The Good, the Bad and the Ugly:

Agent Behavior and Efficiency in Open and Closed Organizations

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**Abstract:** Current literature has largely ignored the fact that some organizations are highly selective when admitting new agents

while others are more open. In addition, some organizations audit or sort agent behavior within the organization more aggressively

than others. One might expect a priori that closed, highly selective, organizations would always be more efficient because they

screen out the worst types, which could lead to better agent behavior. We show that this is not the case. Specifically, when agent

behavior in equilibrium is uniform across organizations (i.e., when the number of agents behaving the same way is identical),

closed organizations are inefficient. However, when agent behavior varies across organizations, closed organizations may or may

not be inefficient, depending on net payoffs to the organization and the agents. Our analysis implies that organizations should

choose the open type when screening or sorting costs are high, when there is a high frequency of good agent types in the population,

when agent misbehavior does not reduce output significantly, and when penalties for misbehavior are large.

**Keywords:** Asymmetric information, organization theory, efficiency, sorting, screening.

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#### 1. Introduction

Why are some organizations or societies more open than other, even seemingly similar, organizations or societies? For example, firms in the same industry, equally developed countries or institutions engaged in the same activities often differ in how selective they are when admitting agents. In addition to this initial screening, organizations frequently monitor, audit or sort their agents *ex post*. Why do some organizations sort more intensively than others, and is there a link between the degree of screening and the intensity of sorting? How does the extent of screening and sorting affect the behavioral choices of agents and the ensuing efficiency of organizations?

We develop a framework in this paper to address these important questions and shed light on the actual practice of organizations. To the best of our knowledge, these questions have been overlooked by current theory. However, these are important questions for new organizations or even for organizations that are undergoing a restructuring process, namely, should these organizations spend resources to screen new agents or to audit agent behavior? We provide answers based on the characteristics of the agent populations, the expected equilibrium behavior of agents, the costs of screening and sorting, and the penalty structure set by the institutional environment. The model is general enough to allow the organization to be a variety of institutions. The organization could be a firm and the agents potential employees, or a country and the agents potential immigrants, or a governmental agency enforcing laws, or a school or licensing authority dealing with applicants. For example, big city law and consulting firms are reputedly more lenient in hiring than small city firms but fewer hires make partner. Large Japanese corporations generally provide long-term employment to their employees, but in doing so they are very selective when they hire people. Some companies give tests to all applicants, for example Microsoft reputedly tests the intelligence and creativity of applicants regardless of credentials, other firms such as Home Depot require all potential employees to take drug tests, and banks routinely obtain credit reports for all job applicants. Some occupations require extensive licensure testing. Some universities admit most students who apply but flunk out a large percentage, while others are very selective but graduate most students who matriculate. It is more difficult to get into a respectable Japanese university than a comparable U.S. university, but reputedly easier to graduate. Several European countries test drivers more thoroughly before granting drivers' licenses than in the United States, but monitoring of good driving behavior (for instance, speeding) is less thorough. In many European countries government job applicants are required to present more documents (for instance criminal records) and undergo greater testing than those in the United States, but there may be less scrutiny once the applicants are hired.

Sorting can take different forms. Organizations may monitor agents while the agents are executing tasks or audit agents after they have taken actions. Examples include observation of worker performance by supervisors, periodic performance evaluations, enforcement of traffic laws, tenure decisions at universities, admission to partnership in law and accounting firms, and school testing. According to Cross (2001), many companies routinely fire 5-10% of their least productive workers over the course of a year. Companies use relative performance evaluations and compare employees to averages or to other employees (as in a yardstick competition or tournament) to identify laggards and weed out these weak links. In some companies or institutions sorting takes the form of up-or-out contracts. While some companies reform, retrain or reassign the bottom ranked workers, other companies simply fire these employees immediately after they are identified. For instance, in the last few years, GE has instituted a program it calls "Organizational Vitality" in which bottom ranked workers are reformed successfully or they are ousted. Cisco has a plan where the bottom ranked 5% employees are put on a "Performance Improvement Plan." Employees who fail to achieve prespecified milestones are simply "PIPped." On the other hand, Siebel routinely turns over the lowest performing employees without spending resources to revitalize them. Tenure denial at academic institutions (in rates that differ among institutions) is another example.

We develop the model of agent behavior in organizations in section 2. Each agent is intrinsically "good," "bad" or "ugly," and can behave as either an achiever or a laggard in equilibrium, as shown in section 3. Organizations can be "open" or "closed." Closed organizations "screen" potential agents before admitting them while open organizations do not. Both types of organization have the option to audit or "sort" individual behavior after observing the aggregate outcome obtained by the agents, and impose disciplinary penalties. The initial screening is designed to weed out those agents whose performance is likely to be unacceptable. In contrast, *ex post* sorting aims at isolating agents whose actual behavior is unacceptable, and one might expect that more thorough screening would reduce the scope for sorting. We show that extensive initial screening does not eliminate the scope for *ex post* sorting, because limited sorting invites opportunistic misbehavior by the agents. Surprisingly, when the sorting cost per agent is constant or exhibits economies of scale, both organizations may sort the same number of agents in equilibrium, which leads agents of the same type to behave uniformly across

<sup>&</sup>lt;sup>1</sup> Specifically, closed organizations deny entry to the worst type; that is, ugly agents. The case of three intrinsic and two behavioral types is the smallest case in type space that yields general organizational efficiency results. Using two intrinsic and two behavioral types leads to the conclusion that the open organization is almost always more efficient in equilibrium. The three-intrinsic and three-behavioral type case does lead to a few additional social efficiency results compared to the model analyzed in the paper, but it is much more cumbersome.

organizations. However, agent behavior across intrinsic types may or may not be uniform. Interestingly, agents in equilibrium will never behave uniformly as achievers unless even the ugly types find it beneficial to behave as achievers and the sorting cost is very small. When all agent types behave uniformly as laggards or as achievers across organizations, and an equilibrium exists in both organizations, closed organizations are less efficient from the organization's perspective and socially than open ones, as shown in section 4. This follows because both organizations sort the same number of agents, so the screening costs incurred by the closed organization are unnecessary. If agent behavior is a mixture of types, then closed organizations can be efficient because they screen out some of the worst agent types in advance.

Our analysis yields the following implications. Organizations facing sufficiently high screening costs or, interestingly, sufficiently high sorting costs per agent relative to penalties recouped for agent misbehavior, will choose the open organization type. If the organization believes that the proportion of good intrinsic types in the population from which agents are drawn is sufficiently high, or the proportion of ugly intrinsic types is sufficiently low, it will choose the open organization type again. Organizations facing a sufficiently high outcome from agents who behave as laggards relative to the outcome received from agents who behave as achievers will chose the open organization type. Lastly, organizations that can recoup sufficiently high penalties for laggard behavior will choose the open type, provided that the sorting cost is sufficiently low.

We show in section 5 that most of the findings above extend to the case in which organizations can precommit to a sorting frequency. However, more equilibria are then possible; that is, equilibria exist in cases where no equilibria exist without precommitment, and there are more cases in which closed organizations sort fewer agents than open organizations. Thus, precommitment may or may not have value. Precommitment has no value when the sorting cost is less than the penalties expected to be recouped through sorting, because the organization will still find it optimal to sort the same number of agents. By contrast, precommitment has value when there are equilibria under precommitment while no equilibria exist under no-precommitment, and in the case in which the sorting cost exceeds the penalties expected to be recouped, because instead of sorting no agents, the organization may precommit to sorting some agents at a loss in order to induce better behavior.

We also extend the analysis to allow for decreasing returns to scale in sorting, or for sequential rather than simultaneous sorting. In both cases we find that the number of sorted agents can be intermediate; that is, other than all or none, which was the case with constant or increasing returns to scale and simultaneous sorting. Under decreasing returns to scale we also find that closed organizations will never sort more agents in equilibrium than open organizations, and when the organizations sort a

different number of agents, closed organizations induce better agent behavior even though they sort fewer agents.

This paper draws from related work on auditing and agent behavior in organizations. For instance, Kahlil (1997) examines a principal-agent model in which the principal can audit the agent's compliance with a contract but cannot precommit to auditing. He showed that lack of commitment to auditing when information is asymmetric can lead to production above the level that would be obtained with full information in order to reduce the probability of agent noncompliance. Maskin and Tirole (2004) consider "accountable" democratic systems in which government officials are screened and disciplined by voters and "unaccountable" systems in which government officials are appointed and hence neither screened nor disciplined by voters. Their interest is in determining the circumstances under which each type of system works best.

