Thus, the pursuit of sustainability needs to take international economic relations into account.

International trade is analyzed in Elíasson & Turnovsky (2004) from the point of view of a small open economy in which the renewable resource is used

[1] Coe et al. (1997) reports that in 1990, industrial countries accounted for 96% of the world's R&D expenditure. Countries like United States, Japan, Germany, France and UK originate 90% of the patents in the world. The rest of the countries in the world are considered technological followers.

to purchase imports of a consumption good. They prove the existence of a sustainable growth path, that is, the coexistence of a limited natural resource sector with an unlimited growth of the economy. Their growth model is based on the AK model which allows growth even after the exhaustion of the renewable resource. This cannot explain the economies of many developing countries whose productive process depend on the extraction of the resource. One of the main results of Elíason & Turnovsky (2004) is that resource abundance reduces long-run growth rate. This result follows without invoking other explanatory variables given in the literature such us rent-seeking, sub-optimal allocation of resources, terms of trade or political incentives (Stevens (2003), Papyrakis & Gerlagh (2004), Robinson et al. (2006)). An economy having access to a more bountiful natural resource allocates more labor in the resource sector at the expense of less employment in the final output sector and consequently a lower long-run growth rate. This result critically depends on the harvesting function of the resource, which only requires the use of labor but it is not affected by the stock of the natural resource. A more bountiful natural resource could enhance the productivity of labor in the resource sector, as it is typically assumed in environmental models, (see Clark (1990) for further discussions on this topic), making unnecessary larger employment in the resource sector and without harming the long-run growth rate of the economy. The models presented in this paper consider both specifications for the harvesting function.

Contrary to the model of Elíasson and Turnovsky, a bilateral trade model is analyzed in Cabo et al. (2005). The natural resource extracted in one country is sold abroad, where it is used as an input. The resource-dependent country acts as the supplier of the natural resource but it has no industrial structure and the final output for consumption must be imported. The consumption growth in this country is a direct consequence of the economic growth in the industrialized country. The work developed in this paper represents a more realistic situation.

The models we propose in this paper extend the literature on endogenous growth and environment in several ways. The closed economy studied first allows to clarify the mechanism through which an economy can take advantage of resource abundance to increase its growth rate in the long-run. The second model addresses the problem of sustainability in an economy endowed with a natural resource but with no investment in technological progress. FDI is shown as a key element in the process of achieving sustainable growth in resource-dependent economies.

The rest of the paper is organized as follows. In section 2, we present a closed economy endowed with a natural resource which also invests on technological innovation. In section 3, the economy does not carry out R&D activities, but relies on FDI. In both sections we concentrate on steady-state equilibria and study their existence, uniqueness and stability. We provide also a sensitivity analysis of the steady-state equilibrium. In section 4, we compare the long-run growth rates and the consumers' welfare obtained with domestic innovation and FDI. In section 5, we conclude.

2 Sustainable growth with domestic innovation

In this section we deal with a closed economy endowed with a stock of a renewable natural resource which is harvested and used as an essential input in the production of final output, combined with labor and intermediate nondurable goods. The total labor force, which is assumed to be constant, is allocated between the harvesting of the natural resource and the production of final output. Intermediate goods are invented and produced by monopolistic entrepreneurships. Let us provide a detailed description of each sector of this economy.

2.1 Resource sector

At any point of time, the net growth rate of the renewable natural resource, S, is given by the natural reproduction of the resource minus the harvesting, that is,

$$\dot{S} = G(S) - R, \quad S(0) = S_0, \tag{1}$$

where $G(S)$ describes the gross reproduction rate of the resource, R is the rate of harvest and S_0 is the initial stock of the resource.[2] The reproduction function is assumed to be of the well-known logistic or Verlhust type (see, for example, Clark, 1990):

$$G(S) = gS\left(1 - \frac{S}{C}\right),$$

where g denotes the intrinsic growth rate of the natural resource and C represents the carrying capacity or saturation level.

The harvesting of the natural resource R depends upon labor and the size of the renewable resource (its stock). In its general specification, the harvesting function presents decreasing marginal returns to the effort (in our case identified by labor) and the stock level. Thus, the harvest rate can be represented by

$$R(L_S, S) = BL_S^{1-\delta}S^{\theta}, \quad B > 0, \quad 0 < \delta < 1, \quad 0 \le S \le C, \quad 0 \le \theta \le 1, \tag{2}$$

where L_S is the amount of labor employed in the resource sector. The decreasing marginal return to the stock of the natural resource comes as a result of the hypothesis of congestion; while the decreasing marginal return to labor is a consequence of ultimate gear saturation. One particular case is given by $\theta = 0$, which implies that harvesting is independent of the stock size (this case is studied by Elíasson & Turnovsky, 2004). Another particular case is $\theta = 1$, when the harvest rate corresponds to the well-known Schaefer pattern used in many other models. The main hypothesis is that the harvest is proportional to the stock of the renewable resource.[3] In what follows we shall name the harvest flow as R, omitting the arguments L_S and S.

[2] The time argument is eliminated when no confusion can arise.

[3] The hypothesis $\theta = 0$ is appropriate for forests or fish leaving close to the surface; whereas, $\theta = 1$ is suitable for bottom-dwelling fish (see, Elíasson & Turnovsky, 2004 and references therein).

2.2 Final output sector

The economy comprises a large number of identical firms, each of which produces final output using labor, the natural resource and nondurable intermediate inputs. The output production function of a representative firm is given by

$$Y = AL_Y^{1-\alpha-\beta} \sum_{j=1}^{N} X_j^{\alpha} R^{\beta}, \quad A > 0, \quad 0 < \alpha,\, \beta,\, \alpha+\beta < 1, \tag{3}$$

where L_Y is the labor input, X_j is the amount of nondurable input of type $j \in \{1, \dots, N\}$, and R is the resource input. This output production function is based on Spence (1976), Dixit & Stiglitz (1977) and Ethier (1982), but in our case it is subject to an environmental restriction. In addition to labor and intermediate goods, the natural resource is a necessary factor for production and growth in this economy. Output production has diminishing marginal productivity in each input, L_Y, X_j and R, and constant returns to scale in all inputs together.

Competitive firms equate net marginal products to factor prices:

$$w = (1-\alpha-\beta)\frac{Y}{L_Y}, \qquad p_R = \beta\frac{Y}{R}, \tag{4}$$

$$X_j = L_Y \left(\frac{\alpha A}{p_j}\right)^{\frac{1}{1-\alpha}} \left(\frac{R}{L_Y}\right)^{\frac{\beta}{1-\alpha}}, \tag{5}$$

where w is the wage rate, p_R is the price of the natural resource and p_j is the price of intermediate good j.

2.3 Behavior of innovators

At a point in time, the existing technology allows the production of N varieties of intermediate goods. Technological progress takes the form of an expansion in this number of varieties and follows the model of Barro & Sala-i-Martin (1999, Chapter 6). The production of each type of intermediate good is monopolized by a single firm. Assuming that, once invented, an intermediate good of type j costs σ units of Y to produce, the monopolist sets the price p_j, at each date, to maximize his instantaneous profits, $\pi_j = (p_j - \sigma)X_j$, where X_j is given in equation (5). The maximum is obtained at $p_j = \sigma/\alpha > \sigma$. Using this price in (5) we obtain

$$X_j = X = L_Y \left(\frac{A}{\sigma}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left(\frac{R}{L_Y}\right)^{\frac{\beta}{1-\alpha}}, \tag{6}$$

$$Y = AL_Y^{1-\alpha-\beta} N X^{\alpha} R^{\beta} = \frac{\sigma}{\alpha^2} N X. \tag{7}$$

Note that the amount of intermediate good X_j is the same for all $j \in \{1, \dots, N\}$ and depends on variables L_Y and R.

The cost to invent a new type of product is fixed at η times the production cost, that is, $\eta\sigma$ units of output Y. We assume free entry into the business of being an inventor so that, in equilibrium, the present value of the profits for each intermediate good must equal $\eta\sigma$, that is:

$$\eta\sigma = \int_t^{\infty} (p_j - \sigma) L_Y \left(\frac{A}{\sigma}\right)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left(\frac{R}{L_Y}\right)^{\frac{\beta}{1-\alpha}} e^{-\bar{r}(s,t)(s-t)} ds, \tag{8}$$

where $\bar{r}(s,t) = [1/(s-t)] \int_t^s r(w)dw$ is the average interest rate between times t and s.

Note that differentiating (8) with respect to t and taking into account that L_Y, R and r are time-dependent, it follows that

$$r = \frac{1}{\eta}\frac{1-\alpha}{\alpha} X = \frac{1}{\eta}\frac{1-\alpha}{\alpha} \alpha^{\frac{2}{1-\alpha}} L_Y \left(\frac{A}{\sigma}\right)^{\frac{1}{1-\alpha}} \left(\frac{R}{L_Y}\right)^{\frac{\beta}{1-\alpha}}. \tag{9}$$

2.4 Consumers

Consumers accumulate assets and receive financial interest income from them, together with the income derived from their labor. A representative consumer has one unit of labor per unit of time which must be allocated between the production of final output and the harvesting of the natural resource. She receives a wage income derived from her labor services in the final output sector, plus the profits from the extraction of the natural resource. We assume that the exploitation of the resource is managed as a cooperative of identical members with perfect property rights. The income from the sale of the resource is equally distributed between all members. Thus, per capita budget constraint for a representative consumer is

$$\dot{a} = ra + vw + p_R \frac{R}{\bar{L}} - c, \quad a(0) = a_0, \tag{10}$$

where a is per capita assets, $v \in [0,1]$ is the fraction of labor in the final output production, c is per capita consumption of the final good, $\bar{L}$ is the constant labor force and $p_R R/\bar{L}$ is per capita income derived from the extraction of the resource. Given previous definition of v, the labor employed either in the final output sector or in the resource sector can be redefined as $L_Y = v\bar{L}$ and $L_S = (1-v)\bar{L}$. The initial amount of per capita assets is denoted by a_0.

A representative consumer has to decide her consumption, c, and the fraction of labor, v and $1-v$, employed either in the final output production or in harvesting, to maximize her utility:

$$\max_{c,v} \int_0^{\infty} \ln(c) e^{-\rho t} dt, \qquad \rho > 0,$$

subject to (1) and (10). Performing the maximization problem leads to the necessary conditions for an interior solution:

$$\frac{1}{c} = \mu, \tag{11}$$

$$(1-\delta)\left(-\lambda + \mu\frac{p_R}{\bar{L}}\right)\frac{R}{1-v} = \mu w, \tag{12}$$

$$\dot{\mu} = \mu\left[\rho - r\right], \tag{13}$$

$$\dot{\lambda} + \lambda\left[g\left(1 - 2\frac{S}{C}\right) - \theta\frac{R}{S} - \rho\right] = -\mu\frac{p_R}{\bar{L}}\theta\frac{R}{S}, \tag{14}$$

together with the transversality conditions

$$\lim_{t\to\infty}\mu(t)a(t)e^{-\rho t} = 0, \ \lim_{t\to\infty}\lambda(t)S(t)e^{-\rho t} = 0, \tag{15}$$

where μ and λ are the shadow values of assets and the renewable resource, respectively.

Condition (11) equates the marginal utility of consumption to the shadow value of assets. Condition (12) equates the marginal returns of labor in the two sectors. The return of labor in the final output sector is just its salary, whereas the marginal benefit yielded by labor in the resource sector equals the marginal income derived by the extraction of the resource minus the value of the resource foregone in the process. This is a consequence of the assumption of perfect property rights over the resource. Under an open access regime consumers would not deduce the value of the used resource and the resource will not be efficiently used.

Condition (13) is the well-known Ramsey rule of optimal saving. Condition (14) equates the rate of return on investing in the resource to the loss on asset accumulation.

From (11) and (13) it follows that

$$\frac{\dot{c}}{c} = r - \rho. \tag{16}$$

The consumption growth rate is given by the well-known gap between the rate of return on assets, r, and the discount rate, ρ.

Considering the economy closed to international asset exchange, total households' assets, $a\bar{L}$, equal the market value of the firms that produce the intermediate goods, $\eta\sigma N$. Taking into account (4), (7), (9), and (10) the dynamics of the number of intermediate goods, N, is

$$\dot{N} = \frac{1}{\eta}\left[\frac{Y - c\bar{L}}{\sigma} - NX\right], \quad N(0) = N_0, \tag{17}$$

where N_0 is the initial quantity of existing intermediate inputs.

2.5 Steady-state equilibrium

Definition 1 *Given $N(0)$ and $S(0)$, an equilibrium consists of time paths for N, S, c and v that maximize the utility of a representative consumer subject to (1) and (10), where the wage rate, w, and the price of the resource, p_R, are given by (4) and the amount of intermediate goods, X, by (6).*

Definition 2 *A steady-state equilibrium would be an equilibrium where all variables grow at constant rates (that could be zero for some variables).*

The following proposition characterizes a steady-state equilibrium.

Proposition 3 *If a steady-state equilibrium exists, the different variables along this path behave as follows:*

- *The stock of the natural resource, S, the labor share devoted either to the final output sector, v, or to the resource sector, $1-v$, the harvesting, R, and the interest rate, r, remain constant.*
- *Output, Y, consumption, c, the price of the natural resource, p_R, and the salary, w, all grow at the same rate as N.*

Proof. See Appendix A. ■

A steady-state equilibrium can be seen as a sustainable growth path. In such a solution, the economy will be continuously growing maintaining constant the stock of the renewable resource.

The assumption of perfect property rights over the natural resource introduced into this model, leads consumers to take into account the resource dynamics (1) in their decision making process. This environmental restriction is effective as long as the share of labor devoted to the extraction of the resource is lower under perfect property rights than under an open access regime. Otherwise, natural resource dynamics would not restrict consumers' decisions and the environmental restriction will not be binding. The behaviour of consumers will not be affected by natural resource scarcity. The following proposition determines the effort devoted to harvest the resource under open access.

Proposition 4 *If the natural resource is an open access resource, the representative consumer would allocate a fraction of labor $v^{oa} = 1/\phi$ to output production (and correspondingly $1-1/\phi$ to harvesting), where*

$$\phi = \frac{1-\alpha-\delta\beta}{1-\alpha-\beta} > 1$$

Proof. See Appendix A. ■

In what follows, we concentrate on equilibria with an extraction effort below the harvesting effort under open access, i.e. $v > v^{oa} = 1/\phi$. These are the equilibria which would appear if the environmental restriction is binding.

Note that if v and S remain constant, then the harvest rate, $R = B\left[(1-v)\bar{L}\right]^{1-\delta} S^{\theta}$, and the interest rate, r, given by (9), will be also constant. Moreover, output Y, given by (7), will grow at the same rate as the number of intermediate goods, N. This rate will be constant if and only if $\tilde{c} = c/N$ is also constant. Therefore, a steady-state equilibrium, as it is described in Proposition 3, will be obtained if and only if variables v, S and $\tilde{c}$ remain constant. The dynamics of these three variables are given in Lemma 22 in Appendix A.

The following proposition collects all the hypotheses needed to guarantee a unique steady-state for variables v, S and $\tilde{c}$. In both cases, when the stock of the natural resource does and does not affect labor productivity in harvesting, $\theta = 1$ or $\theta = 0$, conditions on the intrinsic growth rate of the resource guarantee the existence and uniqueness of a steady-state equilibrium with an extraction effort below the open access harvesting effort, that is, $v > 1/\phi$.

Proposition 5 *The existence and uniqueness of a steady-state equilibrium with $\tilde{c}^* > 0$, $1/\phi < v^* < 1$ and $0 < S^* < C/2$ have been proven:*

- *for $\theta = 1$, under sufficient condition:*

$$g \geq \rho; \tag{18}$$

- *for $\theta = 0$, under necessary and sufficient condition:*

$$g \in (\rho, g^+), \tag{19}$$

where g^+ is the upper bound given in (54) in Appendix A.

Proof. See Appendix A. ∎

When the stock of the resource does not affect harvesting, $\theta = 0$, steady-state values S^* and v^* can be explicitly found (see Appendix A):

$$S^* = \frac{g-\rho}{2g}C < \frac{C}{2}, \qquad v^* = 1 - \frac{1}{\bar{L}}\left[\frac{(g^2-\rho^2)C}{4gB}\right]^{\frac{1}{1-\delta}}.$$

Thus, a necessary condition for the positivity of S^* is $g > \rho$, which also guarantees that $v^* < 1$. That is, the intrinsic growth rate of the resource must be greater than the rate of temporal discount for the existence of a feasible interior steady-state.

Moreover, when $\theta = 0$, condition $v > 1/\phi$ says that the harvesting is below the open access extraction, $R^{oa} = B(1 - 1/\phi)^{1-\delta}$. At the steady-state, this inequality is equivalent to $G(S^*) < R^{oa}$. If this condition is not fulfilled, even the harvesting under open access will not be high enough to maintain motionless the natural resource. Therefore, the economy will not be facing an environmental shortage. Condition $G(S)^* < R^{oa}$ is equivalent to condition $g < g^+$, stated on previous proposition, which establishes that the intrinsic growth rate of the natural resource must be upper bounded for the environmental restriction to be effective. Proposition 5 proves that this upper bound, together with condition $g > \rho$, are necessary and sufficient conditions for the existence of a unique steady-state with $v^* > 1/\phi$.

When the stock of the resource affects harvesting, $\theta = 1$, a closed-form for the stock of the resource and the labor share in the final output sector at the steady-state cannot be found. However, the assumption of a intrinsic growth rate larger than the discount rate, $g \geq \rho$, ensures the existence of a unique equilibrium with an extraction effort below the open access harvesting effort.

The lack of complete stability is a typical property of balanced paths in endogenous growth models (Martínez-García, 2003). This is also the case for our model for all $\theta \in [0, 1]$, as it is proved in Lemma 23 in Appendix A. The following proposition proves conditional stability when $\theta = 0$ or $\theta = 1$.

Proposition 6 *The steady-state equilibrium is a saddle-point with a one-dimensional stable manifold.*

Proof. See Appendix A. ■

In what follows we shall concentrate on the particular cases $\theta = 0$ and $\theta = 1$, where the existence, uniqueness, and saddle-point stability are proved.

The following proposition presents the responses of the steady-state equilibrium values of the stock of the natural resource and the labor allocated to each sector upon changes in the environmental parameters.

Proposition 7 *When a unique steady-state equilibrium exists, the stock of the resource, S^*, and the labor share in the final output sector, v^*, increases and decreases, respectively, with the intrinsic growth rate of the natural resource, g. Likewise, S^* increases with the carrying capacity, C, while its effect on v^*, is negative for $\theta = 0$, although it is null for $\theta = 1$.*

Proof. See Appendix A. ■

The previous results conduct to the following interpretations. A higher intrinsic growth rate, g, leads consumers to devote a larger labor share to the resource sector, $1 - v^*$, which pushes the harvesting of the natural resource up. Nevertheless, since the resource grows faster, this situation is compatible with a larger stock of the resource in the steady-state equilibrium.

The carrying capacity of the natural resource, C, represents the size of the resource sector. It has a positive effect on the equilibrium resource stock, S^*. The effect of C on the labor share devoted to each productive sector depends on the value of θ. When the stock of the natural resource does not affect harvesting, $\theta = 0$, a greater carrying capacity needs an increment in harvesting to maintain constant the stock of the resource , which requires a higher extraction effort, $1 - v^*$. However, when the stock of the natural resource does affect harvesting, $\theta = 1$, the increment in C also raises the stationary resource stock, S^*. The increment in C increases harvesting in the same proportion as the increment in S^*, which makes unnecessary an augment in the extraction effort, $1 - v^*$. Thus, for $\theta = 1$, the labor share in each sector is unaffected by the carrying capacity.

From the growth rate of consumption, in (16), and the rate of return, in (9), the growth rate of the economy, γ, along a steady-state equilibrium follows:

$$\gamma = \frac{1}{\eta}\frac{1-\alpha}{\alpha}X - \rho = \frac{1}{\eta}\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}\left(\frac{A}{\sigma}\right)^{\frac{1}{1-\alpha}}(v^*\bar{L})^{\frac{1-\alpha-\beta}{1-\alpha}}R(v^*, S^*)^{\frac{\beta}{1-\alpha}} - \rho, \quad (20)$$

where $R(v^*, S^*) = B\left[(1-v^*)\bar{L}\right]^{1-\delta} S^{*\theta}$. The results in Proposition 7 can be used to compare the long-term growth rates of two economies that differ in resource abundance. The next proposition states the results of this comparison.

Proposition 8 *The long-term growth rate of the economy, γ, decreases with the cost of innovation, η, and it is positively related with the carrying capacity, C, and with the intrinsic growth rate of the natural resource, g.*

Proof. The results immediately follow taking the partial derivatives of γ with respect to η, the intrinsic growth rate, g, and the carrying capacity, C, and taking into account the results established in Proposition 7. ■

An increment in the cost of innovation, η, and reduces the rate of return on assets for investors, which lessens the growth rate of the economy.

As equation (20) states, the stock of the resource at the steady-state, S^*, only affects the growth rate through its effect on harvesting. Let recall that when $\theta = 0$, the resource extraction is independent of the resource stock. Therefore, the resource abundance, measured by the carrying capacity, C, and the intrinsic growth rate, g, influences the growth rate of the economy in the long-term through its effect on v^*, when $\theta = 0$, or both on v^* and S^*, when $\theta = 1$.

When $\theta = 0$, resource abundance has two opposite effects on the growth rate in the long-term. On the one hand, a higher C (or a higher g) leads to devote a lower share of labor to the final output sector, v^*, which has a negative effect on the growth rate in the long run. This is the only effect that the model of Ellíasson & Turnovsky (2004) takes into account. In their model, where the extracted resource is traded to obtain foreign consumption goods, harvesting has no effect on the production of final output. This fact leads the authors to conclude that if the economy has access to a more bountiful natural resource, it chooses more consumption today, at the cost of slower growth in the long run. However, in our model, harvesting of the resource has a positive influence on the final output production. Therefore, we can deal with a second effect of the environmental conditions on the growth rate. A higher C (or a higher g) leads to a larger labor share to the resource sector, harvesting a higher amount of the resource, which is used to increase the final output production, enlarging the growth rate of the economy. These two effects have opposite signs. If open access to the resource was allowed, consumers would choose the labor share were these two effects compensate, $v^{oa} = 1/\phi$. However, the assumption of perfect property right over the resource leads consumers to devote a higher share of labor to the final output sector, that is, $v^* > v^{oa}$, and consequently $R(v^*, S^*) < R^{oa}$. In this situation the environmental restriction is forcing to overenlarge the labor force in the final output and to underuse the resource. Any movement correcting these distortions will have a positive effect on the long-run growth rate. The second effect will be stronger, and we can conclude that an economy having access to a more bountiful natural resource will grow faster.

When $\theta = 1$, a larger intrinsic growth rate, g, lowers the labor share in the final output sector, and raises the labor share devoted to harvesting, which pushes extraction up. In addition, the increment in the stationary stock of the resource, S^*, associated with a higher g, pushes extraction further up. Thus, the net effect of a higher intrinsic growth rate on the economy growth rate is positive, likewise as for $\theta = 0$. With regard to the effect of the carrying capacity, it has no effect on the labor share devoted to each sector of the economy. However, the stock of the natural resource, which has now a positive effect on the growth rate, will be greater, leading to a higher growth rate.

From expression (46) in Appendix A it is straightforward to show that the effect on $\tilde{c}^*$ of changes in the environmental parameters C and g, are the same as that on the economy growth rate, γ. Therefore, the following corollary results.