#### 2. Model

We develop a model with one organization and a finite number of agents. Agents are born with *intrinsic* types but then select *behavioral* types. An intrinsic type can reflect innate ability, competence, or a predisposition to, for example, work hard, behave legally or conform to organizational norms. The agents make endogenous decisions about their behavioral types by considering the benefits and costs of these decisions. The costs include the expenses associated with adopting various behavioral types contingent on the intrinsic types, and the penalty from being caught (sorted) weighted by its likelihood. Assume three intrinsic types: *good*, *bad* and *ugly*; also assume two behavioral types: *achiever* and *laggard*. We consider two types of organization, *closed* and *open*. A closed organization screens its agents before they are admitted into the organization more extensively than does an open organization. For simplicity, we assume a closed organization screens all agents before the agents are admitted to the organization, however, an open organization does no screening at all so all agents are allowed to join. Both a closed and an open organization have the option to sort their agents after the agents select behavioral types.<sup>2</sup>

The timing of events is as follows: First, nature selects an intrinsic type  $t_i \in \{g,b,u\}$  for each agent i, where g stands for good, b for bad and u for ugly, with probability  $p_i(t_i) > 0$  and *anonymity*, that is  $p_i(t_i) = p(t_i)$ , for all i. Then agents privately learn their intrinsic types. Second, the closed organization

<sup>&</sup>lt;sup>2</sup> Note that for concreteness we model sorting as taking place once after the agents choose behavioral types. The model, however, could easily be extended to include sorting while behavioral types are being adopted (i.e., monitoring of agents' activities) without changing the results qualitatively.

we consider screens the agents at a fixed cost of s per agent, rejecting those who are found to be the worst type, ugly, so that only n good and bad types are let in.<sup>3</sup> Assuming that the closed organization lets good and bad types in makes the analysis more interesting than if the organization lets in only good types (in which case all agents would behave the same way in equilibrium). The open organization, by contrast, does no screening and n agents, who can be good, bad or ugly, are let in. For either type of organization, we denote the true number of each intrinsic type in the organization as n(t<sub>i</sub>). Third, each agent i chooses a behavioral type or action  $\tau_i \in \{A,L\}$ , where A stands for achiever and L stands for laggard, at an adjustment cost of  $k(\tau_i|t_i)$ ; that is, the cost of adopting a behavioral type depends on the agent's intrinsic type. Let  $\tau = (\tau_1, ..., \tau_n)$  be the vector of behavioral types adopted by the agents. Each action  $\tau_i$  leads deterministically to a payoff to the agent of  $v(\tau_i)$ , with v(L) > v(A) > 0, and to an outcome  $x_i(\tau_i)$  for the organization, with  $x_i(A) = x(A)$ ,  $x_i(L) = x(L)$ ,  $\forall i$ , and x(A) > x(L) > 0. Note that the outcome for the organization does not depend on the agent's intrinsic type. Fourth, even though the organization does not observe individual outcomes, it does observe the aggregate outcome,4 X =  $\sum_{i=1}^{n} x_i(\tau_i)$ . Fifth, the organization sorts  $\ell$  agents, and  $\ell/n$  is the probability (frequency) that an agent is sorted. When an agent is sorted, his behavioral type is publicly revealed. The administrative cost to sort an agent is assumed to be the same for both types of organization and is fixed and equal to z > 0, where z is the cost of determining whether an agent has adopted laggard behavior. An agent who is found to be  $\tau_i$  pays a penalty  $r(\tau_i)$ , with r(L) > r(A) = 0. We assume the penalties are predetermined. This is a reasonable assumption because the legal system, standard industry practices, organizational and social norms, and outside agencies such as accrediting or overseeing bodies commonly predetermine or place an upper bound on the severity of penalties for various types of behavior. For instance, organizations cannot execute their agents for sleeping on the job. The penalties we model can therefore be thought of as the maximum allowed by social norms and institutions.<sup>5</sup> Naturally, extremely severe penalties could

<sup>&</sup>lt;sup>3</sup> For simplicity, we assume that organizations can determine through screening whether agents are ugly, but determining precisely whether agents are good or bad is prohibitively costly. In a more complex model we could endogenize the quality of screening by the closed organization. We conjecture that the results would not change qualitatively.

<sup>&</sup>lt;sup>4</sup> Note that the model can easily be expanded to include the moral hazard case in which the organization observes individual contributions, but those are stochastically dependent on unobservable actions taken by the agents. The model can also be expanded, without changing the results qualitatively, to the case in which X is verifiable to outsiders so that penalties can be imposed without sorting costs when all agents behave as laggards.

<sup>&</sup>lt;sup>5</sup> Given that the penalties are the maximum allowable, the organization *ex post* can control only the number of agents to be sorted, but this is not sufficient to induce high achievement by all agents as the analysis below will indicate. Therefore we cannot include incentive compatibility constraints that will ensure high achievement by all

make all agents behave as achievers, but penalties cannot be severe enough due to factors extraneous to the organization. As Shapiro and Stiglitz (1984) point out (p. 433), in the case of labor markets, "...the worst that can happen to a worker who shirks on the job is that he is fired." Even though the penalties in our model are parameterized, we do consider different possible values and their relative impact on behavior. Depending on the organization, the penalty could take many forms; for example, it could be a reduction in salary or a fine for illegal behavior. For simplicity we assume that the penalty imposed on the agents is paid to the organization. Qualitatively similar results would be obtained if the payoff to the organization did not coincide with the penalty but was systematically dependent on it. For instance, if a firm determines that an employee should be reformed by taking more extensive training or should be fired, the employee suffers a welfare loss while the firm enjoys an increase in productivity.

We make the following assumptions:

**Assumption 1a.** 
$$k(L|g) = k(L|b) = k(L|u) = 0.$$

**1b.** 
$$k(A|u) > k(A|b) > k(A|g) > 0.$$

Assumption 1a means there is no cost to the agent of any type in becoming a laggard.<sup>6</sup> We assume in 1b that the cost of becoming an achiever increases when moving from good to bad to ugly types.

Note that Assumption 1 implies that  $[v(L) - k(L|t_i)] - [v(A) - k(A|t_i)] > 0$ ,  $\forall t_i$ . That is, without ex post sorting, and hence penalties, any agent type prefers to behave as L rather than as A. Thus agents have an inherent incentive for misbehavior and there may be scope for sorting.

If an agent of intrinsic type  $t_i$  chooses behavioral type  $\tau_i$ , his expected payoff given his adjustment cost and, if sorted, his penalty is

(2.1) 
$$E(\tau_i|t_i) = v(\tau_i) - k(\tau_i|t_i) - \frac{\ell}{n} r(\tau_i).$$

The open organization's expected payoff after observing the aggregate outcome X is

agents.

<sup>&</sup>lt;sup>6</sup> We considered alternative cost structures in which the cost of behaving as a laggard depended on the agent's intrinsic type, with good types facing the highest cost. For example, an agent who is born a hard worker may find it as difficult to adjust to being a laggard as it is for a born shirker to behave as an achiever. However, this only complicated the analysis without changing the results qualitatively.

(2.2) 
$$\Pi_{o}(\ell) = X + \ell [\Phi(L|X) - z],$$

where  $\Phi(L|X) = \varphi(L|X)/n$  is the organization's assessment of the frequency of agents behaving as L, made after observing X, with  $\varphi(L|X)$  being the assessment of the number of agents behaving as L. The assessed frequency  $\Phi(L|X)$  is given by Bayes rule

(2.3) 
$$\Phi(L|X) = \frac{\Phi(X|L) \Phi(L)}{\sum_{\tau_i} \Phi(X|\tau_i) \Phi(\tau_i)},$$

where

(2.4) 
$$\Phi(\tau_i) = \sum_{t_i} p(\tau_i | t_i) p(t_i).$$

Note that in pure strategies, the probability  $p(\tau_i|t_i)$  that agent i chooses action  $\tau_i$  given his intrinsic type  $t_i$ , which enters (2.4), equals either 1 or 0.

In a closed organization, the expected payoff after observing the aggregate outcome X is

(2.5) 
$$\Pi_c(\ell) = -\operatorname{ms} + X + \ell[\Phi(L|X) - z],$$

where m is the number of agents that had to be screened to find n good and bad agents,  $\Phi(L|X)$  is determined by Bayes rule as above, and

(2.6) 
$$\Phi(\tau_i) = \sum_{t_i} p(\tau_i|t_i) \, \mu(t_i),$$

with the probability of agent i being of type t, in a closed organization given by

(2.7) 
$$\mu(t_i) = \frac{p(t_i)}{p(g) + p(b)}, \forall t_i \in \{g, b\}.^7$$

The rationale for (2.7) is that the closed organization screens the agents and removes all the ugly types u. Note that this implies

<sup>&</sup>lt;sup>7</sup> Note that for ease of exposition we have abused the notation; for instance, we use the same notation for all  $\Phi(\cdot)$  probabilities regardless of the type of organization, even though the probabilities depend on the type of organization.