Corollary 9 *The steady-state equilibrium of consumption per variety of intermediate good, $\tilde{c}^*$, depends positively on the cost of innovation, η, the carrying capacity, C, and the intrinsic growth rate of the natural resource, g.*

Since an increment in the cost of innovation, η, reduces the rate of return on assets, consumers tend to increase their consumption with respect to investment, augmenting the ratio of consumption per variety of intermediate good, $\tilde{c}^*$. Moreover, a more bountiful resource increases harvesting, and so, consumers attain a larger income from their extraction activities in the resource sector, which increases consumption and the ratio $\tilde{c}^*$.

3 Sustainable growth with FDI

Developing economies tend to rely on FDI rather than on domestic innovation as the source of technological development. With this idea, the following model assumes that technological improvements in the country endowed with the renewable natural resource come imported from a technological leader country. It is an extension of the two-country endogenous growth model described in Barro & Sala-i-Martin (1999, Chapter 8). We shall see how, although no technological investments are carried out in the country endowed with the natural resource, the trade relationship with a technological leader enables a sustained economic growth maintaining constant the stock of the resource.

We present a model of bilateral trade between a technological follower country which manages the extraction of the natural resource, called country F, and a technological leading country, called country L. We assume that final output producers in country F buy the new intermediate inputs to the innovators in country L, whereas consumers of this latter buy domestic consumption as well as goods produced in country F. We shall study two scenarios depending on the market power of the trading countries. The terms of trade is unaffected by countries' decisions when small open economies with no market power are considered. Conversely, their decisions determine prices when either one country is the unique supplier and its counterpart the only demander of the interchanged goods. Alternatively, the terms of trade is determined by their actions when L (resp. F) is a representative economy of many clones technological leading (resp. follower) economies. In our formulation, although countries determine the term of trade, they do not incorporate the mechanism of price formation in their decision process. Thus, we are considering myopic large economies or small representative economies. For simplicity, we refer to this as large open economies (LOE) scenario, while the scenario with no market power is known as small open economies (SOE). In what follows we shall describe the problem each country faces.

3.1 The technological follower country

Country F manages the natural resource of the logistic type, where the harvest rate is given by (2). This country does not invest on technological improvements. Final output producers import the intermediate goods invented and produced in the leading country.

The final good production of a representative firm presents the same functional form as (3):[4]

$$Y_F = A_F L_{FY}^{1-\alpha-\beta} \sum_{j=1}^{N} X_{Fj}^{\alpha} R^{\beta}. \tag{21}$$

Following the same reasoning as for the closed economy, net marginal products are equated to factor prices:

$$w_F = (1-\alpha-\beta)\frac{Y_F}{L_{FY}},\ p_R = \beta\frac{Y_F}{R},\ X_{Fj} = L_{FY}\left(\frac{\alpha A_F}{p_j^F}\right)^{\frac{1}{1-\alpha}}\left(\frac{R}{L_{FY}}\right)^{\frac{\beta}{1-\alpha}}, \tag{22}$$

where p_j^F is the price paid for the intermediate goods to the leading country entrepeneurships.

Since no innovative activity exists in country F and there is no international trade on financial assets, consumers of country F do not accumulate assets in the form of ownership claims on innovative firms, and do not receive financial interest income from them. The only asset that consumers of this country can hold is the ownership of the natural resource, whose accumulation law is given by (1). A representative consumer in country F has to decide the fraction of labor, $1-v$, employed to cooperative harvesting, attaining a portion $1/L_F$ of total returns, $p_R R$. Correspondingly, the consumer allocates a fraction, v, of her labor to the final output sector.[5] Therefore, she receives and consumes:

$$p_R\frac{R}{L_F} + v w_F = c_F. \tag{23}$$

Thus, the optimization problem of a representative consumer is:

$$\max_{v} \int_0^{\infty} \ln(c_F) e^{-\rho t} dt \tag{24}$$

$$\text{s.t.} \qquad \dot{S} = gS(1-S/C) - B(L_F(1-v))^{1-\delta} S^{\theta}, \quad S(0) = S_0. \tag{25}$$

[4]Subscript F denotes variables corresponding to the follower country.

[5]Likewise as for the closed economy, labor in the final output sector and the resource sector is redefined as $L_{FY} = vL_F$ and $L_{FS} = (1-v)L_F$.

Proposition 24 in Appendix B characterizes the optimal time paths in the technological follower country.

3.2 The technological leading country

Production of final output of a representative firm is described by [6]:

$$Y_L = A_L L_L^{1-\alpha} \sum_{j=1}^{N} X_{Lj}^{\alpha}. \tag{26}$$

By equating the marginal product to input prices, the wage rate and the total demand of intermediate good j by producers can be written as:

$$w_L = (1-\alpha)\frac{Y_L}{L_L}, \qquad X_{Lj} = L_L \left(\frac{\alpha A_L}{p_j}\right)^{\frac{1}{1-\alpha}}, \tag{27}$$

where p_j is the price of intermediate input j in this country.

The intertemporal maximization problem for a representative consumer reads:

$$\max_{c_L, c_{LF}} U = \int_0^{\infty} [\ln(c_L) + \ln(c_{LF})]\, e^{-\rho t} dt, \tag{28}$$

$$\text{s.t.} \quad : \quad \dot{a}_L = r a_L + w_L - c_L - p_F c_{LF}, \qquad a_L(0) = a_{L0}, \tag{29}$$

where a_L is per capita assets, c_L is per capita consumption of domestic final good, and c_{LF} is per capita consumption of the good imported from country F at a price p_F.

We consider the price of the domestic final good as a numeraire, $p_L = 1$. Consequently, p_F not only represents the price of the good imported from F, but also, the terms of trade that defines commerce between these two countries, i.e. the units of country L's output paid for one unit of consumption imported from country F.

As it is proved in Proposition 25 in Appendix B, the following conditions are necessary for consumer's optimization:

$$\frac{\dot{c}_L}{c_L} = r - \rho, \qquad \frac{\dot{c}_{LF}}{c_{LF}} = r - \rho - \frac{\dot{p}_F}{p_F}. \tag{30}$$

The growth rate of the domestic good consumption is again as in (16). The difference between this rate and the growth rate of the terms of trade gives the growth rate of the imported good consumption.

As there are no innovators in the follower country, production of intermediate goods is carried out in the leader country. This situation applies as long as intellectual property rights are protected both domestically and internationally.

[6] Subscript L denotes variables corresponding to the leading country. Parameters, A_F, and L_F, may differ from their corresponding parameters for country L. Differences between A_F and A_L could reflect differences in government policies. The gap between L_F and L_L reflects the differences in scale between the two economies.

Once invented, an intermediate good of type j costs σ_L units of Y_L to produce, while innovator who produces this intermediate good obtains p_j unit of Y_L. For simplicity, we normalize $\sigma_L = 1.$. The monopolist decides the price p_j to maximize his instantaneous profits from sales to final output producers in L and F:

$$\pi_j = (p_j - 1)\left(X_{Lj} + X_{Fj}\right),$$

where X_{Lj} and X_{Fj} are given in equations (27) and (22).

The maximum price for this problem is:

$$p_j = 1/\alpha > 1, \tag{31}$$

then, the units of final output of country F paid for one unit of the intermediate good j, p_j^F, is equal to $1/(\alpha p_F)$. Therefore, the amount of every intermediate in each country is:

$$X_{Lj} = X_L = L_L A_L^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}}, \tag{32}$$

$$X_{Fj} = X_F = vL_F(p_F A_F)^{\frac{1}{1-\alpha}} \alpha^{\frac{2}{1-\alpha}} \left(\frac{R}{vL_F}\right)^{\frac{\beta}{1-\alpha}}. \tag{33}$$

Note that although X_L is constant, the quantity X_F depends on v and R (which is a function of v and S), and also on p_F.

Likewise as in the closed economy, the cost to create a new intermediate is supposed η times the cost of producing it, that is, η units of Y_L. However, an innovator must pay a cost beyond the initial R&D outlay to transfer and adapt his product for use in country F, ν, with $0 < \nu < \eta$. Once more, the free entry assumption equates the present value of the profits for each intermediate to $\eta + \nu$, and following the same reasoning carried out in the closed economy, we obtain the rate of return on assets:

$$r = \frac{(1-\alpha)(X_L + X_F)}{\alpha(\eta + \nu)}. \tag{34}$$

Investment returns in the technological leader country are linked to the monopolistic benefits in the intermediate good sector. Considering an economy closed to international asset exchange, total households' assets, $a_L L_L$, equal the market value of the firms that produce these intermediate goods, $(\eta + \nu)N$. Therefore, households' assets run parallel to the number of varieties of intermediate inputs, N. The dynamics of the number of intermediate goods, N, can be obtained from the equality $a_L L_L = (\eta + \nu)N$, taking into account the salary in the technological leader country given in (27), the relationship

$$\alpha^2 Y_L = N X_L, \tag{35}$$

and the dynamics of the assets in (29):

$$\dot{N} = \frac{1}{\eta + \nu}\left[Y_L - (c_L + p_F c_{LF})L_L - N\left(X_L - \frac{1-\alpha}{\alpha}X_F\right)\right], \; N(0) = N_0. \tag{36}$$

As we will show, the permanent increment in this number fuels growth of production of final output sector not only in the technological leader country, but also in the follower, where final output producers also use intermediate inputs invented in the leader country.

3.3 Steady-state equilibrium

Before defining an equilibrium for the two-trading economies described above, we briefly consider the problem being solved in each country. The problem for the leader country, PL: A representative consumer of country L has to choose c_L and c_{LF} to maximize (28) subject to (29). The salary w_L will be given by (27) and the rate of return r will be (34). In a symmetric fashion, the problem for the follower country, PF: A representative consumer of country F has to choose v to maximize (24) subject to (25). The wage rate, w_F, and the price of the resource, p_R, will be given by (22).

Two type of equilibria may appear depending on the market power of the trading economies. The terms of trade, p_F, is exogenously fixed and supposed constant in the scenario that considers two small open economies. By contrast, when large open economies are considered, the price, p_F, for a bilateral trade, is determined by equating the value of the final good traded from F to L, to the value of the intermediate goods sold from innovators in L to producers in F:

$$L_L p_F c_{LF} = p_j N X_F. \tag{37}$$

Definition 10 *Given $N(0)$ and $S(0)$, and considering time paths for N, S, c_L, c_{LF} and v such that PL and PF are solved, two type of equilibria may appear:*

- *Small open economies equilibrium (SOEE): p_F is exogenously fixed in the international market and supposed constant at the value $\hat{p}_F$.*
- *Large open economies equilibrium (LOEE): p_F is endogenously determined from equation (37).*

In what follows, we concentrate exclusively on the steady-state equilibria. The first step is to describe the behavior of the different variables along a steady-state equilibrium.

Proposition 11 *If a steady-state equilibrium exists, along this path,*

- *v, S, R, p_F and r remain constant.*
- *Y_L, Y_F, c_L, c_{LF}, c_F, p_R, w_L, and w_F grow at the same rate as N.*

Proof. See Appendix B. ■

The steady-state equilibrium corresponds to a constant growth path in the leading country. Furthermore, although the follower country does not invest in technological improvements, the trade relationship with the leader allows a sustainable growth path in this country. Both trading economies grow at the same constant rate.

As in previous section, the steady-state equilibrium corresponds with a steady-state of variables, $\tilde{c}_L \equiv c_L/N$, v and S. In Lemma 26 in Appendix B presents the dynamical system that characterizes the motion of these three variables. The next two propositions, which are proved in Appendix B, answer these questions.

Proposition 12 *Under conditions in Proposition 5 there exists a unique steady-state equilibrium with $\tilde{c}_L^* > 0$, $1/\phi < v^* < 1$ and $0 < S^* < C/2$.*[7] *Furthermore, values v^* and S^* coincide with those obtained for the closed economy.*

Proposition 13 *The steady-state equilibrium is a saddle-point with a one-dimensional stable manifold.*

Let us note that the steady-state values of the stock of the resource, S^*, and the labor share in the final output sector, v^*, are solutions of the same equation system as those obtained for the closed economy, as it is explained in the proof of Proposition 12. Thus, the effect of changes in the carrying capacity and the intrinsic growth rate collected in Proposition 7 remains valid. However, depending on the market power of the two trading economies, the effect of resource abundance on the growth rate may not be the same. The reason is that the terms of trade, which have a significant influence on the economic growth rate, remain fixed when both economies are small whereas they are determined by the balance trade condition (37) when both are large open economies. The following proposition studies this second case.

Proposition 14 *When a unique steady-state equilibrium exists for LOE, the terms of trade along this equilibrium, p_F^*, increases with the cost of innovation, η, and the cost of adaptation, ν; and decreases with the carrying capacity, C, and the intrinsic growth rate, g.*

Proof. See Appendix B. ■

An increment in either the cost of innovation, η, or the cost of adaptation, ν, implies a reduction in the rate of return on assets for investors in the leading country, r. Lower returns lead consumers to increase their consumption (domestic and imported) with respect to investment, augmenting the ratio of foreign consumption per variety of intermediate good, $\tilde{c}_{LF} = c_{LF}/N$, in the leading country at the steady-state. As long as η and ν do not affect the demand for intermediate inputs in F, bilateral trade equilibrium leads to a gain in the follower terms of trade, p_F^*.

An increment in either the carrying capacity, C, or the intrinsic growth rate, g, leads consumers in the follower country, who own the resource, to reduce the labor share in the final output sector in favor of a higher harvesting rate of the resource, which pushes final output production up. The second effect is

[7] Conditions are still valid, replacing $\bar{L}$ by L_F in g^+ (expression (54))

stronger both, when $\theta = 0$ and S^* does not affect harvesting, and when $\theta = 1$ and the increment in S^* fuels harvesting and final output production further. A higher final output production in the follower country requires higher imports of intermediate inputs. As long as C and g do not affect the demand for foreign consumption in the leading country, the equilibrium in bilateral trade leads to a lose in the follower's terms of trade.

These changes in the terms of trade may also affect consumption. The next proposition studies the effects upon consumption per variety of intermediate good along the steady-state equilibrium under the SOE and LOE scenarios.

Proposition 15 *When a unique steady-state equilibrium exists, along this equilibrium, the ratios of consumption per variety of intermediate good $\tilde{c}_L^*, \tilde{c}_{LF}^*$ and $\tilde{c}_F^*$,[8] increase with the cost of innovation, η, and the cost of adaptation, ν, except $\tilde{c}_F^*$ in SOE scenario which remains constant.*

The effect of an increment in C or g on the ratio of consumption per variety of intermediate good also depends on the size of the open economies:

- *SOE: $\tilde{c}_L^*$ and $\tilde{c}_{LF}^*$ remain constant, while $\tilde{c}_F^*$ increases.*
- *LOE: $\tilde{c}_L^*$ remains constant, while $\tilde{c}_{LF}^*$ and $\tilde{c}_F^*$ increase.*

Proof. See Appendix B. ■

As it has been previously explained, an increment in either η or ν leads consumers in the leading country to increase their consumption (domestic and imported) with respect to investment, augmenting the ratios $\tilde{c}_L^* = c_L^*/N$ and $\tilde{c}_{LF}^* = c_{LF}^*/N$ in the same proportion, when p_F is fixed and constant (SOE). In the LOE scenario η and ν have positive effects on $\tilde{c}_L^*$ and p_F^*. Better trading position for country F reduces imported consumption in country L. This reduction cuts down the previous rise, increasing $\tilde{c}_{LF}^*$ but in a lower proportion.

As long as η and ν do not affect the demand for intermediate inputs in F, their effect on consumers income in this country is null, and so it is on $\tilde{c}_F^*$when p_F is fixed (SOE). However, in the LOE scenario η and ν lead to a gain in the follower country's terms of trade, p_F^*, increasing their income and consumption, and then $\tilde{c}_F^*$.

Proposition 15 states that, for SOE, the stationary ratios of the domestic and imported consumption per variety of intermediate goods in L, $\tilde{c}_L^*$ and $\tilde{c}_{LF}^*$, are unaffected by C or g, whereas the ratio of consumption per variety of intermediate goods in F, $\tilde{c}_F^*$, increases. For SOE, the terms of trade are constant, and $\tilde{c}_{LF}^*$ remains unaffected by changes in the resource bounty. However, for LOE, the relative price for the follower country drops with C and g, increasing the consumption of imported goods in the leading country.

The effect of resource bounty on the consumption per variety of intermediate good in F, $\tilde{c}_F^*$, is twofold. On the one hand, a higher C or g increases harvesting, $R(v^*, S^*)$, and so, consumers in the follower country attain larger income from their extraction activities in the resource sector. Conversely, resource abundance

[8] Recall that $\tilde{c}_i = c_i/N$, $i \in \{L, LF, F\}$.

also means a lower relative price for F. The trading position of the follower country worsens, pushing down net revenues from bilateral trade. The first effect, which boosts consumption in F, surpasses in size the negative effect of lower terms of trade.

Finally, the next proposition states the long-term growth rates for small and large open economies and shows the results of the sensitivity analysis.

Proposition 16 *Along a steady-state equilibrium the economies in both the technological leading and follower countries grow at rates given by:*

- *Small open economies (SOE):*

$$\gamma^{soe} = \frac{(1-\alpha)\alpha^{\frac{2}{1-\alpha}}}{\alpha(\eta+\nu)} \left[L_L A_L^{\frac{1}{1-\alpha}} + (L_F v^*)^{\frac{1-\alpha-\beta}{1-\alpha}} A_F^{\frac{1}{1-\alpha}} R(v^*, S^*)^{\frac{\beta}{1-\alpha}} \hat{p}_F^{\frac{1}{1-\alpha}} \right] - \rho.$$

- *Large open economies (LOE):*

$$\gamma^{loe} = (1+\alpha) \left[\frac{(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}}{2(\eta+\nu)} L_L A_L^{\frac{1}{1-\alpha}} - \rho \right]. \tag{38}$$

Both, γ^{soe} and γ^{loe} decrease with the cost of innovation, η, and the cost of adaptation, ν. Furthermore, γ^{soe} increases with the carrying capacity and the intrinsic growth rate, whereas γ^{loe} is independent of these parameters.

Proof. See Appendix B. ■

An increment in the cost of innovation, η, or the cost of adaptation, ν, reduces net benefits of innovators and then, the rate of return on assets for investors in the leading country, r. Thus, by usual definition of the growth rate presented in (30), the negative effect on γ^{soe} and γ^{loe} follows.

The proposition states that the resource bounty, described both by the carrying capacity or by the intrinsic growth rate, raises the growth rate of small trading countries, with a constant terms of trade. Conversely, resource bounty has no effect on the growth rate of large open economies.

When the two trading economies are small, and the relative price, $\hat{p}_F$, is given and constant, the effect of C and g upon the growth rate is the same that it was in the model of a closed economy with domestic innovation. Resource bounty fuels final output production in country F. A higher final output production requires higher imports of each type of intermediate inputs. This rise implies a higher rate of return in the leading country and, in consequence, a higher growth rate in both economies.

However, when trading economies are large, resource bounty lessens the terms of trade, pushing down the imports of intermediate goods. This negative effect exactly compensates the previous pressure to rise of imports of intermediates in country F to keep invariant the value of imports in L, $p_F^* \tilde{c}_{LF}^* = \tilde{c}_L^*$. Since resource bounty has no influence on the traded amount of intermediate goods, neither it affects the rate of return in L nor the growth rate of both countries.

4 Domestic innovation vs. FDI

We have proved that both domestic innovation and FDI, can fuel technological innovation so that to attain sustainable economic growth in a country endowed with a renewable natural resource, with a limited regeneration rate and a bounded carrying capacity. The main question to answer in this section is whether the country is better off when innovation is carried out within its borders or when technology is imported from abroad. This section compares the long-run growth rates and the representative consumer's consumptions and utilities under both scenarios: a closed economy with domestic innovation and an open economy with FDI.

In the case of two open economies with FDI, innovators in the leading country pay one unit of Y_L to produce an already invented intermediate good. Under this assumption if there were inventors in the follower country they should face the same production cost. That is, one unit of Y_L, or equivalently, $1/p_F$ units of the good produced in this country, Y_F. Thus, to compare domestic innovation and FDI scenarios, parameter σ equates $1/p_F$ in the former, where p_F represents the constant and given price in the SOE scenario, $\hat{p}_F$, or the endogenously determined terms of trade in the LOE scenario, p_F^* in (72). For comparison purposes $\overline{L} = L_F$, $A = A_F$, and thus, $X(v^*, S^*) = X_F(v^*, S^*)$.

Proposition 17 *The gap between long-run growth rates with domestic innovation, γ, and FDI, γ^{oe}, characterizes as follows*

$$\gamma > \gamma^{oe} \Leftrightarrow \frac{\nu}{\eta} > \frac{X_L}{X_F(v^*, S^*)}. \tag{39}$$

Proof. See Appendix C ■

By condition (39), the long-run growth rate under FDI coincides with the growth rate under domestic innovation if and only if:

$$\frac{X_F(v^*, S^*)}{\eta} = \frac{X_L + X_F(v^*, S^*)}{\eta + \nu}.$$

For a specific variety of intermediate good, the employed amount over the cost of innovation in the domestic scenario matches the employed amount over the costs of innovation and adaptation under FDI. Under this condition, the return to asset holders is the same under both scenarios. Since this rate of return equally determines the growth rate of consumption both under domestic innovation in (16) and FDI in (60) the economies grow at the same rate.

Furthermore, condition (39) shows that the greater the cost of adaptation in terms of the cost of innovation, the stronger the incentive to switch from FDI to domestic innovation. Equivalently, this incentive is stronger, the greater the amount of intermediate good needed in the country which has to decide whether to innovate or to import intermediate goods from the leading country.

Corollary 18 *For a large open economy, the shift from foreign direct investment to domestic innovation enhances the long-run growth rate if the ratio ν/η*

is greater or equal to $2\alpha/(1-\alpha)$. On the contrary, the long-run growth rate decreases if the output elasticity of the intermediate good is sufficiently large, specifically, $\alpha \geq 2/3$.

Proof. See Appendix C. ■

Corollary 18 establishes two sufficient conditions to ensure that it is more (or less) profitable for the economy to innovate rather than to import new intermediate goods. The economy would grow faster with domestic innovation if the cost of adaptation with respect to the cost of innovation surpasses a lower bound, which depends positively on the output elasticity of the intermediate goods, α. Since $\nu < \eta$ this first sufficient condition can only occur if $\alpha < 1/3$, being more likely the smaller is α. The smaller is the output elasticity of the intermediate goods, the less worthy is to import them from abroad. Conversely, domestic innovation slows down growth when the output elasticity of the intermediate good is large enough.

Resource bounty differently affects the growth rate of the economy and the consumption per variety of intermediate good with domestic innovation or with FDI. These effects are collected in the next two propositions.

Proposition 19 *When the economies are small, the increment (resp. decrease) in the long-run growth rate after a shift from foreign direct investment to domestic innovation is higher (resp. softer) the more bountiful the natural resource. When economies are large, the gap in long-run growth rates is unaffected by resource abundance.*

Proof. See Appendix C. ■

Proposition 20 *A switch from FDI to domestic innovation leads to a greater consumption per variety of intermediate good at the steady-state. This increment is larger the lower the terms of trade. Furthermore, for LOE, resource abundance enhances this increment in consumption per variety, while for SOE this gap remains constant.*

Proof. See Appendix C. ■

For the previous propositions a question arises: Is resource bounty an incentive for an economy to switch from FDI to domestic innovation?

Regardless of the size of the economies, a switch from foreign direct investment to domestic innovation does not have an utterly determined effect on the long-run growth rate, although it increases the consumption per variety of intermediate good.