(2.8) 
$$\mu(t_i) > p(t_i), \forall t_i \in \{g,b\},$$

because agents of type u are replaced with agents of types g and b. Of course, if the closed organization draws from a different population than does the open organization (e.g., a European versus a US pool), it is not possible to compare the distributions of the intrinsic types in the two organizations (that is, neither (2.7) nor (2.8) apply in this case).

In analyzing the extensive-form game played between the organization and the agents, given the type of organization, we adopt perfect Bayesian equilibrium (PBE) as the solution concept and we focus on pure strategy equilibria.<sup>8</sup> The equilibrium of the game is characterized by: (i) a choice of behavioral type  $\tau_i$  by each intrinsic type  $t_i$  of agent, that is,  $\tau_i(g)$ ,  $\tau_i(b)$  and  $\tau_i(u)$ , with  $\tau_i(t_i) \in \{A,L\}$ , given the probabilities  $p(t_i)$ , in an open organization, and  $\mu(t_i)$ , in a closed organization; (ii) a function  $\Phi(\tau_i|X) \in$ [0,1] denoting the organization's beliefs about agent i's behavioral type after observing the aggregate outcome X; (iii) a strategy  $\ell(X)$  for the organization determining the number of agents to be sorted based on the observed value of aggregate outcome X. The agents' strategies and the organization's strategy and beliefs must satisfy the following conditions. First, an agent's choice of behavioral type given his intrinsic type and the probabilities  $p(t_i)$  or  $\mu(t_i)$  is sequentially rational, that is, it maximizes his expected payoff in (2.1), given the organization's strategy and belief function. Second, given the agents' strategies, the organization's observation of aggregate outcome and its belief function, the organization chooses the strategy that maximizes its expected payoff shown in (2.2) for an open organization or in (2.5) for a closed organization. Third, the organization's belief function is derived from Bayes' rule according to (2.3). Note that the PBE is determined by backward induction because the organization observes X prior to choosing the number of agents to be sorted,  $\ell$ .

# 3. Equilibrium

To characterize the equilibrium, first note that the following two cases are possible regarding belief probabilities about behavioral types: (i) If X = nx(A) or X = nx(L), the organization receives a fully informative signal about each agent's behavioral type; because when X = nx(A) the organization knows that  $\tau_i(t_i) = A$ ,  $\forall i$ , and when X = nx(L),  $\tau_i(t_i) = L$ ,  $\forall i$ . (ii) If nx(L) < X < nx(A), the organization infers the number of agents behaving as either type exactly by solving two linear equations with two unknowns. That is,  $\phi(A|X) + \phi(L|X) = n$  and  $\phi(A|X)x(A) + \phi(L|X)x(L) = X$ , where  $\phi(A|X)$  is the number of

<sup>&</sup>lt;sup>8</sup> See the concluding section for a brief discussion of how the analysis could be extended to mixed strategies.

agents who behave as achievers and  $\phi(L|X)$  is the number of laggards. Hence, the organization can also infer the total number of intrinsic types who behave as laggards or as achievers. Note, however, that the organization cannot identify the actual agents who have behaved as laggards unless it sorts.

Because the equilibrium is obtained by backward induction, we first determine the organization's strategy,  $\ell(X)$ , in Lemma 1.

**Lemma 1.** In an open or closed organization, if X = nx(L), then

(3.1) 
$$\ell(X) = \begin{cases} n, & \text{if } z < r(L) \\ 0, & \text{if } z \ge r(L) \end{cases}$$

If X = nx(A), then

$$\ell(X) = 0.$$

If nx(A) < X < nx(L), then

(3.3) 
$$\ell(X) = \begin{cases} n, & \text{if } z < E(r) \\ 0, & \text{if } z \ge E(r) \end{cases}$$

where  $E(r) = \sum_{\tau_i} \Phi(\tau_i^{}|X) \, r(\tau_i^{}(t_i^{}))$  is the expected penalty from sorting agent i.

**Proof.** The proof is straightforward and is therefore omitted.

The rationale behind Lemma 1 is that the organization either sorts all agents or none depending on whether the cost per agent z is less than or greater than the expected penalty. It sorts all agents if all agents behave as laggards or as a mixture of laggards and achievers, provided that the sorting cost is smaller than the expected penalty, and it sorts no agents when they all behave as achievers (because the expected penalty is zero) or when the sorting cost is larger than or equal to the expected penalty.

Agent behavior in equilibrium will be shown to depend on whether the sorting frequency  $\ell/n$  lies in various intervals defined by the parameter below. Let

<sup>&</sup>lt;sup>9</sup> This is used in Cases (iii) and (iv) of Proposition 3 and in Case (iii) of Proposition 5 below.

(3.4) 
$$F(t_i) = \frac{v(L) - [v(A) - k(A|t_i)]}{r(L)}.$$

 $F(t_i)$  shows the marginal benefit to the agent of behaving as L rather than as A, divided by the marginal penalty imposed on the agent if he is sorted and found to have behaved as L rather than as  $A^{10}$ . Thus,  $F(t_i)$  measures the marginal benefit over the marginal cost of misbehaving for intrinsic type  $t_i$ . Assumption 1b then implies that 0 < F(g) < F(b) < F(u), which is used in Lemmas 2 and 4 (note, however, that F(u) is irrelevant for the closed organization because no ugly types are let in). Further, to characterize tie breaking cases we make the following regularity assumption.

**Assumption 2.** 
$$\tau_i(t_i) = L \text{ iff } E(\tau_i = L | t_i) > E(\tau_i = A | t_i).$$

This assumption means that the agent behaves as a laggard only when his payoff is strictly larger, and the agent behaves as an achiever otherwise.

### 3A. Equilibrium in an Open Organization

Lemma 2 characterizes agent behavior in an open organization for given sorting frequencies,  $\ell/n$ , in relation to the F(·) values. These results are useful in determining the equilibrium in Proposition 3.

**Lemma 2.** In an open organization, for  $0 \le \ell/n \le 1$  and given F(g), F(b) and F(u):

$$(3.5a) \hspace{1cm} \text{If } \frac{\ell}{n} < F(g), \text{ then } \tau_i(t_i) = L, \hspace{0.3cm} \forall i, \hspace{0.3cm} \forall t_i;$$

$$(3.5b) \hspace{1cm} \text{if } F(g) \leq \frac{\ell}{n} < F(b), \text{ then } \tau_i(g) = A, \ \forall i, \text{ and } \tau_i(t_i) = L, \ \ \forall i, \ \forall t_i \in \{b,u\};$$

$$(3.5c) \hspace{1cm} \text{if } F(b) \leq \frac{\ell}{n} < F(u), \text{ then } \tau_i(t_i) = A, \ \forall i, \ \forall t_i \in \{g,b\}, \text{ and } \tau_i(u) = L, \ \ \forall i;$$

$$(3.5d) \qquad \qquad \text{if } F(u) \leq \frac{\ell}{n}, \text{ then } \tau_i(t_i) = A, \ \forall i, \ \forall t_i.$$

**Proof.** See Appendix.

Recall that  $k(L|t_i) = 0$ ,  $\forall t_i$ , and r(A) = 0.

The behavior of agents can be summarized in the following table.

		$\ell/n < F(g)$	$F(g) \leq \ell/n < F(b)$	$F(b) \leq \ell/n < F(u)$	$F(u) \leq \ell/n$
	g	L	A	A	A
t <sub>i</sub>	b	L	L	A	A
	u	L	L	L	A

**Table 1**. Behavioral type chosen by agent in an open organization for various  $\ell/n$  values.

Lemma 2 shows that agent behavior is determined by whether the  $F(\cdot)$  value for any given type is larger or equal to the sorting frequency ( $\ell/n$ ). As conditions (A3) and (A4) in the appendix indicate, a relatively high  $F(\cdot)$  value makes agents misbehave, and a relatively low  $F(\cdot)$  value makes agents behave. In general, behavior improves as the sorting frequency increases relative to given  $F(\cdot)$  values. At low sorting frequencies all agent types behave as laggards; clearly if even the good intrinsic types prefer to behave as laggards, then all will behave as laggards. As the sorting frequency is increased the good types are the first to switch to achiever, the bads next and finally the uglies. This is so because the ugly agents face the highest cost of behaving as achievers, the bad types face an intermediate cost and the good types face the lowest cost. Clearly, then, if even the ugly intrinsic types prefer to behave as achievers, then all agent types will behave as achievers.