Resource wealth differently modifies the effect of a switch from FDI to domestic innovation, depending on the size of the economies. When economies are small, the more bountiful the natural resource the economy has access to, the higher the amplitude of the increment in the long-run growth rate, or the smaller the amplitude of the decrease in this rate. Furthermore, resource wealth does not affect the increment in the consumption per variety of intermediate good. When economies are large, resource wealth does not modify the amplitude of the gap in the long-run growth rates, although it increases the consumption per variety of intermediate good.

From previous reasoning the following result can be established.

Corollary 21 *The gap between the steady-state utilities after a switch from foreign direct investment to domestic innovation is positively affected by resource abundance.*

5 Concluding remarks

For a country endowed with a natural resource and with a resource dependent economy, two models have been analyzed, depending on whether the economy invests in new technology or adopts technology developed abroad. The main concern of the paper is the analysis of the sustainability of the economic growth, for both models. Furthermore, we have focused on the effect of resource abundance on the growth rate of the economy, the terms of trade, the stationary level of the resource stock, and the consumers' welfare.

Our findings are compared for domestic innovation and foreign direct investment. Under both scenarios the existence, uniqueness and saddle-point stability of a steady-state equilibrium that allows a sustained economic growth maintaining constant the stock of the natural resource have been proved. On the steady-state equilibrium, technological innovation, consumption and the price of the natural resource all grow at the same constant rate. Correspondingly the harvesting and the stock of the natural resource remain constant.

The first model assumes a resource-dependent economy that develops its own R&D sector. Resource wealth, measured either by the carrying capacity or the intrinsic growth rate, enhances the long-run growth rate of the economy. This increment occurs despite of a larger share of labor devoted to harvesting and due to higher harvestings associated with a higher level of the resource. This effect on the growth rate is softer when resource abundance is measured by the carrying capacity, and the harvest rate is proportional to the stock of the resource. A more bountiful natural resource also increases the consumption per variety of intermediate good. Both effects lead to a higher consumers' welfare.

This result differs from the negative relationship between an economy's resource abundance and its long-term growth rate in Ellíasson and Turnovsky (2004). In their model, the stock of the resource does not influence extraction, and the extracted resource is used to purchase imports of a foreign consumption good, avoiding technological innovation, or any other mechanism, which enlarges the resource returns. In our model, the natural resource is invested on output production, and technological innovation enhances its productivity. Literature relies on technological innovation to achieve sustainable growth. In this study we go further and assert that if technological improvements enhance the resource returns on income, the economy will avoid the "resource curse" and will receive a "blessing". This conclusion applies regardless of whether the stock of the resource does or does not affect harvesting.

In the second model, the economy endowed with the natural resource can obtain new technology from abroad by FDI. A technological leading country invests on technological progress, which is adopted by the technological follower. To the best of our knowledge, this is the first attempt to tackle simultaneously trade, technology transfer and natural resource management in the context of endogenous growth economies. In our opinion, this is an appropriate framework to describe trade relationships between developing countries and industrialized and technological developed countries. Moreover, while pioneer models of technological change and environmental problems are applicable to industrialized countries, our approach allows us to study the existence of sustainable growth in economies of developing countries, typically linked to the extraction of a natural resource, with an underdeveloped or non-existent R&D sector.

Our results prove that technological innovation in the leader country is a sufficient condition for sustainable economic growth in both countries. The technological diffusion by FDI permits the reconciliation between unlimited economic growth and bounded natural resource in developing countries. In our context trade relationships between these two countries allow the transmission of growth from the technological leader to the follower country.

We have proved that the effect of resource bounty on the long-run growth rate depends on the size of the economies. For small open economies, with a fixed and constant terms of trade, the growth rate is positively affected as in the case of domestic innovation. Conversely, for large open economies, a more bountiful natural resource reduces the terms of trade of the country owning the natural resource, cancelling out the previous positive effect and making the growth rate independent on the resource wealth. For small economies, and during time periods of constant terms of trade, different resource endowments can generate differences in the growth rates. However for large economies, where the prices balance the trade, the asymptotic growth rate does not depend on resource bounty. Empirical evidence by Evans (1996) supports this result.

Consumption per variety of intermediate good increases with resource wealth in the follower country. However, this abundance does not affect consumption in the leader, except in the case of large open economies, when its imports increase, associated with a lower terms of trade.

The adaptation and innovation costs and the amounts of intermediate good employed in each country establish a condition that determines if the long-run growth rate is larger under domestic innovation or foreign direct investment. However, the switch from domestic innovation to foreign direct investment always implies a higher consumption per variety of intermediate good.

When comparing long-run growth rates before and after a shift from foreign direct investment to domestic innovation, resource wealth enhances gains and smoothes losses for small open economies, but has no influence if economies are large. Conversely, the increment in consumption per variety of good due to this jump is widen for large open economies and unaffected for small economies.

In consequence, the increment in welfare associated with a change to a non-dependent policy of technology innovation is larger, the better the economy is supplied with natural resource. Thus, the incentive to carry out R&D investment activities is strengthen by resource bounty.

Up to now, we have concentrated on the existence, uniqueness and stability of the steady-state equilibrium representing a sustainable growth solution. We have proved that it is a saddle point with a one-dimensional stable manifold. Thus there exists a unique transition path to sustainability. Further research will include the transitional dynamics to this sustainable solution, in order to characterize the growth rates of the relevant variables along the transition period.

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7 Appendix A: Sustainable growth with domestic innovation

Proof of Proposition 3. The labor share devoted to the final output sector takes value between zero and one, while the stock of the natural resource is lower and upper bounded by 0 and C, respectively. Thus, v and S cannot grow indefinitely at a non-zero constant rate. Therefore, these variables must be constant on a steady-state equilibrium. Provided that the harvesting of the natural resource depends on the labor and the stock of the natural resource, which are motionless, the harvesting also must remain constant on a steady-state equilibrium. Consequently, by (6), the amount of intermediate good, X, is also constant.

By (9) immediately follows that the rate of return on assets is constant. Consequently, the growth rate of consumption is also constant by (16).

From the expression of Y in (7), provided that the amount of intermediate good remains constant on steady-state, the production function grows at the same rate as N.

Taking into account (17) and (7), the growth rate of the number of intermediate goods is:

$$\frac{\dot{N}}{N} = \frac{1}{\eta}\left[\left(\frac{1}{\alpha^2} - 1\right) X - \frac{c}{N}\frac{\bar{L}}{\sigma}\right]. \tag{40}$$

As long as X remains constant along the steady-state equilibrium, the growth rate of N is constant if consumption, c, grows at the same rate as the number of intermediate goods, N.

Finally, expressions in (4) show that the price of the natural resource, p_R, and the salary, w, grow at the same rate as Y and N. ■

Proof of Proposition 4. Under open access, consumers do not take into account the dynamics of the natural resource. They solve the maximization problem:

$$\max_{c,v} \int_0^\infty \ln(c) e^{-\rho t}\, dt,$$

subject to their budget constraint given by (10). From the necessary conditions for optimality and the definition of R in (2), it follows:

$$w - \frac{1-\delta}{1-v} R \frac{p_R}{\bar{L}} = 0,$$

which, taking into account expressions in (4), can be rewritten as:

$$\frac{1-\alpha-\beta}{v} = \beta \frac{1-\delta}{1-v}.$$

The fraction of labor allocated to the final output sector, v^{oa}, immediately follows:

$$v^{oa} = \frac{1-\alpha-\beta}{1-\alpha-\delta\beta} = \frac{1}{\phi}.$$

■

Lemma 22 *Any steady-state equilibrium for the model described in Section 2 corresponds to a steady-state of the following three differential equations:*

$$\dot{\tilde{c}} = \tilde{c}\left\{\frac{1}{\eta}\left[\frac{\bar{L}\tilde{c}}{\sigma} - \left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{1+\alpha}{1-\alpha}}(v\bar{L})^{\frac{1-\alpha-\beta}{1-\alpha}}\left(\frac{A}{\sigma}\right)^{\frac{1}{1-\alpha}} R^{\frac{\beta}{1-\alpha}}\right] - \rho\right\}, \quad (41)$$

$$\dot{v} = f_1(v,S) = \Omega(v)\left\{\Theta(v,S) + (\phi v - 1)\frac{\dot{\tilde{c}}}{\tilde{c}}\right\}, \quad (42)$$

$$\dot{S} = f_2(v,S) = G(S) - R, \quad (43)$$

where $\tilde{c} = c/N$ *and*

$$\Omega(v) = \frac{(1-\alpha)v(1-v)}{(1-\alpha)(1-v) + (1-\alpha-\beta)(1-\delta v)(\phi v - 1)},$$

$$\Theta(v,s) = \left[\rho - g\left(1 - 2\frac{S}{C}\right) + \theta\frac{1-\alpha-\beta}{1-\alpha}\frac{\dot{S}}{S}\right](\phi v - 1) - \theta\frac{R}{S}(1-v),$$

and

$$\phi = \frac{1-\alpha-\delta\beta}{1-\alpha-\beta} > 1.$$

Proof. Note that if v and S remain constant, then $R = B\left[(1-v)\bar{L}\right]^{1-\delta} S^{\theta}$ and the interest rate r, given by (9), will be constant. Moreover, Y given by (7) will grow at the same rate as N, which will be constant if $\tilde{c}$ is also constant. Therefore, a steady-state of system (41)-(43) corresponds with a steady-state equilibrium of the model in Section 2.

The dynamics of the new variable, $\tilde{c}$, is:

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{\dot{c}}{c} - \frac{\dot{N}}{N} = r - \rho - \frac{\dot{N}}{N},$$

which substituting the interest rate by its expression in (9) and using (40), can be written as (41).

To derive the dynamics for variable v let replace prices p_R and w in (12) by their expressions given in (4), obtaining

$$\lambda R(v,S) = (\phi v - 1)\frac{(1-\alpha-\beta)}{(1-\delta)v}\frac{Y}{c\bar{L}}.$$

Differentiating we obtain:

$$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{R}}{R} = \frac{1}{\phi v - 1}\frac{\dot{v}}{v} + \frac{\dot{Y}}{Y} - \frac{\dot{c}}{c}. \quad (44)$$

Expression (2) leads to:

$$\frac{\dot{R}}{R} = -(1-\delta)\frac{\dot{v}}{1-v} + \theta\frac{\dot{S}}{S}.$$

Replacing in (44) the time derivatives of $R(v,S)$ and λ, after several computations we can write the growth rate of variable v as follows:

$$\frac{\dot{v}}{v}=\frac{1-v}{1-v+(1-\delta)v(\phi v-1)}\left\{\left[\rho-g\left(1-2\frac{S}{C}\right)+\theta\frac{\dot{S}}{S}-\frac{\dot{Y}}{Y}+\frac{\dot{c}}{c}\right](\phi v-1)-\theta\frac{R}{S}(1-v)\right\}. \tag{45}$$

From (6) and (7) we obtain that

$$\frac{\dot{Y}}{Y}-\frac{\dot{c}}{c}=-\frac{(1-\alpha-\beta)\,(\phi v-1)}{(1-\alpha)\,(1-v)}\frac{\dot{v}}{v}+\frac{\theta\beta}{1-\alpha}\frac{\dot{S}}{S}-\frac{\dot{\tilde{c}}}{\tilde{c}}.$$

Using this last expression in (45) equation (42) is obtained. ■

Proof of Proposition 5. We have to prove that the dynamical system (41)-(43) admits a unique steady-state, denoted by $(\tilde{c}^*, v^*, S^*)$ with $\tilde{c}^* > 0$, $1/\phi < v^* < 1$ and $0 < S^* < C/2$.

By equation (41), in a steady-state equilibrium

$$\tilde{c}^* = \frac{\sigma}{\bar{\bar{L}}}\left[\frac{1-\alpha}{\alpha}\alpha^{\frac{1+\alpha}{1-\alpha}}\left(v^*\bar{L}\right)^{\frac{1-\alpha-\beta}{1-\alpha}}\left(\frac{A}{\sigma}\right)^{\frac{1}{1-\alpha}}R(v^*,S^*)^{\frac{\beta}{1-\alpha}}+\rho\eta\right], \tag{46}$$

which expresses the value of $\tilde{c}^*$ as a function of v^* and S^*.

Note that the steady-state equilibrium values of v^* and S^* are the values that solve the following equation system

$$\left[\rho-g\left(1-2\frac{S^*}{C}\right)\right](\phi v^*-1)-\theta B\left[(1-v^*)\bar{L}\right]^{1-\delta}(S^*)^{\theta-1}(1-v^*)=0, \tag{47}$$

$$gS^*\left(1-\frac{S^*}{C}\right)-B\left[(1-v^*)\bar{L}\right]^{1-\delta}(S^*)^{\theta}=0. \tag{48}$$

If v^* and S^* solve the system (47)-(48) and $\tilde{c}^*$ is given by (46), it is clear that this three values simultaneously vanish the equations (41)-(43). On the other hand, any three values of v^*, S^* and $\tilde{c}^*$ that simultaneously vanish equations (41)-(43) must satisfy (46) and (47)-(48). Therefore, obtaining the steady-states of (41)-(43) is equivalent to solve the system (47)-(48) and take $\tilde{c}^*$ as given by (46). We focus now on the existence and uniqueness of values (v^*, S^*) in the feasible region which correspond to a steady-state of the dynamical system (47)-(48). We prove this result separately for cases $\theta = 1$ and $\theta = 0$.

- For $\theta = 1$:

  Equation (47) can be rewritten as[9]:

$$S_I(v)=\frac{g-\rho}{2g}C+\frac{CB\bar{L}^{1-\delta}(1-v)^{1-\delta}}{2g}\frac{1-v}{\phi v-1}. \tag{49}$$

[9]This expression implicitly avoids the open access case, $v^{oa} = 1/\phi$, so that denominator in the second term of $S_I(v)$ never vanishes.

Correspondingly, equation (48) is equivalent to

$$S_{II}(v) = C - \frac{CB\bar{L}^{1-\delta}(1-v)^{1-\delta}}{g}. \tag{50}$$

Function $S_I(v)$ presents a vertical asymptote for $v = 1/\phi$ (see Figure 1). We are looking for a solution satisfying $v \in (1/\phi, 1)$. Note that within this interval $S_I'(v) < 0$, the right branch of function $S_I(v)$ decreases from $\lim_{v \to 1/\phi^+} S_I(v) = \infty$ to $S_I(1) = (g-\rho)C/g$. Correspondingly, $S_{II}'(v) > 0$, and $S_{II}(v)$ grows from $S_{II}(1/\phi)$ to $S_{II}(1) = C$. A sufficient condition for the existence of a unique equilibrium within this interval and a positive stock of resource is given by: $S_I(1) \geq 0$ which is equivalent to $g \geq \rho$.[10]

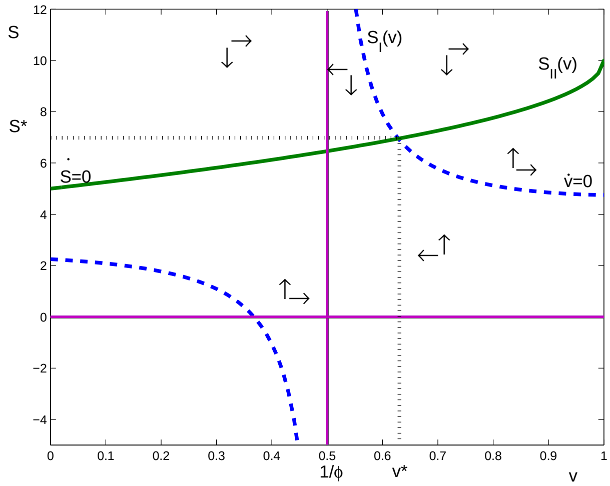

Figure 1: Phase diagram

Moreover, from (47)-(48) the following relationship between v^* and S^* can be derived:

$$v^* = v(S^*) = \frac{\rho - g + g\frac{S^*}{C}}{\phi\rho - g(\phi - 1) + g(2\phi - 1)\frac{S^*}{C}}. \tag{51}$$

An easy computation shows that

$$\text{sign } v'(S^*) = \text{sign } (1 - \phi).$$

[10]Note that if $g < \rho$, a unique equilibrium with $S^* \geq 0$ still exists if $S_{II}(1/\phi) \geq 0$, or equivalently, $g \geq B\bar{L}^{1-\delta}(1 - 1/\phi)^{1-\delta}$. The right hand side of last inequality represents the average and the marginal rate of harvesting per unit of the resource stock, S, under an open access natural regime.

Since $\phi > 1$, it follows that $v'(S)^* < 0$. Therefore, differentiating (47) with respect to S^*,

$$g\left(1 - 2\frac{S^*}{C}\right) = \frac{\partial R}{\partial v}(v^*, S^*)v'(S^*) + \frac{\partial R}{\partial S}(v^*, S^*) > 0.$$

Then, the gross reproduction rate of the resource presents a positive slope at the steady-state. That is, $S^* < C/2$.

- For $\theta = 0$:

  Assuming interior solutions, equation (47) is equivalent to:

$$\left[\rho - g\left(1 - 2\frac{S}{C}\right)\right][\phi v - 1] = 0.$$

We remove the open access solution $v^{oa} = 1/\phi$, because we are interesting in a solution within the interval $v \in (1/\phi, 1)$. Thus, the stock of the natural resource at the steady-state would be:

$$S^* = \frac{g - \rho}{2g}C < \frac{C}{2}, \tag{52}$$

which takes a positive value if and only if $g > \rho$.

For this stationary stock of natural resource, equation (48) gives the steady-state value for the share of labor devoted to the final output sector:

$$v^* = 1 - \frac{1}{\bar{L}}\left[\frac{(g^2 - \rho^2)C}{4gB}\right]^{\frac{1}{1-\delta}}. \tag{53}$$

Condition $g > \rho$ ensures that $v^* < 1$. Moreover, v^* will be higher than $1/\phi$ if and only if

$$G(S^*) \equiv \frac{g^2 - \rho^2}{4g}C < B(\bar{L}(1 - 1/\phi))^{1-\delta},$$

which means that the gross reproduction rate at the steady-state does not surpass the harvesting under open access. The above inequality is equivalent to inequality $g < g^+$, where g^+ is given by

$$g^+ = \frac{2B(\bar{L}(1 - 1/\phi))^{1-\delta}}{C} + \sqrt{\frac{4B^2(\bar{L}(1 - 1/\phi))^{2(1-\delta)}}{C^2} + \rho^2}. \tag{54}$$

■

Lemma 23 *The trace of the Jacobian matrix associated with the dynamical system (41)-(43) evaluated at the steady state* $(\tilde{c}^*, v^*, S^*)$, J^*, *is positive for all* $\theta \in [0, 1]$.

Proof. The Jacobian matrix associated with the dynamical system (41)-(43) is:

$$J(\tilde{c},v,S)=\begin{pmatrix}\frac{\partial\dot{\tilde{c}}}{\partial\tilde{c}} & \frac{\partial\dot{\tilde{c}}}{\partial v} & \frac{\partial\dot{\tilde{c}}}{\partial S}\\ \frac{\partial\dot{v}}{\partial\tilde{c}} & \frac{\partial\dot{v}}{\partial v} & \frac{\partial\dot{v}}{\partial S}\\ \frac{\partial\dot{S}}{\partial\tilde{c}} & \frac{\partial\dot{S}}{\partial v} & \frac{\partial\dot{S}}{\partial S}\end{pmatrix}=\begin{pmatrix}\frac{\dot{\tilde{c}}}{\tilde{c}}+\tilde{c}\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial\tilde{c}} & \tilde{c}\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v} & \tilde{c}\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial S}\\ v\frac{\partial(\dot{v}/v)}{\partial\tilde{c}} & \frac{\dot{v}}{v}+v\frac{\partial(\dot{v}/v)}{\partial v} & v\frac{\partial(\dot{v}/v)}{\partial S}\\ S\frac{\partial(\dot{S}/S)}{\partial\tilde{c}} & S\frac{\partial(\dot{S}/S)}{\partial v} & \frac{\dot{S}}{S}+S\frac{\partial(\dot{S}/S)}{\partial S}\end{pmatrix},$$

which evaluated at the steady-state equilibrium is

$$J^*=J(\tilde{c}^*,v^*,S^*)=\begin{pmatrix}\tilde{c}^*\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial\tilde{c}} & \tilde{c}^*\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v} & \tilde{c}^*\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial S}\\ v^*\frac{\partial(\dot{v}/v)}{\partial\tilde{c}} & v^*\frac{\partial(\dot{v}/v)}{\partial v} & v^*\frac{\partial(\dot{v}/v)}{\partial S}\\ S^*\frac{\partial(\dot{S}/S)}{\partial\tilde{c}} & S^*\frac{\partial(\dot{S}/S)}{\partial v} & S^*\frac{\partial(\dot{S}/S)}{\partial S}\end{pmatrix}=\begin{pmatrix}w_{11} & w_{12} & w_{13}\\ w_{21} & w_{22} & w_{23}\\ 0 & w_{32} & w_{33}\end{pmatrix}.$$

Note that

$$\begin{aligned}
w_{11} &= \frac{\bar{L}\tilde{c}^*}{\eta\sigma}>0,\\
w_{22} &= \Omega(v^*)\left\{\phi\left[\rho-g\left(1-2\frac{S^*}{C}\right)\right]+\Gamma\frac{R(v^*,S^*)}{S^*}+(\phi v^*-1)\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v}\right\}\\
&= \Omega(v^*)\left\{\Lambda\frac{R(v^*,S^*)}{S^*}+(\phi v^*-1)\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v}\right\},\\
w_{33} &= -\frac{gS^*}{C}-(\theta-1)\frac{R(v^*,S^*)}{S^*}=g\left[\left(1-2\frac{S^*}{C}\right)-\theta\left(1-\frac{S^*}{C}\right)\right],
\end{aligned}$$

where

$$\begin{aligned}
\Gamma &= \theta\left[1+(1-\delta)+\frac{(1-\delta)(\phi v^*-1)(1-\alpha-\beta)}{(1-\alpha)(1-v^*)}\right],\\
\Lambda &= \theta\frac{\phi(1-v^*)}{\phi v^*-1}+\Gamma>\theta\left[\frac{1}{v^*}+(1-\delta)+\frac{(1-\delta)(\phi v^*-1)(1-\alpha-\beta)}{(1-\alpha)(1-v^*)}\right]>0.
\end{aligned}$$

Note that,

$$\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v}=\frac{(\phi v^*-1)}{v^*(1-\alpha)(1-v^*)(1-\alpha-\beta)}\left(\frac{\bar{L}\tilde{c}^*}{\sigma}-\eta\rho\right)>0,$$

and therefore, $w_{22}>0$.

However w_{33} does not have a clear sign. For $\theta=0$ the sign of w_{33} is positive, whereas for $\theta=1$ the sign is negative.

Taking into account that $\Omega(v^*)\Lambda > \theta$, then

$$\begin{aligned} w_{22}+w_{33} &> \theta\frac{R(v^*,S^*)}{S^*}+\Omega(v^*)(\phi v^*-1)\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v}+g\left[\left(1-2\frac{S^*}{C}\right)-\theta\left(1-\frac{S^*}{C}\right)\right] \\ &= \Omega(v^*)(\phi v^*-1)\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v}+g\left(1-2\frac{S^*}{C}\right) > 0, \end{aligned}$$

and we conclude that the trace of matrix J^* is positive for all $\theta \in [0,1]$. ■

Proof of Proposition 6. As the previous lemma has stated $trace(J^*) > 0$. To prove the saddle point property with a one-dimensional stable manifold we need to establish a negative sign for the determinant of matrix J^*.