Proposition 3 characterizes the equilibrium of the extensive game played between the open organization and the agents. The proposition highlights the importance of the sorting cost z relative to the expected penalty recouped from the sorted agents. It also emphasizes the significance of the  $F(t_i)$  values which, again, reflect the marginal benefit of behaving as L rather than as A, relative to the penalty imposed if sorted.

**Proposition 3.** Given Lemmas 1 and 2, in an open organization and in the PBE of the extensive-form game, the following cases are possible:

### Case (i): $z \ge r(L)$

Agent behavior:  $\tau_i(t_i) = L, \forall i, \forall t_i$ 

Outcome observed: X = nx(L)Assessments:  $\Phi(L|X) = 1$  Expected penalty: E(r) = 0

Number of agents sorted:  $\ell(X) = 0$ 

Case (ii): F(g) > 1 and z < r(L)

Agent behavior:  $\tau_i(t_i) = L, \forall i, \forall t_i$ 

Outcome observed: X = nx(L)

Assessments:  $\Phi(L|X) = 1$ 

Expected penalty: E(r) = r(L)

Number of agents sorted:  $\ell(X) = n$ 

Case (iii):  $F(g) \le 1 < F(b)$ 

Agent behavior:  $\tau_i(g) = A, \ \forall i, \ \text{and} \ \ \tau_i(t_i) = L, \ \forall i, \ \forall t_i \in \{b,u\}$ 

Outcome observed:  $nx(L) < X = \phi(A \mid X)x(A) + \phi(L \mid X)x(L) < nx(A)$ 

Assessments:  $\Phi(A|X) = n(g)/n, \ \Phi(L|X) = [n(b)+n(u)]/n$ 

 $Expected \, penalty: \qquad E(r) = \begin{cases} \Phi(L \, | \, X) r(L), \, \, \text{if} \, \, z < [[n(b) + n(u)]/n] r(L) \\ 0, \, \, \text{if} \, \, [[n(b) + n(u) + 1]/n] r(L) > z \, \geq \, [[n(b) + n(u)]/n] r(L) \end{cases}$ 

 $\text{Number of agents sorted:} \qquad \text{ $(X)$= } \begin{cases} n, & \text{if } z < [[n(b) + n(u)]/n]r(L) \\ 0, & \text{if } [[n(b) + n(u) + 1]/n]r(L) > z \ge [[n(b) + n(u)]/n]r(L) \end{cases}$ 

Case (iv):  $F(b) \le 1 < F(u)$ 

Agent behavior:  $\tau_i(t_i) = A, \ \forall i, \ \forall t_i \in \{g,b\} \ \text{and} \ \ \tau_i(u) = L, \ \forall i$ 

Outcome observed:  $nx(L) < X = \phi(A \big| X) x(A) + \phi(L \big| X) x(L) < nx(A)$ 

Assessments:  $\Phi(A|X) = [n(g)+n(b)]/n, \Phi(L|X) = n(u)/n$ 

Expected penalty:  $E(r) = \begin{cases} \Phi(L \mid X) r(L), \text{ if } z < [n(u)/n] r(L) \\ 0, \text{ if } [[n(u)+1]/n] r(L) > z \geq [n(u)/n] r(L) \end{cases}$ 

 $\mbox{Number of agents sorted:} \qquad \ell(X) = \left\{ \begin{array}{l} n, \ \ \mbox{if} \ \ z < [n(u)/n]r(L) \\ 0, \ \ \mbox{if} \ \ [[n(u)+1]/n]r(L) > z \ \ge \ [n(u)/n]r(L) \end{array} \right.$ 

Case (v):  $F(u) \le 1$  and z < r(L)/n

Agent behavior:  $\tau_i(t_i) = A, \ \forall i, \ \forall t_i$ 

Outcome observed: X = nx(A)

Assessments:  $\Phi(A|X) = 1$ 

Expected penalty: E(r) = 0

Number of agents sorted:  $\ell(X) = 0$ 

## **Proof.** See Appendix.

The rationale behind Proposition 3 is as follows. In deciding whether to sort agents or not, after observing the aggregate outcome X, the organization compares the sorting cost z to the penalties it expects to collect, E(r). Since z is a constant, the organization will sort either all agents or none depending on the relative value of z. 11 When the sorting cost exceeds r(L), the organization will never sort any agents, and all agents will behave as laggards. This is case (i). By contrast, in all remaining cases, when equilibrium E(r) exceeds z, the organization will sort all agents in equilibrium so that  $\ell/n$ equals 1. Then agent behavior in equilibrium depends on whether the  $F(\cdot)$  value is larger or smaller than 1 for any given type. Agent behavior in equilibrium is consistent with Lemma 2 which is summarized in Table 1. When  $F(\cdot)$  is larger than 1, the marginal benefit of behaving as L rather than as A exceeds the marginal cost if sorted, and agents will prefer to behave as L rather than A. The opposite is true when  $F(\cdot)$  is smaller than or equal to 1. If even the good agents have strong incentives to behave as laggards rather than as achievers when all agents are sorted, then all agents will behave as laggards. This is case (ii) and corresponds to the first column of results in Table 1. Clearly the other cases occur when bad or ugly agent types have inherent incentives to misbehave, but good types do not. In cases (iii) - (v), when z is smaller than the penalty to be recouped from any unilateral deviation, but z exceeds the E(r) that would be recouped if all agents were sorted, then the organization will not sort any agents in equilibrium,  $\ell(\cdot) = 0$ , and the agents will not deviate because they would be sorted if they did. Note that in cases (iii) and (iv) the agents behave the same way regardless of whether all or none of them are sorted. This is because those agents who behave as laggards when all agents are sorted will also behave as laggards when none are sorted (i.e.,  $0 < 1 < F(\cdot)$ ). And those agents who behave as achievers when all agents are sorted, and would behave as laggards if no agents were sorted (i.e., types for which 0 <  $F(\cdot) \le 1$ ), will also behave as achievers when none are sorted because if they deviated they would be sorted.

<sup>&</sup>lt;sup>11</sup> The analysis below will demonstrate that our results are quite general and apply to cases where z is not constant as well (see the extensions in section 5).

Note that case (v) fits this explanation because 0 < z < r(L)/n, and 0 represents the E(r) that would be recouped if all agents were sorted.

Proposition 3 characterized the cases in which an equilibrium exists. Naturally, there are other cases in which no equilibrium exists as shown below in the Corollary to Proposition 3.

Corollary to Proposition 3. If z < r(L), and  $z \ge E(r)$  in both the putative equilibrium and any unilateral deviations, then the putative equilibrium is not an equilibrium.

### **Proof.** See Appendix.

The intuition is that if in the putative equilibrium  $E(r) \le z < r(L)$ , then the organization would not sort any agents and all agent types would behave as laggards, which contradicts the condition above because E(r) would equal r(L) in the putative equilibrium.<sup>13</sup>

# 3B. Equilibrium in a Closed Organization

Recall that the closed organization we consider screens all agents, rejecting those who are found to be ugly so that only n good and bad types are let in. Therefore, the analysis of a closed organization is similar to that of an open organization, modified to eliminate u as an acceptable intrinsic type. Lemma 4 below characterizes agent behavior for given sorting frequencies,  $\ell/n$ , in relation to the  $F(\cdot)$  values.

**Lemma 4.** In a closed organization, for  $0 \le \ell/n \le 1$  and given F(g) and F(b):

$$(3.6a) \hspace{1cm} If \frac{\ell}{n} < F(g), \text{ then } \tau_i(t_i) = L, \hspace{0.3cm} \forall i, \hspace{0.3cm} \forall t_i;$$

$$(3.6b) \hspace{1cm} \text{if } F(g) \leq \frac{\ell}{n} < F(b), \text{ then } \tau_i(g) = A, \ \forall i, \text{ and } \tau_i(b) = L, \ \ \forall i;$$

$$(3.6c) \hspace{1cm} \text{if } F(b) \leq \frac{\ell}{n}, \text{ then } \tau_i(t_i) = A, \ \forall i, \ \forall t_i.$$

**Proof.** The proof is analogous to the proof of Lemma 2 in the Appendix.

Note that the case in which z exceeds E(r), so that the organization does no sorting and all agents behave as laggards, does not fit the condition above because  $z \ge E(r) = r(L)$ . This is equilibrium case (i) in Proposition 3.

The intuition behind the results is similar to the intuition behind the results in Lemma 2. The behavior of agents can be summarized in the following table.

	,	$\ell/n < F(g)$	$F(g) \leq \ell/n < F(b)$	$F(b) \leq \ell/n$
$t_{i}$	g	L	A	A
	b	L	L	A

**Table 2**. Behavioral type chosen by agent in a closed organization for various ℓ/n values.

Proposition 5 characterizes the equilibrium of the extensive game played between the closed organization and the agents. Similar to Proposition 3, Proposition 5 highlights the importance of the sorting cost z relative to the expected penalty recouped from the sorted agents, and the significance of the marginal benefit of behaving as L rather than as A, relative to the penalty imposed if sorted (i.e., the significance of the  $F(\cdot)$  values).

**Proposition 5.** Given Lemmas 1 and 4, in a closed organization and in the PBE of the extensive-form game, the following cases are possible:

## Case (i): $z \ge r(L)$

Agent behavior:  $\tau_i(t_i) = L, \forall i, \forall t_i$ 

Outcome observed: X = nx(L)

Assessments:  $\Phi(L|X) = 1$ 

Expected penalty: E(r) = 0

Number of agents sorted:  $\ell(X) = 0$ 

# Case (ii): F(g) > 1 and z < r(L)

Agent behavior:  $\tau_i(t_i) = L, \forall i, \forall t_i$ 

Outcome observed: X = nx(L)

Assessments:  $\Phi(L|X) = 1$ 

Expected penalty: E(r) = r(L)

Number of agents sorted:  $\ell(X) = n$ 

**Case (iii):**  $F(g) \le 1 < F(b)$ 

Agent behavior:  $\tau_i(g) = A, \forall i, \text{ and } \tau_i(b) = L, \forall i.$ 

Outcome observed:  $nx(L) < X = \phi(A|X)x(A) + \phi(L|X)x(L) < nx(A)$ 

Assessments:  $\Phi(A|X) = n(g)/n, \ \Phi(L|X) = n(b)/n$ 

 $Expected \ penalty: \qquad E(r) = \begin{cases} \Phi(L|X)r(L), \ if \ z < [n(b)/n]r(L) \\ 0, \ if \ [[n(b)+1]/n]r(L) > z \ \ge \ [n(b)/n]r(L) \end{cases}$ 

 $\text{Number of agents sorted:} \qquad \ell(X) = \begin{cases} n, & \text{if } z < [n(b)/n]r(L) \\ 0, & \text{if } [[n(b)+1]/n]r(L) > z \geq [n(b)/n]r(L) \end{cases}$ 

Case (iv):  $F(b) \le 1$  and z < r(L)/n

Agent behavior:  $\tau_i(t_i) = A, \ \forall i, \ \forall t_i$ 

Outcome observed: X = nx(A)

Assessments:  $\Phi(A|X) = 1$ 

Expected penalty: E(r) = 0

Number of agents sorted:  $\ell(X) = 0$ 

**Proof.** The proof is similar to the proof of Proposition 3 in the Appendix.

Similar to the open organization, there are certain cases in which no PBE exists for the closed organization, as the corollary below demonstrates.

Corollary to Proposition 5. If z < r(L), and  $z \ge E(r)$  in both the putative equilibrium and any unilateral deviations, then the putative equilibrium is not an equilibrium.

**Proof.** The proof is identical to the proof of the Corollary to Proposition 3.

# 4. Open versus Closed Organization

In this section we focus on the efficiency of open versus closed organizations from the organization's perspective, which has implications for the choice of organizational type faced by a would-be organization drawing agents from a given pool, and from a social perspective. Proposition 6 characterizes the relative efficiency in organizations.

**Proposition 6.** Open organizations are more efficient than closed organizations from the organization's perspective and socially in the following cases:

$$(4.1a) z \ge r(L),$$

(4.1b) 
$$z < r(L) \text{ and } 1 < F(g),$$

$$(4.1c) z < r(L)/n \text{ and } F(u) \le 1.$$

In these cases, all agent types adopt the same behavioral type in both organizations. Open organizations may or may not be more efficient than closed organizations from the organization's perspective and socially when:

$$(4.2) F(g) \le 1 < F(u)$$

and z is either smaller than equilibrium E(r) or larger than equilibrium E(r) but smaller than E(r) from any deviations. In this case, agent types do not all adopt the same behavioral type.

**Proof.** When the sorting cost per agent, z, exceeds r(L), all agents behave as L in both types of organization, and the organization sorts no agents ( $\ell = 0$ ). This is Case (i) in Propositions 3 and 5. Thus open organizations are clearly more efficient from the organization's perspective and socially when condition (4.1a) holds.

In the remaining cases, when z is smaller than the equilibrium E(r), the organization sorts all agents in equilibrium ( $\ell/n=1$ ). When z exceeds the equilibrium E(r), but it is smaller than the E(r) in any unilateral deviations, then the organization will not sort any agents in equilibrium ( $\ell=0$ ). Regardless of  $\ell/n$ , agent behavior in equilibrium is summarized in the following tables, which draw from Propositions 3 and 5.

		1 < F(g) Case (ii)	$F(g) \le 1 < F(b)$ Case (iii)	$F(b) \le 1 < F(u)$ $Case (iv)$	F(u) ≤ 1 Case (v)
$\mathbf{t}_{\mathrm{i}}$	g	L	A	A	A
	b	L	L	A	A
	u	L	L	L	A

**Table 3.** Equilibrium behavior chosen by agent in an open organization.

	·	1 < F(g) Case (ii)	$F(g) \le 1 < F(b)$ Case (iii)	F(b) ≤ 1 Case (iv)
t <sub>i</sub>	g	L	A	A
	b	L	L	A

**Table 4.** Equilibrium behavior chosen by agent in a closed organization.

If 1 < F(g), then the marginal benefit to agents of intrinsic type  $\,g\,$  of behaving as L rather than as A exceeds the marginal cost if sorted. If even the good types have incentives to misbehave, then all types will have incentives to misbehave. Hence all agent types behave as L regardless of the organization type. This is Case (ii) in Propositions 3 and 5. Hence open organizations are more efficient than closed organizations when condition (4.1b) holds.

If  $F(u) \le 1$  (which is Case (v) in Proposition 3 and (iv) in Proposition 5), then all agent types will behave as achievers in both organization types, and the open organization is clearly more efficient when condition (4.1c) holds.

If  $F(g) \le 1 < F(b)$ , then the g types have incentives to behave as A and the other types have incentives to misbehave in both types of organizations. This is Case (iii) in Propositions 3 and 5. Conditions (2.8) (which state that the probabilities of being good or bad in a closed organization are larger than the corresponding probabilities in an open organization) imply that more agents behave as achievers in a closed organization than in an open organization. Therefore, the closed organization is expected to make a larger outcome, X, and to recoup less in penalties, E(r), from any sorted agents. But

agents incur greater behavioral adjustment costs,  $k(\cdot|\cdot)$ , because some of the ugly types (who would have behaved as laggards and would have faced a cost of k(L|u)=0) are replaced by good types who behave as achievers and therefore incur adjustment costs k(A|g)>0 that the ugly types would not have incurred. If  $F(b) \le 1 < F(u)$ , then agents of type u (if present) will misbehave while all other agents will behave as achievers. This is case (iv) in Propositions 3 and 5. Because all agent types in a closed organization behave as achievers, the closed organization is expected to make a larger outcome and to recoup less in penalties from any sorted agents. But agents (similar to the previous case) incur greater behavioral adjustment costs because some of the ugly types (who would have behaved as laggards) are replaced by good types who behave as achievers and therefore incur adjustment costs that the ugly types would not have incurred. Thus open organizations may or may not be efficient from the organizations perspective and socially when condition (4.2) holds.

The intuition behind the results in Proposition 6 is that when all agents behave as the same type in both organizations, the closed organization is wasting resources by screening agents because there is no gain in agent performance. This happens when sorting costs are prohibitively high or when all agent types behave as laggards or as achievers. By contrast, when sorting costs are not prohibitively high and agent behavior is a mixture of types, closed organizations may be more efficient than open because they lead to better agent behavior which results is higher output. However, closed organizations may be less efficient because they require screening costs, yield lower expectations of penalties to be recouped and cause agents to incur higher behavioral adjustment costs.

Proposition 6 implies the following, arguably testable, implications regarding the choice of organizational type by any institutional entity.<sup>16</sup> These implications presume that an equilibrium exists

Note that the rest of the ugly types are replaced by bads who behave as laggards and, similar to the uglies, face zero adjustment costs k(L|b).

Note that when (4.2) holds, given Propositions 3 and 5, specifically cases (iii) and (iv) in Proposition 3 and case (iii) in Proposition 5, then if  $\ell = n$  in closed then  $\ell = n$  in open, and when  $\ell = 0$  in open,  $\ell = 0$  in closed. Thus it is possible that the sorting frequencies differ between the two organization types when (4.2) holds. However, when we interpret the results obtained above to make policy implications, for ease of exposition, we ignore this possibility.