Its determinant is given by

$$|J^*| = \begin{vmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ 0 & w_{32} & w_{33} \end{vmatrix},$$

where[11]

$$\begin{aligned} w_{21} &= \Omega(v^*)\,(\phi v^*-1)\,\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial \tilde{c}}, \\ w_{22} &= \Omega(v^*)\left\{\phi\left[\rho-g\left(1-2\frac{S^*}{C}\right)\right]+\theta(2-\delta)\frac{R^*}{S^*}+[\phi v^*-1]\left[\theta\frac{1-\alpha-\beta}{1-\alpha}\frac{\partial(\dot{S}/S)}{\partial v}+\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v}\right]\right\}, \\ w_{23} &= \Omega(v^*)\left\{\frac{2g}{C}[\phi v^*-1]+\theta(1-\theta)\frac{R^*(1-v^*)}{S^{*2}}+[\phi v^*-1]\left[\theta\frac{1-\alpha-\beta}{1-\alpha}\frac{\partial(\dot{S}/S)}{\partial S}+\frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial S}\right]\right\}. \end{aligned}$$

Then,

$$\begin{aligned} |J^*| &= S^*\tilde{c}^*\Omega(v^*)\times \\ &\begin{vmatrix} \frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial \tilde{c}} & \frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial v} & \frac{\partial(\dot{\tilde{c}}/\tilde{c})}{\partial S} \\ 0 & \phi\left[\rho-g\left(1-2\frac{S^*}{C}\right)\right]+\theta(2-\delta)\frac{R^*}{S^*} & \frac{2g}{C}\left[\phi v^*-1\right]+\theta(1-\theta)\frac{R^*(1-v^*)}{S^{*2}} \\ 0 & \frac{\partial(\dot{S}/S)}{\partial v} & \frac{\partial(\dot{S}/S)}{\partial S} \end{vmatrix} \\ &= S^*\tilde{c}^*\Omega(v^*)\frac{\bar{L}}{\eta\sigma}\begin{vmatrix} \theta\frac{R^*}{S^*}\left[\frac{\phi-1}{\phi v^*-1}+(1-\delta)\right] & \frac{2g}{C}\left[\phi v^*-1\right]+\theta(1-\theta)\frac{R^*(1-v^*)}{S^{*2}} \\ (1-\delta)\frac{R^*}{(1-v^*)S^*} & -\frac{g}{C}-(\theta-1)\frac{R^*}{S^{*2}} \end{vmatrix} \end{aligned}$$

[11] Along this proof and in order to simplify the notation as much as possible, we will denote $R^* = R(S^*,v^*)$.

$$= S^*\tilde{c}^*\Omega(v^*)\frac{\bar{L}}{\eta\sigma}\left\{\frac{gR^*}{CS^*}\begin{vmatrix} \theta\left[\frac{\phi-1}{\phi v^*-1}+(1-\delta)\right] & 2\left[\phi v^*-1\right] \\ \frac{1-\delta}{1-v^*} & -1 \end{vmatrix}\right.$$

$$\left.+\theta(1-\theta)\frac{R^{*2}}{S^{*3}}\begin{vmatrix} \frac{\phi-1}{\phi v^*-1}+(1-\delta) & 1-v^* \\ \frac{1-\delta}{1-v^*} & 1 \end{vmatrix}\right\}.$$

It is clear that the first term of the last sum is always negative. Therefore, when $\theta = 0$ or $\theta = 1$ the determinant of the Jacobian matrix will be negative. ■

Proof of Proposition 7. The effect of changes in parameters C and g on S^* and v^* for case $\theta = 0$, can be easily obtained, taking partial derivatives in expressions (52) and (53). Therefore, from now on in this proof we concentrate on case $\theta = 1$. The proof is based on the effect of each parameter on curves $S_I(v)$ and $S_{II}(v)$, given by (49) and (50), and depicted in Figure 1.

- Derivatives with respect to g: $(\partial S_I/\partial g)(v) < 0$ (curve $\dot{v} = 0$ moves down), $(\partial S_{II}/\partial g)(v) > 0$ (curve $\dot{S} = 0$ shifts up). By Figure 1, these shifts of curves $S_I(v)$ and $S_{II}(v)$ imply a reduction in the percentage of labor devoted to final output at the steady-state, v^*, while the effect on S^* is ambiguous.

  If expression $v^* = v(S^*)$, given in (51) is incorporated in equation (47), taking derivatives with respect to g, it follows:[12]

  $$\frac{\partial S^*}{\partial g}\left[-\frac{g}{C}+B(1-\delta)[\bar{L}(1-v^*)]^{-\delta}\bar{L}v'(S^*)\right] = -\left(1-\frac{S^*}{C}\right)$$
  $$-B(1-\delta)[\bar{L}(1-v^*)]^{-\delta}\bar{L}\frac{\partial v}{\partial g}$$

  It is easy to prove that $\partial v/\partial g > 0$. Thus, the right hand side in this equation is negative and, since $v'(S^*) < 0$, the derivative $\partial S^*/\partial g$ must be positive.

- Derivatives with respect to C: $(\partial S_I/\partial C)(v) > 0$ and $(\partial S_{II}/\partial C)(v) > 0$, curves $\dot{v} = 0$ and $\dot{S} = 0$ shift up. The stock of the resource at the steady-state, S^*, increases, while the effect on v^* is unknown.

  Elasticity of curves $\dot{v} = 0$ and $\dot{S} = 0$ with respect to the carrying capacity are both equal to one. Therefore, v^* is unaffected by the carrying capacity, $\partial v^*/\partial C = 0$.

[12] Equation (51) could be rewritten as $v^*(\Phi) = v(S^*(\Phi), \Phi)$, where $\Phi = \{\rho, g, C, \delta, \eta, \nu\}$ is a set of parameters affecting equilibria (v^*, S^*). For simplicity we omit these parameters to define $v^* = v(S^*)$. For any parameter $x \in \Phi$, the total derivative is denoted as $\partial v^*/\partial x$, and can be decomposed in terms of partial derivatives: $\partial v/\partial x + v'(S^*)\partial S^*/\partial x$.

■

8 Appendix B: Sustainable growth with FDI

Proposition 24 *The optimal time paths in the technological follower country can be characterized as follows:*

$$\beta - \frac{1-v}{1-\delta}\frac{1-\alpha-\beta}{v} = \frac{1-\alpha}{2}\lambda_F R, \tag{55}$$

$$\dot{\lambda}_F = \left(\rho - G'(S) + \theta\frac{R}{S}\right)\lambda_F - \frac{\beta\theta}{S(1-\alpha)}. \tag{56}$$

Furthermore, the growth rate of national good consumption is given by:

$$\frac{\dot{c}_F}{c_F} = \frac{\dot{Y}_F}{Y_F} = \frac{\alpha}{1-\alpha}\frac{\dot{p}_F}{p_F} + \frac{(1-\alpha-\beta)(1-\phi v)}{(1-\alpha)(1-v)}\frac{\dot{v}}{v} + \frac{\dot{N}}{N} + \frac{\beta\theta}{1-\alpha}\frac{\dot{S}}{S}. \tag{57}$$

Proof. The current-value Hamiltonian associated with the dynamic optimization problem that consumers in the follower country are facing (24)-(25) reads:

$$H_F(v, S, \lambda_F) = \ln\left(p_R\frac{R}{L_F} + vw_F\right) + \lambda_F\left[G(S) - B(L_F(1-v))^{1-\delta}S^{\theta}\right],$$

where λ_F denotes the shadow price associated with the stock of the natural resource S.

Assuming interior solutions, first-order optimality conditions are:

$$\frac{\partial H_F}{\partial v} = 0 \Leftrightarrow w_F L_F + p_R\frac{\partial R}{\partial v} = \lambda_F\frac{\partial R}{\partial v}(p_R R + vL_F w_F); \tag{58}$$

$$\dot{\lambda}_F = \rho\lambda_F - \frac{\partial H_F}{\partial S} = \rho\lambda_F - \left[\frac{p_R\frac{\partial R}{\partial S}\frac{1}{L_F}}{p_R\frac{R}{L_F} + vw_F} + \lambda_F\left(G'(S) - \frac{\partial R}{\partial S}\right)\right]. \tag{59}$$

From the definition of the harvesting of the natural resource R given in (2):

$$\frac{\partial R}{\partial v} = -\frac{1-\delta}{1-v}R, \qquad \frac{\partial R}{\partial S} = \theta\frac{R}{S},$$

and replacing $\partial R/\partial v$ in equation (58), after several computations, we get:

$$p_R R - \frac{1-v}{1-\delta}w_F L_F = \lambda_F R[p_R R + vL_F w_F].$$

Likewise, replacing $\partial R/\partial S$ in (59) the dynamics of the costate variable λ_F can be rewritten as:

$$\dot{\lambda}_F = \left[\rho - \left(G'(S) - \theta\frac{R}{S}\right)\right]\lambda_F - \frac{p_R\frac{R}{S}}{p_R R + vL_F w_F}.$$

Taking into account (22), expressions (55) and (56) immediately follow.

From (22) and (23) one gets:

$$\frac{\dot{c}_F}{c_F} = \frac{\dot{Y}_F}{Y_F}.$$

Taking into account (21), (22), (2) and (31) the growth rate of consumption and final output production of the follower country, in (57) follows. ■

Proposition 25 *In the technological leader country, along the optimal time paths, the growth rates of consumption of national and imported goods are given by:*

$$\frac{\dot{c}_L}{c_L} = r - \rho, \qquad \frac{\dot{c}_{LF}}{c_{LF}} = r - \rho - \frac{\dot{p}_F}{p_F}. \tag{60}$$

Proof. The current-value Hamiltonian associated with the dynamic optimization problem that consumers in the leader country are facing (28)-(29) is given by:

$$H_L(c_L, c_{LF}, a_L, \lambda_L) = \ln(c_L) + \ln(c_{LF}) + \lambda_L(ra_L + w_L - c_L - p_F c_{LF}),$$

where λ_L denotes the shadow price of the assets, a_L.

Assuming interior solutions, the first-order optimality conditions are:

$$\frac{\partial H_L}{\partial c_L} = 0 \Leftrightarrow c_L = \frac{1}{\lambda_L}; \tag{61}$$

$$\frac{\partial H_L}{\partial c_{LF}} = 0 \Leftrightarrow p_F c_{LF} = \frac{1}{\lambda_L}; \tag{62}$$

$$\dot{\lambda}_L = \rho\lambda_L - \frac{\partial H_L}{\partial a_L} = (\rho - r)\lambda_L. \tag{63}$$

From (61) and (62) the following relationship between the consumption of the two different final goods can be derived:

$$c_L = p_F c_{LF}, \tag{64}$$

and therefore,

$$\frac{\dot{c}_L}{c_L} = \frac{\dot{p}_F}{p_F} + \frac{\dot{c}_{LF}}{c_{LF}}.$$

Furthermore, (61) implies:

$$\frac{\dot{c}_L}{c_L} = -\frac{\dot{\lambda}_L}{\lambda_L},$$

and together with (63) establishes (60). ■

Proof of Proposition 11. Following the same reasoning as in the proof of Proposition 3, v, S and R are constant on a steady-state equilibrium.

The growth rate of the number of intermediate goods, N, replacing the expression (64) of the consumption of imported goods in the leader country given in (36), can be written as:

$$\frac{\dot{N}}{N} = \frac{1}{\eta+\nu}\left[A_L L_L^{1-\alpha} X_L^{\alpha} - 2\frac{c_L}{N} L_L - X_L + \frac{1-\alpha}{\alpha} X_F(v,S)\right]. \tag{65}$$

The growth rate of N is constant along the steady-state equilibrium if the consumption of national good in the leader country, c_L, grows at the same rate as the number of intermediate goods, N, and at the same time, the amount of intermediate goods used in the follower final output sector, $X_F(v,S)$, is also stationary. From equation (33) for X_F to be constant, since v and R are motionless, also the terms of trade, p_F, must remain constant.

Taking into account (34), provided that p_F, v and R remain constant along the steady-state equilibrium, the interest rate r is also constant and equal to:

$$r = \frac{\alpha^{\frac{2}{1-\alpha}}(1-\alpha)\left[L_L A_L^{\frac{1}{1-\alpha}} + L_F v (p_F A_F)^{\frac{1}{1-\alpha}} R^{\frac{\beta}{1-\alpha}}\right]}{(\eta+\nu)\alpha}. \tag{66}$$

Along the steady-state equilibrium, constants v, S and p_F allow us to rewrite the growth rate of the final output production, Y_F, and the consumption, c_F, in the follower country, in (57), equal to the growth rate of the number of intermediate goods, N (which coincides with the growth rate of the national good consumption in the leading country). Furthermore, by (64), since p_F remains constant along the steady-state equilibrium, the growth rate of the imported good, c_{LF}, equals the growth rate of the national good consumption in the leading country.

Expression (35) implies that the growth rate of the production in the leader country also equals that of N, since X_L is constant.

Finally, provided that R remains constant (22) shows that the price of the natural resource grows as the same rate as Y_F along the steady-state equilibrium. ■

Lemma 26 *Any steady-state equilibrium for the trade model described by the dynamic problems for countries L and F, corresponds to a steady-state of the following three differential equations:*

$$\dot{\tilde{c}}_L = \tilde{c}_L \left\{\frac{1}{\eta+\nu}\left[2L_L\tilde{c}_L - (1-\alpha)L_L A_L^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}\right] - \rho\right\}, \tag{67}$$

$$\dot{v} = f_1^{oe}(v,S) = \Omega^{oe}(v)\Theta^{oe}(v,S), \tag{68}$$

$$\dot{S} = f_2(v,S) = G(S) - R(v,S), \tag{69}$$

where

$$\Omega^{oe}(v) = \frac{v(1-v)}{1-v+(1-\delta)v[\phi v-1]},$$

$$\Theta^{oe}(v,S) = \left[\rho - G'(S) + \theta\frac{\dot{S}}{S}\right][\phi v - 1] - \theta\frac{R(v,S)}{S}(1-v).$$

Proof. Following the same reasoning as in Lemma 22, the dynamics of $\tilde{c}_L$ can be obtained from the expression of r in (66) and the dynamics of N in (65). Furthermore, considering the expressions of p_R and w given in (22), in the necessary condition in (55), the dynamics of the labor share, v, in (67) arises likewise as in the proof of Lemma 22. ■

Proof of Proposition 12. We have to prove that the dynamical system (67)-(69) admits a unique steady-state, $(\tilde{c}_L^*, v^*, S^*)$, with $\tilde{c}_L^* > 0$, $1/\phi < v^* < 1$, $0 < S^* < C/2$. Notice that the dynamics of $\tilde{c}_L$ does not depend on v or S, neither the last two equations of the dynamical system do depend on $\tilde{c}_L$. Therefore, we can study the existence of the steady-state isolating the two last dynamic equations.

By equation (67), assuming interior solutions, in a steady-state equilibrium:

$$\tilde{c_L}^* = \frac{\rho(\eta+\nu) + L_L A_L^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha)}{2L_L}, \tag{70}$$

which does not depend on v^* or S^*.

Considering interior solutions, $\dot{v} = f_1^{oe}(v,S) = 0$ is equivalent to $\Theta^{oe}(v,S) = 0$. Furthermore, the equations system $f_2(v,S) = \Theta^{oe}(v,S) = 0$ is equivalent to system (47)-(48). Thus, the proof of Proposition 5 is valid. ■

Proof of Proposition 13. Following the same reasoning as in the proof of Proposition 6, the Jacobian matrix evaluated at the steady-state reads:

$$(J^{oe})^* = J^{oe}(\tilde{c}_L^*, v^*, S^*) = \begin{pmatrix} w_{11}^{oe} & 0 & 0 \\ 0 & w_{22}^{oe} & w_{23}^{oe} \\ 0 & w_{32} & w_{33} \end{pmatrix},$$

where

$$w_{11}^{oe} = \frac{2\tilde{c}_L^*}{\eta+\nu} > 0,$$

$$w_{22}^{oe} = \Omega(v^*)\left\{\phi\left[\rho - g\left(1-2\frac{S^*}{C}\right)\right] + \theta(2-\delta)\frac{R(v^*,S^*)}{S^*} + \theta[\phi v^* - 1]\frac{\partial(\dot{S}/S)}{\partial v}\right\},$$

$$w_{23}^{oe} = \Omega(v^*)\left\{\frac{2g}{C}[\phi v^* - 1] + \theta(1-\theta)\frac{R(v^*,S^*)(1-v^*)}{S^{*2}} + \theta[\phi v^* - 1]\frac{\partial(\dot{S}/S)}{\partial S}\right\}.$$

One of the three eigenvalues of this matrix is given by $w_{11}^{oe} > 0$. Furthermore, the determinant of this Jacobian matrix can be written as:

$$w_{11}^{oe} \begin{vmatrix} w_{22}^{oe} & w_{23}^{oe} \\ w_{32} & w_{33} \end{vmatrix},$$

which has the same sign as $|J^*|$ in the proof of Proposition 6. Therefore, for $\theta = 0$ or $\theta = 1$ this determinant will be negative. This ensures that the Jacobian matrix presents two positive eigenvalues and one negative eigenvalue, and therefore the saddle-point stability is proved. ■

Proof of Proposition 14. From equation (37) and taking into account the optimal consumption decisions in the leading country given in (64), at the steady-state it follows:

$$L_L \tilde{c}_L^* = p_j X_F(v^*, S^*). \tag{71}$$

From (71) and the expression of X_F in (33), we get the terms of trade at the steady-state:

$$p_F^* = \frac{(\tilde{c}_L^* L_L)^{1-\alpha}}{\alpha^{1+\alpha} A_F (L_F v^*)^{1-\alpha-\beta} R(v^*, S^*)^{\beta}}.$$

Replacing the expression of the steady-state value of variable $\tilde{c}_L$, $\tilde{c}_L^*$, given in (70), we get the final expression for the price p_F along the steady-state equilibrium:

$$p_F^* = \frac{\left[\rho(\eta + \nu) + L_L A_L^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} (1-\alpha)\right]^{1-\alpha}}{2^{1-\alpha} \alpha^{1+\alpha} A_F (L_F v^*)^{1-\alpha-\beta} R(v^*, S^*)^{\beta}}. \tag{72}$$

From the expression of the terms of trade at the steady-state in (72), the positive effect of η and ν, on p_F^* immediately follows.

The effect of parameters C and g on the denominator in expression (72) has the same sign that the effect of these parameters on γ^{soe}. Moreover, resource abundance does not affect the numerator in this expression. In consequence, by Proposition 16, the terms of trade at the steady-state decrease with C and g. ■

Proof of Proposition 15. The ratio of domestic consumption per variety of intermediate goods in L, $\tilde{c}_L^*$, in (70) is independent of resource abundance, regardless of the size of the trading economies.

The ratio of consumption of the imported good per variety of intermediate good in L, reads $\tilde{c}_{LF}^* = \tilde{c}_L^*/p_F^*$. For SOE, the terms of trade at the steady-state is an exogenous constant, $p_F^* = \hat{p}_F$, and thus, $\tilde{c}_{LF}^*$ will also be independent of C and g. Conversely, for LOE, by Proposition 14 it follows a positive relationship.

Taking into account (21) and (22), the ratio of consumption in F per variety of intermediate good, $\tilde{c}_F^*$, in (23), can be rewritten as:

$$\tilde{c}_F^* = \frac{1-\alpha}{\alpha^2} \frac{X_F(v^*, S^*)}{p_F^*}.$$

For SOE, with a constant $p_F^* = \hat{p}_F$, an increment in C or g increases $X_F(v^*, S^*)$ and thus, $\tilde{c}_F^*$.. For LOE, $X_F(v^*, S^*)$ is unaffected by resource abundance, but implies that p_F^* decreases, which rises $\tilde{c}_F^*$. ■

Proof of Proposition 16. The growth rate at the steady-state can be obtained by the growth rate of consumption of national good in the technological leader country in (30). This growth rate equals $r - \rho$, where r is given in (66). Therefore, denoting by γ^{oe} this growth rate it can be written as follows:

$$\begin{aligned}\gamma^{oe} &= \frac{1-\alpha}{\alpha(\eta+\nu)}\left[X_L + X_F(v^*, S^*)\right] - \rho \\ &= \frac{(1-\alpha)\alpha^{\frac{2}{1-\alpha}}}{\alpha(\eta+\nu)}\left[L_L A_L^{\frac{1}{1-\alpha}} + (L_F v^*)^{\frac{1-\alpha-\beta}{1-\alpha}} A_F{}^{\frac{1}{1-\alpha}} R(v^*, S^*)^{\frac{\beta}{1-\alpha}} p_F{}^{\frac{1}{1-\alpha}}\right] - \rho.\end{aligned} \tag{73}$$

Last expression above shows the growth rate of the economies in the SOE scenario, γ^{soe}, when the price is constant and exogenously given, $\hat{p}_F$. Conversely, in the LOE scenario, the growth rate of the economies is obtained replacing the terms of trade, p_F, by its stationary value, p_F^*.

Substituting the value of p_F^*, given by (72), in (73) and simplifying, the growth rate γ^{loe} in (38) follows.

The effect on γ^{soe} of changes in the environmental parameters C and g can be established along the same lines as for the closed economy (see Proposition 8). ■

9 Appendix C: Domestic innovation vs. FDI

Proof of Proposition 17. The result immediately follows from the expressions of the long-run growth rates in (20) and (73). ■

Proof of Corollary 18. From the expressions of the long-run growth rates in (20) and (38), and taking into account p_F^* in (72), $\gamma > \gamma^{loe}$ if and only if the expression below is negative:

$$(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L_L A_L^{\frac{1}{1-\alpha}}[2\alpha\eta - (1-\alpha)\nu] - \rho(\eta+\nu)[(1+\alpha)\eta + (1-\alpha)\nu]. \tag{74}$$

If $\dfrac{\nu}{\eta} \geq \dfrac{2\alpha}{1-\alpha}$, then $2\alpha\eta - (1-\alpha)\nu \leq 0$, and expression (74) is negative. Thus, $\gamma > \gamma^{loe}$.

On the contrary, expression (74) takes positive values under sufficient condition:

$$(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} L_L A_L^{\frac{1}{1-\alpha}}(3\alpha - 1) - 2\rho(\eta+\nu) \geq 0.$$

It is easy to prove that the LHS of this inequality is no lower than $(3\alpha-1)\gamma^{loe}$ if and only if $\alpha \geq 2/3$. Furthermore, for these values of α, and under the assumption of a positive long-run growth rate in LOE, it follows that $(3\alpha - 1)\gamma^{loe} > 0$. Thus, expression (74) is positive and $\gamma < \gamma^{loe}$. ■

Proof of Proposition 19. From the expressions of the long-run growth rates in (20) and (73) it follows that the effect of resource abundance on the gap $\gamma - \gamma^{oe}$, presents the same sign as its effect on $X_F(v^*, S^*)$. For SOE, an increment in C or g increases $X_F(v^*, S^*)$, while for LOE, $X_F(v^*, S^*)$ does not change with resource abundance. ■

Proof of Proposition 20. From (21), (22) and (23) the consumption per variety of intermediate good in the follower country reads:

$$\tilde{c}_F^* = (1-\alpha)\frac{A_F}{L_F}\alpha^{\frac{2\alpha}{1-\alpha}}(v^* L_F)^{\frac{1-\alpha-\beta}{1-\alpha}}(p_F A_F)^{\frac{\alpha}{1-\alpha}} R(v^*, S^*)^{\frac{\beta}{1-\alpha}}.$$

The gap between $\tilde{c}^*$ in (46) and $\tilde{c}_F^*$ is $\dfrac{\rho\eta}{p_F L_F}$. This increment in consumption per variety of intermediate good decreases with p_F. For SOE this price is exogenous and constant, $\hat{p}_F$, and so, independent on C or g. For LOE the terms of trade, p_F^*, decreases with resource abundance, leading to a wider gap. ■