<sup>&</sup>lt;sup>16</sup> Since we are looking at these implications from the organization's point of view, we do not take agents' behavioral adjustment costs into consideration when making these testable implications.

for both organization types.<sup>17</sup>

- (a) Organizations facing sufficiently high screening costs (because the screening cost per agent is high) will choose the open type. First, if all agent types choose the same behavioral type regardless of organization, the open organization is more efficient regardless of screening cost. Second, if different agent types choose different behavioral types, open organizations are more efficient provided that screening costs are sufficiently high.
- (b) Organizations facing sufficiently high sorting costs relative to penalties expected to be recouped for agent misbehavior will choose the open type, because all agents will behave as laggards regardless of the organization type.
- (c) Organizations facing a sufficiently high frequency of good types in the population will choose the open type. First, if all agent types choose the same behavioral type regardless of organization type, the open organization is more efficient regardless of the frequency of good or ugly types. Second, if different agent types choose different behavioral types, and the frequency of good types is sufficiently high, then the benefits of a closed organization are outweighed by the costs because the increase in total output is low, but the organization faces screening costs and lower expected penalties.
- (d) Organizations facing a sufficiently high outcome from agents who behave as laggards relative to the outcome received from agents who behave as achievers will choose the open organization type. The rationale is the same as in (c) above.
- (e) Organizations that can recoup sufficiently high penalties for laggard behavior and face sufficiently low sorting costs (so that agents do not behave uniformly as laggards) will choose the open type. First, if all agent types choose the same behavioral type regardless of organization type, the open type is more efficient regardless of penalties. If different agent types choose different behavioral types, open organizations are more efficient, provided the penalties are sufficiently high.

One word of caution is in order. In the preceding analysis, we assume that organizations of all

<sup>&</sup>lt;sup>17</sup> It is possible that both organization types face the same sorting cost per agent, and this cost exceeds the penalty expected to be received per agent in one organization type but not in the other type. This can occur only when agent behavior differs across agent types, in which case the expected penalty is always larger for the open organization. Hence, if the sorting cost exceeds the expected penalty for one organization type only, it must be the closed type. The Corollary to Proposition 5 demonstrates that there is no equilibrium for the closed organization in this case, and we cannot make comparisons when this occurs.

types draw agents from the same pool. This assumption allows us to infer that there are more good and bad types in a closed organization than in an open organization (condition (2.8)). The implication is that closed organizations expect a larger outcome and less penalties to be recouped when agent behavior is not uniform. However, if the different types of organization draw agents from different pools, then we can never tell a priori how the distributions of intrinsic types differ in the two organization types, hence we cannot compare agent behavior (and hence expected outcome and penalties) in the two organizations at all. As an example, suppose that an open organization draws agents from a pool in which there is a plethora of good types, and the closed organization draws agents from a pool in which good types are scarce. Then the likelihood of good types in a closed organization (even though it screens all agents ex ante) may be smaller than that of an open organization. One important implication of this observation for our analysis, and for any empirical research, is that we must be careful to determine whether agents are drawn from the same pool or not. If we are comparing organizational differences in, say, firms in the same industry and in the same geographical area, then it probably is a safe assumption that they are drawing agents from the same pool. However, if we are comparing US versus European or Japanese firms, they may be drawing agents from different pools, hence, comparing agent behavior may be impractical.

### 5. Extensions

The remaining analysis extends our results by relaxing some of our assumptions. Specifically, first we allow the average sorting cost to vary with the number of agents sorted; second we study the case when the organization has the power to precommit to a sorting frequency; and third we briefly consider sequential rather than simultaneous sorting.

### **5A.** Economies and Diseconomies of Scale in Sorting Costs

The first extension we consider is average sorting costs that depend on the number of agents sorted. Even though we focused on the case of a constant average cost of sorting, z, our analysis applies much more generally. A constant average cost of sorting does imply that the organization will sort either all agents or none, but this would also be the case with economies of scale in the sorting cost. This is so because if it is worth sorting  $0 < \ell < n$  agents (i.e., if the expected benefit per agent from collecting a penalty outweighs the sorting cost per agent when  $0 < \ell < n$  agents are sorted), then it is worth sorting all n agents. In addition, if it is not worth sorting all n agents, then it is not worth sorting any  $0 < \ell < n$  agents.

We now turn to the case in which diseconomies of scale are present. For ease of exposition, we assume there is a continuum of agents, and therefore the marginal sorting cost function is continuous in  $\ell/n$ . It can then be shown that an equilibrium always exists at the  $\ell/n$  such that the marginal cost of sorting equals the marginal benefit E(r).

The rationale for this result is as follows. Lemmas 2 and 4 show that for  $\ell/n$  values in different intervals agents will choose specific behavioral types, which lead to specific outcomes X that are observed by the organization. As shown previously, once the organization observes X it can infer exactly the number of agents behaving as each type and hence expects a unique penalty per sorted agent, E(r). If agents expect the organization to choose an  $\ell$  in a particular interval determined by the  $F(\cdot)$  values, such that the marginal cost of sorting equals the E(r) in that interval, then they behave in a way such that the X observed by the organization will lead the organization to expect the same E(r) as above. The organization will then choose the same  $\ell$  that agents expected. Note, however, that E(r) is a step function; hence, if the marginal sorting cost function crosses E(r) at a point where E(r) is discontinuous, no equilibrium may exist. We demonstrate this by example in Figure 1 for the case of the closed organization (see Table 2). Let the total sorting cost be  $Z = z(\ell)\ell$ . In Figure 1, the Z' curves are different marginal sorting cost functions, and  $\Lambda(L)$  is the true frequency of agents behaving as L. E(r) can be r(L) (when all agents behave as laggards), or  $\Lambda(L)r(L)$  (when only bad types behave as laggards), or  $\Omega(L)r(L)$  (when all types behave as achievers).

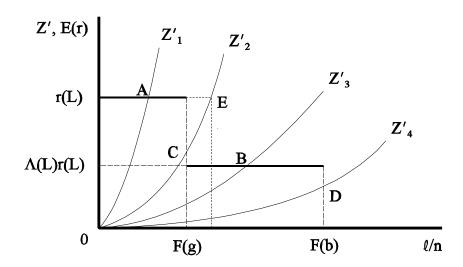


Figure 1

Clearly points A and B are equilibria in Figure 1. However, when Z' crosses the E(r) function where it is discontinuous, such as points C and D, there may be no equilibrium. For instance, at C or any point to the right of C on  $Z'_2$  agents expect  $\ell/n \ge F(g) > 0$ , and therefore the b types behave as L and the g types behave as A. But if the organization expects  $E(r) = \Lambda(L)r(L)$ , it chooses  $\ell/n = 0$  because  $Z'_2$  at F(g) exceeds E(r). Then the good types may have unilateral incentives to misbehave or not depending on the impact of their deviation on E(r) and the associated reaction of the organization. If the organization still finds it profitable to not sort any agents, then there is no equilibrium. At any point to the left of C on  $Z'_2$ , agents expect  $F(g) > \ell/n \ge 0$ , and hence all agents behave as L. But if the organization expects E(r) = r(L), it chooses the  $\ell/n$  corresponding to point E, where  $\ell/n > F(g)$ . Thus there is no equilibrium when the marginal sorting cost is  $Z'_2$ .

As argued earlier, E(r) for the closed organization is always smaller or equal to that for the open organization, at the same  $\ell/n$ . Therefore closed organizations will never sort more agents than open organizations, if both organizations sort agents in equilibrium. This is shown by points A and B in Figure 2 where the solid line represents E(r) for the closed organization and the horizontal dashed line depicts E(r) for the open organization. The organizational efficiency implications are that closed organizations bear screening costs but less sorting costs, and expect to enjoy a larger outcome but recoup less in penalties. Further, similar to the constant sorting cost case, if Z' exceeds E(r) in one organization type, it must be the closed type. Then, no equilibrium exists for the closed organization, and hence no comparison of efficiency between organization types is possible. An example is shown by points C and D in Figure 2.

<sup>&</sup>lt;sup>18</sup> It is straightforward to show that if there is no continuum of agents but, instead, the number of agents is discrete, then the equilibrium occurs at the largest  $\ell/n$  at which  $Z' \leq E(r)$ . Note that if Z' > E(r) for  $\ell = 1$ , then the organization will not sort any agents in equilibrium.

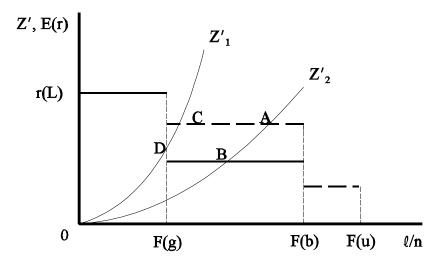


Figure 2

### 5B. Precommitment by the Organization

Next, we turn to the case when the organization has the power to precommit to a sorting frequency, assuming the cost per agent sorted is constant. The equilibrium of the game in this case is characterized by: (i) a strategy  $\ell$  for the organization determining the number of agents to be sorted; and (ii) a choice of behavioral type  $\tau_i$  by each intrinsic type  $t_i \in \{g,b,u\}$  of agent,  $\tau_i(t_i|\ell) \in \{A,L\}$ . The agents' strategies and the organization's strategy and beliefs must satisfy the following conditions. First, the organization chooses the strategy that maximizes its expected payoff shown in (5.1) for an open organization, or in (5.2) for a closed organization, given the agents' responses to the organization's choice:

(5.1) 
$$n \sum_{t_i} \left[ p(t_i) \left[ x_i(\tau_i(t_i|\ell)) + \frac{\ell}{n} \left[ -z + r(\tau_i(t_i|\ell)) \right] \right] \right],$$

$$-E(m)\,s\,+\,n \underset{t_{i}}{\sum}\,\left[\mu(t_{i})\Bigg[x_{i}(\tau_{i}(t_{i}\big|\,\ell))\,+\,\frac{\ell}{n}\Big[-\,z\,+\,r(\tau_{i}(t_{i}\big|\,\ell))\Big]\Bigg]\Bigg],$$

where E(m) = n/[p(g) + p(b)] is the expected number of agents that have to be screened by the closed organization to get n good and bad agents. This is so because the number of good and bad agents

obtained through screening follows a binomial distribution. Second, an agent's choice of behavioral type depending on his intrinsic type, and given the precommitted sorting frequency  $\ell/n$ , maximizes his expected payoff in (2.1).

Clearly, under precommitment, all equilibria in Propositions 3 and 5 where z < E(r) and  $\ell = n$  survive. This is so because if z < E(r), then  $\ell$  should equal n even with precommitment to  $\ell$ . In these cases, precommitment has no value because it does not affect the equilibrium behavior of agents compared to the no-precommitment case. However, when z > r(L), the equilibrium may differ from Case (i) in Propositions 3 and 5 in which  $\ell = 0$ . With precommitment, the organization has to trade off the net cost of sorting (sorting cost minus penalties recouped) against the benefit from better agent behavior (higher output, X) and may sort  $0 < \ell \le n$  agents in equilibrium. Since the trade-off may or may not differ across organization types, they may or may not sort the same number of agents. The rationale for this is that even though closed organizations engage in *ex ante* screening, there may still be scope for extensive sorting to encourage good behavior. Precommitting to a low sorting frequency would invite opportunistic misbehavior by the agents. It is easy to show that, in fact, open organizations may sort the same or more agents than closed organizations.

For ease of exposition, we assume again that there is a continuum of agents. We also assume the  $F(\cdot)$  values are small in the sense that 1 > F(u) for generality (if they were large, the analysis would be simpler). Clearly, to minimize the loss from sorting, the organization will always sort the minimum number of agents necessary to induce the agent behavior it wants to implement. For example, if it is optimal for both organization types to implement high achievement by all agent types, then the open organization will sort exactly F(u) agents, and the closed organization will sort exactly F(b) agents. Thus an open organization chooses the sorting frequency among 0, F(g), F(b), or F(u) that maximizes (5.1), while the closed organization chooses among 0, F(g) or F(b) to maximize (5.2). It is easy to see that precommitment now may have value in both organization types, because the equilibrium can occur at  $\ell > 0$ . For instance, if x(A) is relatively large in the sense that x(A) > x(L) + F(b)z, then  $\ell = 0$  will never be an equilibrium in either type of organization.

Finally, while no equilibrium exists in the absence of precommitment when  $E(r) \le z < r(L)$  as shown in the Corollaries to Propositions 3 and 5, there is always an equilibrium under precommitment in which the organization selects  $\ell$  to maximize (5.1) or (5.2), and the agents behave accordingly.

### **5C. Sequential Sorting**

We now briefly show how the analysis can be extended when we allow for sequential instead of

simultaneous sorting, keeping the cost per agent sorted constant. First note that if  $z \ge E(r)$ , and if an equilibrium exists, no sorting will occur in equilibrium even if sorting is sequential. Thus we focus on the case where z < E(r). We showed in the simultaneous sorting case that, if an equilibrium with sorting exists, the organization sorts all agents,  $\ell = n$ . We now characterize the optimal number of agents to be sorted when sorting is done sequentially. Specifically, we characterize the optimal stopping rule. As argued in the preceding analysis, once the organization observes X, it can infer the actual number of agents behaving as A,  $n\Lambda(A)$ , and/or as L,  $n\Lambda(L)$ . Recall that, in this case, the organization can recoup a penalty from an agent only if it sorts that agent and finds him to be a laggard. After  $\ell$  agents have been sorted and  $L(\ell)$  agents have been found to be laggards, the organization will not find it optimal to sort one more agent if

(5.3) 
$$\frac{n\Lambda(L) - L(\ell)}{n - \ell} r(L) \le z,$$

where  $[n\Lambda(L) - L(\ell)] / [n - \ell]$  is the probability that the next agent sorted is found to be a laggard.<sup>19</sup> The actual  $\ell$  may be less than in the simultaneous sorting case if the organization finds a high proportion of the laggards early in the sorting process, in which case it is not worth sorting additional agents because the probability that the next agent sorted is found to be a laggard is low.<sup>20</sup> In this setting, the agent's choice of behavioral type in equilibrium is best response to his expectation of  $\ell$ , and the organization's choice of  $\ell$  is best response to X in accord with (5.3). Comparing the equilibria in the two organization types and the associated organizational efficiency, when  $\ell$  is a random variable due to the sequential sorting, is the subject of future work. We refer the reader to the literature on stochastic games with stopping.

#### 6. Conclusions

This paper develops a model of agent behavior in two stylized types of organization, open and closed, that differ in the degree to which they scrutinize potential affiliates. An open organization does no screening of agents before they are admitted to the organization, while a closed organization screens

Note that the sequence  $L(1),L(2),...,L(\ell),...$  is a *submartingale* if  $E[L(\ell)]$  exists because with probability  $1 E[L(\ell+1)] \ge L(\ell)$ .

 $<sup>^{20}\,</sup>$  It is theoretically possible that  $\ell=n$  if n - 1 laggards were found in  $\ell$  - 1 sequential sortings. The organization then knows in advance that the last agent to be sorted will be found to be a laggard, and it does sort him when z< r(L).

all agents prior to admitting them. After observing the aggregate outcome, both organization types have the option to engage in *ex post* "auditing" of agent behavior (called sorting in our model) and penalize agents whose behavior is subpar. Agents can be of different intrinsic types (good, bad and ugly in our model) that differ in the degree to which they value misbehavior. Actual agent behavior (called achiever and laggard) depends on the short term net benefit of that behavior versus the expected penalty if caught misbehaving. The model is general enough to allow the organization to be a variety of institutions. For example, the organization could be a firm and the agents potential employees, or the organization could be a country and the agents potential immigrants, or a school or licensing authority dealing with applicants. The focus of the paper is agent behavior in these organization types and the associated efficiency implications. We believe these are important issues that has been overlooked by current theory.

One might expect *a priori* that closed organizations are more efficient than open organizations because one would anticipate better agent behavior and less equilibrium sorting, given that the closed organization screens agents and denies entry to the worst types. Surprisingly, this is not the case. Less sorting by the organization would invite opportunistic misbehavior by agents, and thus there would be a trade-off between payoffs to the organization and the costs of screening and sorting. We show that in almost all cases, and under quite general conditions regarding the sorting costs, in particular when the sorting cost per agent is constant or declining because of economies of scale, the closed organization will engage in the same amount of sorting as the open organization. Specifically, either all agents are sorted or none are sorted in equilibrium. Agents of the same intrinsic type, when they expect the same amount of sorting, will behave identically in the two organization types. When agent behavior is uniform across organizations (specifically, when all agents behave as laggards or all behave as achievers), closed organizations are inefficient because they engage in costly *ex ante* screening without any improvement in agent behavior. If agent behavior differs across organizations, then closed organizations end up with better agent behavior and thus may or may not be inefficient.

Not surprisingly, agent behavior is uniformly laggard if the organization does no sorting. However, when agents are sorted and all agent types behave uniformly, all behave as laggards because even the good intrinsic types find it profitable to misbehave. Interestingly, agents in equilibrium will never behave uniformly as achievers unless even the ugly types find it beneficial to behave as achievers and the sorting cost is very small. If the sorting cost is not very small there is no equilibrium. This is because if all agents behaved as achievers, organizations would never sort agents. But then agents would have unilateral incentives to deviate and behave as laggards, provided that the sorting cost exceeds the

expected penalty recouped from a single deviating agent. A prerequisite for this result is that the organization cannot precommit to a sorting frequency.

When we extend the analysis to the precommitment case, we find that precommitment may or may not have value. Specifically, precommitment has no value when the sorting cost is less than the penalties expected to be recouped through sorting, because the organization will still find it optimal to sort all agents. By contrast, precommitment has value when there are equilibria under precommitment while no equilibria exist under no-precommitment, and in the case in which the sorting cost exceeds the penalties expected to be recouped, because instead of sorting no agents, the organization may precommit to sorting some agents at a loss in order to induce better behavior. We conjecture that closed organizations may sort less agents in equilibrium than open organizations, in order to induce the same agent behavior.

We also extend the analysis to allow for decreasing returns to scale in sorting, or for sequential rather than simultaneous sorting. In both cases we find that the number of sorted agents can be intermediate; that is, other than all or none. Under decreasing returns to scale we also find that closed organizations will never sort more agents in equilibrium than open organizations, and when the organizations sort a different number of agents, closed organizations induce better agent behavior even though they sort fewer agents.

Even though we considered only pure strategy equilibria, note that the analysis applies more generally. First observe that the organization will not randomize when sorting agents, because after observing aggregate output it will be optimal for the organization to sort either all agents with probability one or none with probability one. Second, in the cases in which pure strategy equilibria exist, the agents will not randomize between achiever and laggard behavior given that the organization does not randomize. However, we conjecture that in cases in which pure strategy equilibria do not exist, agents could randomize between achiever and laggard behavior. Further, even though there is only one organization (open or closed) in our model, a more complex model could allow for two or more organizations with agents self-selecting the organization they want to join and organizations deciding to be open or closed. One would expect ugly types in this case to avoid closed organizations because they would be screened away, and thus we expect that the organizations will end up with different pools of agents, which will complicate the analysis considerably.

We raised two main questions in the introduction: First, how does organization type affect agent behavior and organization payoff, and second, what factors determine the choice of organizational type? We answer the first question thoroughly as summarized above. Answers to the second question rely heavily on our analysis of the efficiency of different organization types. If all agent types behave uniformly in equilibrium, then any organization will choose to be open to avoid the screening costs. If agent behavior is a mixture of types, then the choice of organization type depends on the trade-off between payoffs to the organization and the costs of screening and sorting.

Our main analysis yields the following implications. Organizations facing sufficiently high screening costs or, interestingly, sufficiently high sorting costs per agent relative to penalties recouped for agent misbehavior, will choose the open organization type. If the organization believes that the probability of good intrinsic types is sufficiently high, it will choose the open organization type again. Organizations facing a sufficiently high outcome from agents who behave as laggards relative to the outcome received from agents who behave as achievers will choose the open organization type. Organizations that can recoup sufficiently high penalties for laggard behavior will choose the open type, provided that the sorting cost is sufficiently low.

To conclude, our analysis shows that, overall, open organizations are more efficient than closed from the organization's perspective and socially when all agent types behave uniformly across organizations (i.e., when the number of agents behaving the same way is identical). However, when agent behavior is richer, either type of organization can be efficient under the right circumstances.

### **Appendix**

**Proof of Lemma 2.** The proof is straightforward by noting that expected payoffs to the agent from behaving as a laggard or as an achiever are

(A1) 
$$E(L|t_i) = v(L) - \frac{\ell}{n}r(L),$$

and

(A2) 
$$E(A|t_i) = v(A) - k(A|t_i).$$

Therefore,

(A3) 
$$\tau_i(t_i) = L \text{ if } F(t_i) > \ell/n,$$

and

$$\tau_i(t_i) = A \text{ if } F(t_i) \le \ell/n.$$
 Q.E.D.

**Proof of Proposition 3.** First note that the organization's assessments of the frequencies of agents behaving as any of the behavioral types after observing aggregate outcome X,  $\Phi(\tau_i|X) = \phi(\tau_i|X)/n$ , are calculated in accordance with the discussion following the characterization of the equilibrium at the end of section 2. Then the organization's reaction  $\ell(X)$  (as determined in Lemma 1) and the expected penalty E(r), which will be recouped from sorting, are calculated in accordance with these assessments. In the PBE of the extensive-form game, the following cases are possible:

- (i) Given  $z \ge r(L)$ , agents know the organization will never find it worthwhile to sort any agents. Given  $\ell(\cdot) = 0$  all agent types behave as laggards; that is,  $\tau_i(t_i) = L$ ,  $\forall i$ ,  $\forall t_i$ , as shown in Lemma 2.
- (ii) Given F(g) > 1, then  $F(t_i) > 1 \ \forall t_i$ ; that is, all agent types prefer to behave as laggards even if all agents are sorted,  $\ell(\cdot) = n$ , as shown in Lemma 2. Therefore, all agent types behave as laggards regardless of the sorting frequency. Given z < r(L), the organization then finds it profitable to sort  $\ell(\cdot) = n$  agents.
- (iii) If z < [[n(b)+n(u)]/n]r(L), then the organization finds it profitable to sort all agents when only the b and u types behave as laggards. When all agents are sorted,  $F(g) \le 1 < F(b)$  implies that  $\tau_i(g) = A$ ,

 $\forall i,$  and  $\tau_i(t_i) = L, \ \forall i, \forall t_i \in \{b,u\}$ , as in Lemma 2; that is, the b and u types do behave as laggards. Given this behavior, and since z < E(r), the organization will sort  $\ell(\cdot) = n$  agents indeed.

If  $z \ge [[n(b)+n(u)]/n]r(L)$ , the organization would not find it profitable to sort any agents when only the b and u types behave as laggards. However, given Lemma 2 and for 1 < F(b) < F(u), the b and u types will prefer to behave as laggards when no agents are sorted, because they would behave as laggards even if all agents were sorted. Given  $0 < F(g) \le 1$ , Lemma 2 indicates the g types would prefer to behave as laggards if no agents were sorted, and as achievers if all agents were sorted. But, because z < [[n(b)+n(u)+1]/n]r(L), the organization would find it profitable to sort all agents if even a single g type behaves as a laggard in addition to all the b and u types who behave as laggards. Therefore, the g types would not deviate from achiever to laggard if the organization sorts no agents and the b and u types behave as laggards. Thus,  $\tau_i(g) = A$ ,  $\forall i$ , and  $\tau_i(t_i) = L$ ,  $\forall i$ ,  $\forall t_{i'} \in \{b,u\}$ . Given this behavior, and since z > E(r), the organization will not sort any agents indeed,  $\ell(\cdot) = 0$ .

- (iv) The proof is analogous to case (iii). The difference is that only the u types would behave as laggards if all agents were sorted.
- (v) Given  $F(u) \le 1$ , then  $F(t_i) \le 1 \ \forall t_i$ ; that is, all agent types prefer to behave as achievers when all agents are sorted as shown in Lemma 2. Given 0 < F(g) < F(b) < F(u), all agent types would prefer to behave as laggards if no agents were sorted. However, given z < r(L)/n, the organization would sort all agents if just one agent behaved as a laggard. Knowing this, no agent will deviate from achiever to laggard. Therefore,  $\ell(\cdot) = 0$  and all agent types behave as achievers; that is,  $\tau_i(t_i) = L$ ,  $\forall i, \forall t_i$ .

Q.E.D.

**Proof of Corollary to Proposition 3.** Given any putative equilibrium in which  $E(r) \le z < r(L)$ , it follows that  $\ell(X) = 0$  for any X on the equilibrium path. Given any deviation in which  $E(r) \le z < r(L)$ , it follows that  $\ell(X) = 0$  for any X off the equilibrium path. Therefore,  $\ell(X) = 0$  for any X (equilibrium or not). But if agents expect  $\ell(\cdot) = 0$ , then  $\tau_i(t_i) = L \ \forall i, \ \forall t_i$ ; that is, all agents behave as laggards. However, the condition that  $E(r) \le z < r(L)$  rules out putative equilibria in which every agent behaves as L, because E(r) would equal r(L) in that case.  $^{21}$ 

However, recall that there can be an equilibrium in which every agent behaves as L and  $\ell(X) = 0$  if  $z \ge r(L)$ , which is case (i) in Proposition 3.

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